Thermal effects in optical systems

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Abstract. Optical systems often have to operate over a wide temperature range The effects of temperature changes on the performance of some typical optical systems are described in this paper Methods used to counteract such variations in optical properties range from servo-controlled motion of the components and bimetal mounts with reciprocating motions to, as this paper describes, a simple choice of appropriate optical and mount materials The optical systems considered range from single lenses to high quality multielement imaging systems

Keywords: optomechanical design, optical design, thermal effects; adverse environments

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CONTENTS

1. INTRODUCTION

Modern optical equipment for military or aerospace applications is expected to perform satisfactorily in a wide range of temperature environments, typical extremes being -30 C to +50 C. Even commercial photographic optics commonly experience 20 F to 90 F which may be as severe a problem since plastics are often used here for their cost advantages If we design a system for the mean temperature, then the system must substantially maintain focus and aberration correction over ±40 C. Temperature change affects an optical system in a large number of ways—temperature gradients cause strain in optical components and increase the aberrations of the system. Such effects are in general unpredictable unless simplifying assumptions are made with respect to the temperature distribution through the optical system We restrict our considerations to uniform temperature variations and to simple radial temperature gradients A uniform temperature increase causes an increase in radii, element thicknesses and airspaces, and the refractive indices of the optical elements change as does the refractive index of the optical medium which is normally air These changes are summarized in Table I.

TABLE I. Parameter Variations with Temperature

Radii	$R \rightarrow R + dR = R(1 + x_g)\,dt$
Thickness	$D \rightarrow D + dD = D(1 + x_g)\,dt$
Spaces	$L \rightarrow L + dL = L(1 + x_m)\,dt$
Index	$n \rightarrow n + dn = n + \frac{dn}{dt}\,dt$
Air	$n_{air} \rightarrow n_{air} + dn_{air} = n + \frac{dn_{air}}{dt}\,dt$

2. THERMO-OPTICAL PROPERTIES

According to Penndorf[1] the refractive index n_t of air at temperature t, at constant pressure of 760 mms Hg varies with temperature according to the relation:

$$n_t - 1 = (n_{15} - 1)\left(\frac{1.0549}{1 + 0\ 00366t}\right), \tag{1}$$

where n_{15} is the index of air at 15 C and t is in degrees Centigrade. The value of n_{15} as a function of wavelength λ is given by Edlen[2] as

$$(n_{15} - 1) \times 10^8 = 8342\ 1 + \frac{2406030}{130-\nu^2} + \frac{15996}{38.9-\nu^2}, \tag{2}$$

where $\nu = 1/\lambda$ and λ is in microns. The element radii R and thicknesses D change according to the linear expansion coefficient of the glass x_g, and the airspaces according to the expansion coefficient of the mount x_m. The temperature coefficients of refractive index are different for every material; they may be either positive or negative and indeed they vary with wavelength and with temperature. One should distinguish between the absolute values (i e., relative to vacuum) or relative values (i e , relative to air) They are related by:

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$$n_{abs} = n_{rel}\, n_{air} \tag{3}$$

$$\text{i.e.,}\ \frac{d}{dt}(n_{abs}) = \frac{d}{dt}(n_{rel}) + n_{rel}\,\frac{d}{dt}(n_{air})$$

since $n_{air} \simeq 1.0$. From Eq. (1) we have

$$\frac{d}{dt}(n_{air}) = \frac{-0.003861\,(n_{15} - 1)}{(1 + 0.00366t)^2} \tag{4}$$

which for $\lambda = 0.55\ \mu$ and $t = 15$ C, takes the value -0.964×10^{-6}

3. SINGLE LENS

Let us examine the optical effect of these changes in constructional parameters, beginning with the single thin lens. The power K (reciprocal focal length) is given by:

$$K = (n - n_{air})(C_1 - C_2)\,, \tag{5}$$

where C_1 and C_2 are the surface curvatures. Thus, differentiating with respect to temperature gives

$$\frac{dK}{dt} = \left(\frac{dn}{dt} - n\,\frac{dn_{air}}{dt}\right)(C_1 - C_2)$$

