First Order Optics

JORDAN JUR | OPTI 521R

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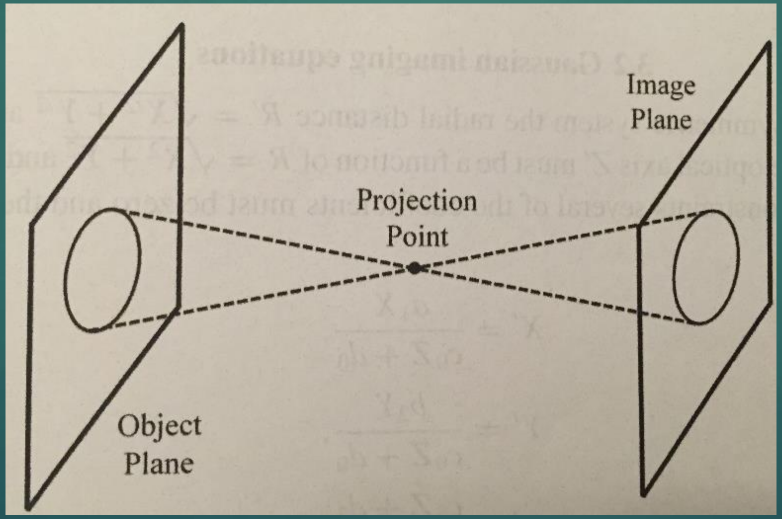
Fundamental Concepts
Central Projection

- This is the fundamental mathematical principle of ray optics.
- The central projection theorem states that an object located in the object plane which is projected through the "projection point" will form an image of the object in the image plane.

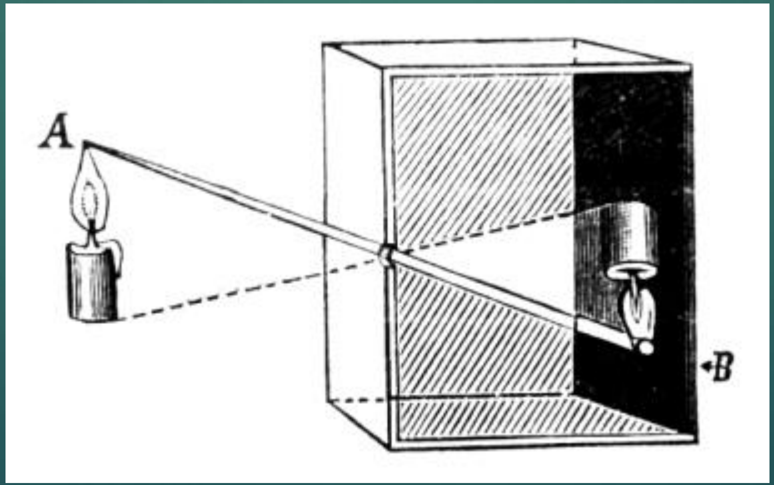


Fundamental Concepts
Collinear Transformation

- A collinear transformation is a one to one mapping between two spaces.
- In the world of optics, the spaces are referred to as object space and image space.
- The projection function of a collinear system states that points map to points, lines map to lines and spaces map to spaces.
- The points, lines and spaces in object space all have corresponding and unique points, lines and spaces in image space. The notion of these corresponding and unique elements describes the basis of conjugate elements.

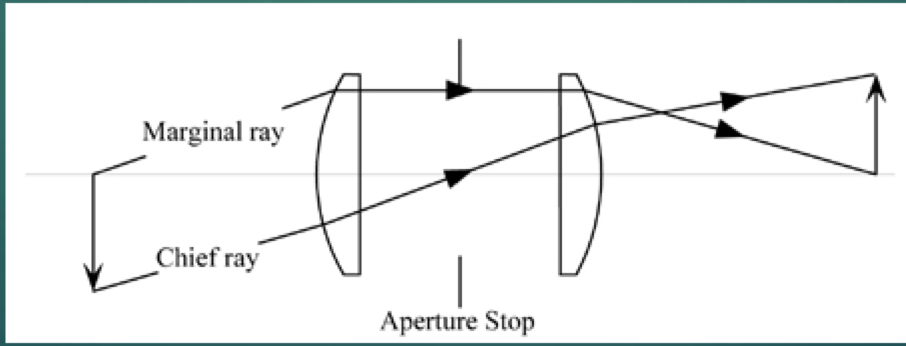
Fundamental Concepts
The Camera Obscura

- The camera obscura is the simplest form of an imaging system. Adaptation of the central projection theorem.
- It consists of a black box with a pinhole in one side. Light from an object placed outside of the box will propagate through the pinhole and form an inverted image on the wall.



Imaging Equations
Ray Definitions

- The imaging equations can be derived by tracing the chief and marginal rays of an optical system.
- The marginal ray is an on-axis ray which travels from the center of an object to the edge of the stop and to the center of the image.
- The chief ray is an off-axis ray which travels from the edge of the object through the center of the stop to the edge of the image.