$$+ (n - n_{air})\left(-\frac{1}{R_1^2}\,\frac{dR_1}{dt} + \frac{1}{R_2^2}\,\frac{dR_2}{dt}\right) \tag{6}$$

where R_1 and R_2 are the surface radii. Now

$$\frac{1}{R_1}\,\frac{dR_1}{dt} = \frac{1}{R_2}\,\frac{dR_2}{dt} = x_g$$

the thermal expansion coefficient of the glass. Relation (6) may be simplified to

$$\frac{dK}{dt} = K\left[\frac{1}{n - n_{air}}\left(\frac{dn}{dt} - n\,\frac{dn_{air}}{dt}\right) - x_g\right]. \tag{7}$$

It is more convenient to convert this to a relationship in f the focal length as

$$x_f = \frac{1}{f}\,\frac{df}{dt} = x_g - \frac{1}{n - n_{air}}\left(\frac{dn}{dt} - n\,\frac{dn_{air}}{dt}\right). \tag{8}$$

The quantity in the left-hand side x_f can be considered as an opto-thermal expansion coefficient, since it is independent of the lens form, being a function only of the properties of the optical material. Just as the lens mount increases in length by an amount determined by x_m for a given increase in temperature, so does the focal length increase by an amount determined by x_f.

It is instructive to examine the values of the opto-thermal expansion coefficient for some optical materials. Thermal data on optical materials are often difficult to obtain, many glass catalogs do not give any at all, and Schott is the only manufacturer who quotes a reasonably comprehensive list. They give the values of dn/dt for various wavelengths and various temperatures. From these we have extracted the value of dn/dt at $\lambda = 546.1$ nm and for the temperature range +20 C to +40 C and computed the values of x_f for a selection of glasses and these are quoted in Table II. The values in the last column thus correspond to the fractional change in focal length per °C. The glasses quoted are not the most common or important glasses, but have been chosen simply to illustrate the range of values of x_f which exist. It is difficult to detect any correlation between the thermal properties and the optical properties except that aberrant optical properties often indicate extreme values of x_f, e.g., FK52, LgSK2. The values in the table are both positive and negative and so the paraxial focus of the lens may either move inwards towards the lens or away from the lens with increasing temperature depending upon the glass used. For BaK4 with a value of x_f of -0.23×10^{-6} the focus is almost constant with temperature. This, however, is no great benefit since the dimensions of the mount are not constant with temperature, i.e., to achieve thermal stability of focus we must match the thermal expansion coefficient of the mount material with the opto-thermal expansion coefficient of the optical material. The thermal expansion coefficient of typical mount materials ranges from Invar at 0.9×10^{-6} to magnesium at 26.0×10^{-6}, the most common being aluminum at 23.6×10^{-6}. Thus using aluminum as the mount, the focus moves faster than the mount for FK52, slower than the mount for BK7, and in the opposite direction for SF11. Obviously no mount material can be found to match a glass with a negative value of x_f.

4. DOUBLET LENS

In practice optical systems are often built up of doublets and we can define an opto-thermal expansion coefficient for a doublet x_f as

$$x_f = \frac{f}{f_1}\,x_{f1} + \frac{f}{f_2}\,x_{f2}\,, \tag{9}$$

where f_1 and f_2 are the focal lengths of the constituents of the doublet. For an achromatic doublet this may be written as

$$x_f = \frac{V_1\,x_{f1} - V_2\,x_{f2}}{(V_1 - V_2)}\,, \tag{10}$$

where V_1 and V_2 are the dispersions of the elements of the doublet.

Athermalization may be achieved either by choosing the mount material to match the x_f of the optical materials or by choosing the optical materials to match the x_m of the mount. Since there are more optical glasses available with a great variety of properties (especially when we can make use of relation (10)), it makes more sense to adopt the latter procedure. In the former approach, a common choice of glasses for an achromatic doublet is BK7 and SF11; then the value of x_f for the doublets is 8.51×10^{-6}, and the best match of mount material would be titanium—an expensive choice.

Choosing aluminum as the most suitable mount material, we search for a pair of glasses which have a value of x_f as close as possible to x_m when combined in an achromatic doublet. To achieve such a high value for x_f we require a large positive x_f for the crown and a large negative value for the flint, a suitable choice being K5 and LaSFN3 which gives a value of x_f of 22.3×10^{-6} for the doublet and results in a focal shift of 0.000124 mms for a 100 mm system and a temperature shift of 10 C.

Towards the end of Table II are included some commonly used infrared transmitting materials. Their values of x_f are very large and no possible mount materials exist to athermalize such systems. For example with germanium, the most commonly used IR material, the paraxial focus moves in towards the lens four times as fast as an aluminum mount moves away from the lens. It is of course possible to conceive of an athermalized mount such as a composite of aluminum and Invar, the total length of the mount being about five times the focal length of the lens. Even if it were possible to provide space for such a mount, it would be very unstable.

In practice, high resolution IR systems employ active focus compensation via a temperature sensing device and servo loop to control a focusing motor.

5. PETZVAL LENS

Well-corrected systems of large relative aperture and long focal length often employ glass types with "abnormal" refractive indices, usually the FK, PK, PSK, KzFS type glasses, and examination

TABLE II. Thermal and Opto-Thermal Properties of Selected Schott Glasses

Glass	Index	$x_g \times 10^6$	$\frac{dn}{dt} \times 10^6$	$x_f \times 10^6$
FK5	1 487	9 2	-2 20	10 77
FK6	1 446	11 2	-5 20	19 73
FK52	1 486	14 4	-7 80	27 50
PK2	1 518	6 9	1 60	0 99
PK50	1 521	8 8	-1 20	8 29
PSK2	1 569	6 4	1 50	1 11
PSK52	1 603	8 5	-1 80	8 92
BK1	1 510	7 7	0 80	3 28
BK3	1 498	5 3	3 70	-5 03
BK7	1 517	7 1	1 70	0 98
BK10	1 498	5 8	2 30	-1 72
BaLK3	1 518	8 3	1 00	3 54
K4	1 519	7 3	2 60	-0 53
K5	1 522	8 2	0 60	4 24
K10	1.501	6 5	3 00	-2 38
K51	1 505	4 3	5 40	-9 27
ZK1	1 533	7 5	2 10	0 79
ZKN7	1 508	4 5	6 10	-10 37
BaK1	1 573	7 6	1 30	2 68
BaK4	1 569	7 0	2 60	-0 23
BaK50	1 568	3 7	7 40	-11 99
SK2	1 607	6 0	3 20	-1 82
SK10	1 623	7 0	1 40	2 24
SKN18	1 639	6 4	4 30	-2 80
SK51	1 621	8 9	-2 30	10 09
KF3	1 515	8 1	2 20	0 99
KF9	1 523	6 8	3 10	-1 93
BaLF4	1 580	6 4	4 10	-3 30
BaLF50	1 589	8 3	0 20	5 36
SSK2	1 622	6 2	3 50	-1 94
SSK4	1 618	6 1	1 70	0 83
SSK51	1 604	7 6	0 60	4 05
LaK3	1 694	8 2	-1 80	8 44
LaKN7	1 652	7 1	-0 50	5 42
LaK8	1 713	5 6	3 30	-1 34
LaKN9	1 691	6 3	2 50	0 32
LaKN16	1 734	5 3	4 90	-3 65
LLF1	1 548	8 1	1 80	2 09
BaF3	1 583	7 8	2 50	0 89
BaF9	1 643	6 5	4 00	-2 18
BaF50	1 683	8 3	1 00	4 46
LF5	1 581	9 1	1 50	3 90
F1	1 626	8 7	2 30	2 52
F2	1 620	8 2	3 10	0 68
F5	1 603	8 0	3 50	-0 37
F7	1 625	9 8	0 40	6 65
BaSF1	1 626	8 5	2 30	2 32
BaSF5	1 603	7 9	8 80	-9 26
LaFN2	1 744	8 2	0 70	5 00
LaF9	1 795	7 2	8 50	-5 67
LaF20	1 682	7 4	1 10	3 41
LaSFN3	1 808	5 9	6 50	-4 30
LaSFN9	1 850	7 6	3 10	1 85
SF1	1 717	8 1	6 40	-3 13
SF2	1 648	8 4	3 50	0 55
SF11	1 785	6 1	11 40	-10 61
SF16	1 646	8 4	1 10	4 24
SF59	1 953	9 4	13 60	-6 85
TiK1	1 479	10 3	-3 50	14 63
TiF4	1 584	8 9	-0 80	7 66
TiF6	1 617	13 9	-5 90	20 94
KzF1	1 551	6 9	2 70	-0 71
KzF4	1 570	7 3	1 70	1 66
KzFS1	1 558	5 0	2 90	-2 89
KzFSN4	1 613	4 5	4 10	-4 73
LgSK2	1 586	12 1	-4 30	16 83
Si	3 420	4 2	162 00	-64 10
Ge	4 000	6 1	270 00	-85 19
ZnSe	2 400	7 7	48 00	-28 24
NaCl	1 490	44 0	-25 00	92 09
KRS5	2 370	58 0	-235 00	227 87
Acrylic	1 495	67 9	-105 00	137 17
Polycarb	1 585	65 5	-107 00	132 04