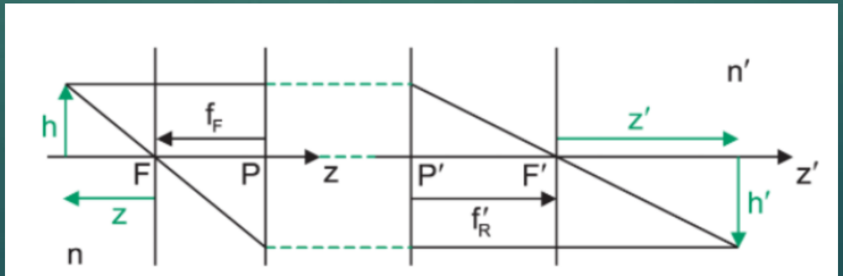
Imaging Equations Newtonian

- The imaging equations were first derived by Sir Isaac Newton in 1666 using similar triangles. In relation to the conjugate object and image planes, the Newtonian equations are referenced to the focal planes.

$$z = -\frac{f_F}{m} \qquad \frac{z}{n} = \frac{f_E}{m}$$

$$z' = -m f'_R \qquad \frac{z'}{n'} = -m f_E$$

$$z z' = f_F f'_R \qquad \left(\frac{z}{n}\right)\left(\frac{z'}{n'}\right) = -f_E^2$$



Imaging Equations Gaussian

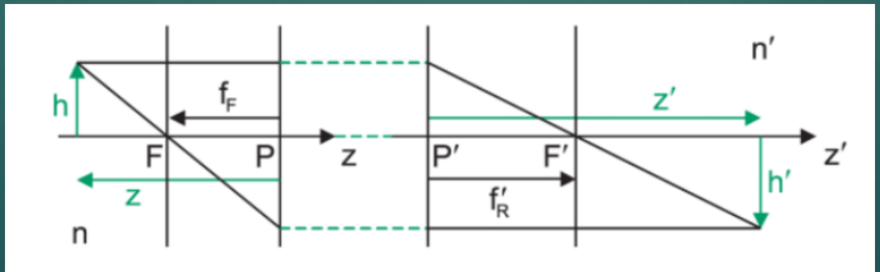
- Gauss later derived similar imaging equations where the conjugate object and image planes are referenced to the principal planes.

$$z = -\frac{(1-m)}{m}f_F \qquad \frac{z}{n} = \frac{(1-m)}{m}f_E$$

$$z' = (1-m)f'_R \qquad \frac{z'}{n'} = (1-m)f_E$$

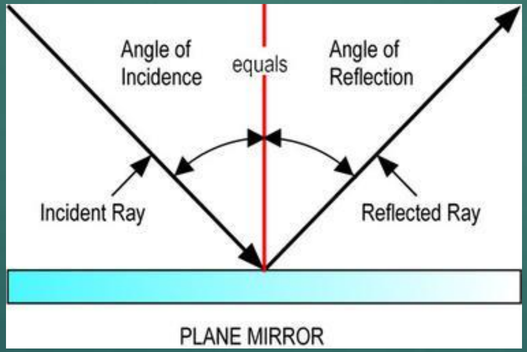
$$m = -\left(\frac{z'}{z}\right)\left(\frac{f_F}{f'_R}\right) \qquad m = \frac{z'/n'}{z/n}$$

$$\frac{f'_R}{z'} + \frac{f_F}{z} = 1 \qquad \frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E}$$



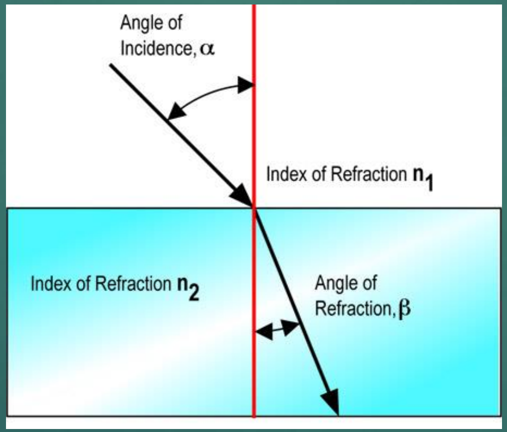
Imaging Equations
Law of Reflection

- The law of reflection states that for a ray incident upon a flat surface at an angle of a with respect to the surface normal reflect away from the flat surface at an angle of –a with respect to the surface normal.