of these in Table II shows that these glasses often have large opto-thermal expansion coefficients as well. Hence, in choosing glasses for such apochromatic designs, we must not only pay attention to the aberrations of the nominal design but also consider the stability of focus with temperature in order that the high degree of aberration correction achieved is not swamped by focus errors due to varying temperature. For instance, an excellent choice of glasses for wide-band spectral correction is FK51 and LaK8. Figure 1(a) shows the transverse ray aberrations for a 250 mm F/2 Petzval type lens using these glasses. The wavelengths traced are from 500 nm to 900 nm and the system shows excellent aberration correction for all wavelengths in this band. If, however, we raise the temperature by 10 C, we completely destroy the performance because of the mismatch between the opto-thermal properties of the glasses and the aluminum mount On the other hand, the glasses PSK52 and KzFS1 give a Petzval lens with inferior nominal aberration correction but much greater thermal stability as shown in Fig 1(b)

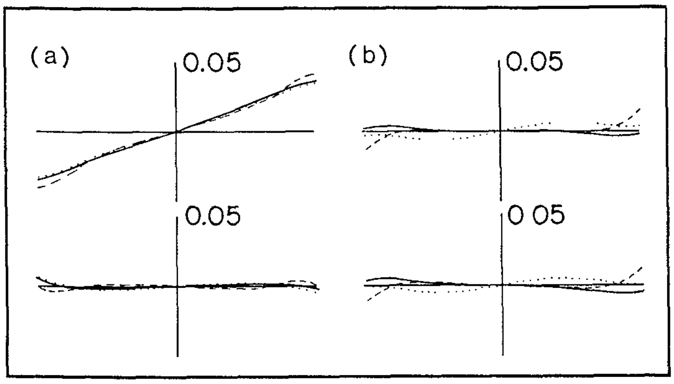

Fig 1 Transverse aberrations of Petzval lenses made of (a) FK51 and LaK8 glasses and (b) PSK52 and KzFS1 glasses at nominal temperature (lower) and with 10 C temperature rise (upper)

6. PHOTOGRAPHIC PERISCOPE

Figure 2(a) shows the aberration correction of a high quality submarine periscope used for photographic purposes. The optical system comprises an objective lens and three image relay stages, the lenses being either three or four element apochromats The glasses were not chosen for their opto-thermal stability so that the focal position varies significantly with temperature. Figure 2(b) shows the aberrations with a 10 C increase in temperature. In the case of the submarine periscope, the focal position is even more uncertain due to variations in pressure within the tube (i.e., the value of n_{air}

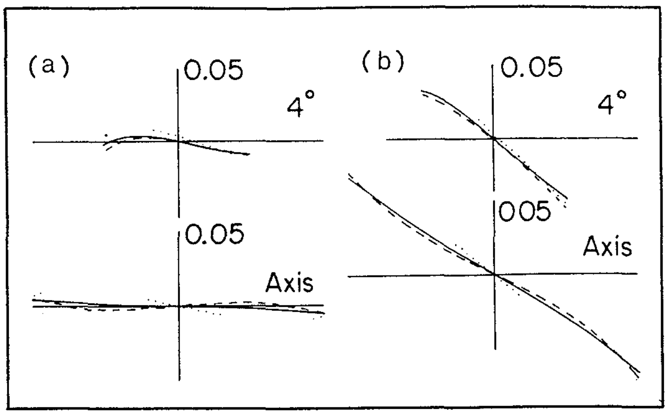

Fig 2 Transverse aberrations of typical submarine periscope optical system on axis and at 4° semifield at (a) nominal temperature and (b) with 10 C temperature rise

is pressure dependent). The solution to both effects in this case is to take a number of photographs in rapid succession, each time moving the film position through focus.

7. TRIPLET LENS

In present day, low-to-medium quality optical systems, plastics are finding the increasing application The low material costs and the ability to mold finished lenses are very attractive to the consumer oriented manufacturer. There are disadvantages, however, such as the softness of plastics, their large temperature coefficients, and the small choice of available materials. Reference to Table II shows that their values of x_f are 10 to 100 times greater than optical glasses Nevertheless, these materials are being used in inexpensive photographic systems where temperature stability of focus is a valid requirement Estelle[3] describes a series of Cooke triplets in which the powers of the elements are chosen such that in addition to the usual constraints on primary aberrations, stability of focal position with temperature was imposed. Two forms are described, either with glass in front and two plastic elements following, or two plastic elements with glass in the rear. Athermalization is achieved primarily by adjusting the relative powers of the positive and negative plastic elements, the glass element having only a small effect. Three-element solutions with acceptable spherical aberration have not been found and four elements are required; indeed an aspheric surface also is necessary.

8. LASER BEAM EXPANDER

A laser beam expander typically consists of a Galilean telescope, the purpose of which is to reduce the divergence of the raw output beam from a laser. The normal makeup is a positive lens group (the objective) K_o and a single negative lens (the eyepiece) K_e separated by the algebraic sum of their focal lengths D, the magnification of the telescope M being equal to K_e/K_o. The exit angle u of a paraxial ray, which enters the telescope parallel to the optical axis at a height h, is given by:

$$u = h(K_o + K_e - D K_o K_e) \qquad (11)$$

Thus differentiating with respect to temperature

$$\frac{du}{dt} = h\left[\frac{dK_o}{dt} + \frac{dK_e}{dt} - \frac{dD}{dt}K_o K_e - D\frac{dK_o}{dt}K_e - D K_o\frac{dK_e}{dt}\right] \qquad (12)$$

From Eq (11) we have $D = (M - 1)/MK_o$, also

$$\frac{dK_o}{dt} = -K_o x_o, \quad \frac{dK_e}{dt} = -K_e x_e = MK_o x_e, \text{ and } \frac{dD}{dt} = x_m D$$

where x_o and x_e are the opto-thermal expansion coefficients of objective and eyepiece and x_m the expansion coefficient of the mount Substituting these relations into Eq (12) gives

$$\frac{du}{dt} = -\frac{1}{2F}\left[M x_o + x_e - x_m(M-1)\right], \qquad (13)$$

where F is the F/number of the objective and eyepiece. To get some idea of the magnitude of typical thermal defocus, consider a system using the common high index flint glass SF11. Then a 10× beam expander, employing an aluminum mount, used at F/1 0 for a 10 C temperature change gives a thermal defocus of 0 164 milliradians. Typically the raw laser beam divergence might be 3 mrds which becomes 0.3 mrds on traversing the beam expander, so that the thermal defocus is 50% of the nominal divergence A better match between the values of x_o, x_e, and x_m can greatly reduce this In fact, du/dt is zero when

$$x_e = x_m(M-1) - M x_o \qquad (14)$$

and a choice of FK6 for the objective and TiK1 for the eyepiece gives a thermal defocus of 0.0122 mrd Changing the mount material to stainless steel with a value of x_m of $10\ 4 \times 10^{-6}$ and a choice of LaK3 for objective and eyepiece results in a thermal defocus of 0 0071 mrd.

9. TEMPERATURE GRADIENTS

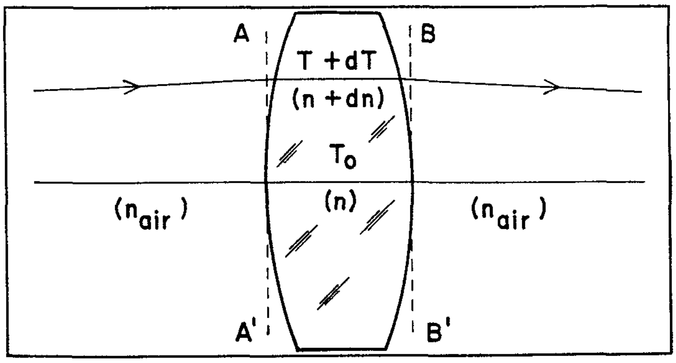

Fig 3 Illustration of a radial thermal gradient in a simple lens

Thermal gradients in optical systems normally occur in transient situations such as changing temperature environments. Their effects are very difficult to calculate but we can make some estimates for some simple cases such as a radially symmetric temperature gradient, as might occur in an optical system mounted in a cylindrical fashion when moving from a cold to a warm environment. The situation is illustrated for a single lens in Fig 3 The center thickness of the lens is T_o and the thickness at the point P where an axial ray passes through the lens is T. A radial temperature gradient is impressed upon the lens such that the center temperature and hence thickness remain the same and the thickness T increases to T + dT because of a temperature change dt. The change in optical path between the planes AA′ and BB′ as a result of the temperature gradient is given by:

$$(n - n_{air})(T_o - T) - nT_o + (n + dn)(T_o - T - dT), \qquad (15)$$

which is approximately equal to

$$T_o\left[(n - n_{air})x_g dt + dn\right] \qquad (16)$$

Note that this depends only on lens thickness (the transverse ray aberrations resulting from this optical path difference will, however, depend on lens diameter and focal length) The magnitude of this effect can be estimated if we take the case of a 10 mm thick lens of SF11 with a temperature difference of 1 C between center and edge which results in a wave aberration of 0.3 λ. This effect can be quite significant in highly corrected systems. Similarly, in a germanium lens of the same thickness, the wave aberration is 0 27 λ (λ is 10.6 μ in this case).

A more accurate analysis of radial thermal gradients can be done by specifying the temperature at a series of radial positions on each optical element, calculating the increase in thickness at each radial position, and fitting each point on the surface to an aspheric form. Similarly, a curve-fitting algorithm is used to generate the refractive index at arbitrary radial positions The index is assumed to be constant along the ray path through the lens element. The image quality of the system is then obtained by ray tracing through a series of general aspheric surfaces with radially varying refractive index This method is applied to a highly corrected triplet objective lens which is used in a high quality submarine periscope system. Figure 4(a) shows the very high degree of aberration correction of the nominal system and Fig. 4(b) shows the results of a 1 C radial temperature gradient, this temperature varying linearly from center to edge. The effect ruins the high quality image

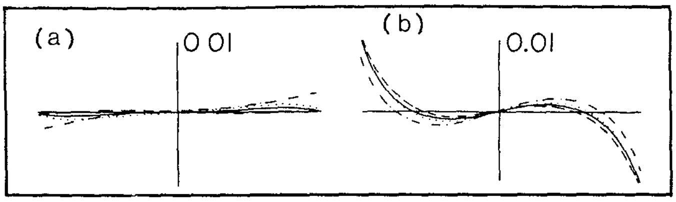

Fig 4 Transverse aberrations of a corrected triplet lens (a) at uniform temperature and (b) with 1 C radial temperature gradient

10. REFERENCES

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