Imaging Equations
Law of Refraction

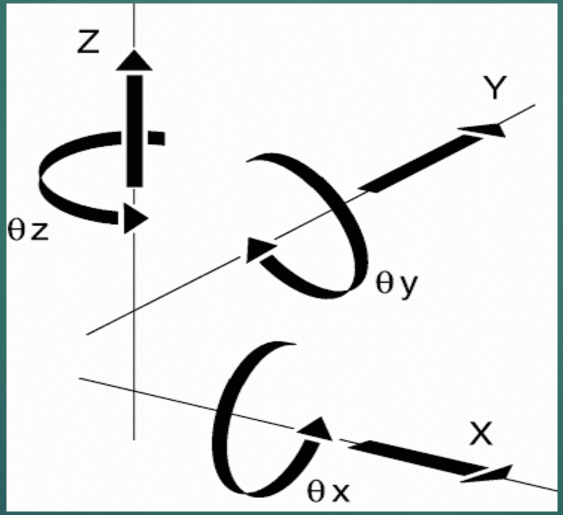
- Snell's Law states that a ray in media 1 which is incident upon a surface at an angle of a with respect to surface normal will alter it's direction into media 2 at angle β with respect to the surface normal.



$$n_1 \sin \alpha = n_2 \sin \beta$$

Object and Image Motion Degrees of Freedom

- Object and image motion is described in relation to a three dimensional Cartesian coordinate system. This coordinate system has six effective degrees of freedom: x, y & z motion and θ_X, θ_Y, & θ_Z rotation.

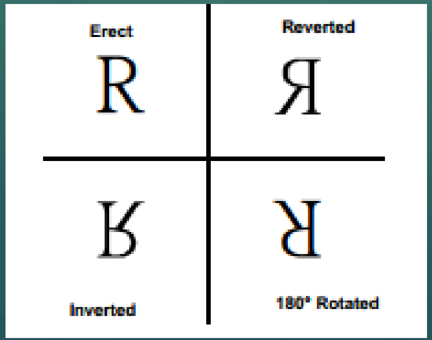


Object and Image Motion Prism Convention Definitions

- Prisms are useful tools which make it possible to compact an optical system into a smaller form factor. Five main categories of prisms: deviation, dispersion, displacement, rotation and expansion prisms.
- Image handedness refers to the number of reflections. There are two types of image handedness- right and left.

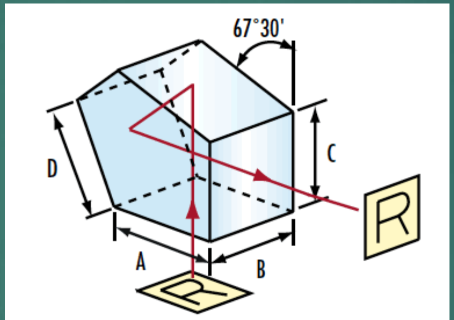
Right and Left Handedness (respectively)

- Image parity refers to an even or odd number of reflections.

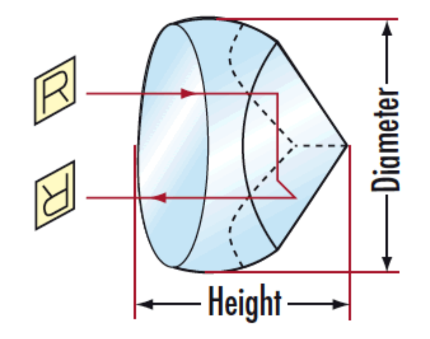


Object and Image Motion Reflective Elements: Prisms I

- The Penta prism has right handedness and even parity; it will invert an image.

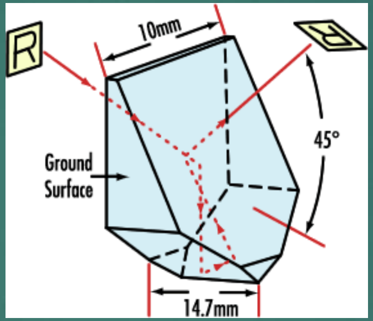


- The Porro prism has right handedness and even parity; it will invert an image.

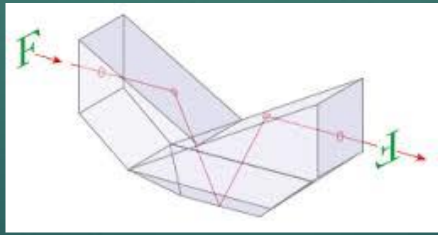


Object and Image Motion Reflective Elements: Prisms II

- The Schmidt prism has right handedness and even parity; it will rotate the image by 180°.

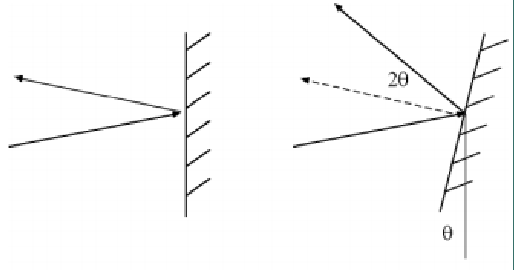


- The abbe-Koenig prism has righthandedness and even parity; it will rotate an image by 180°.

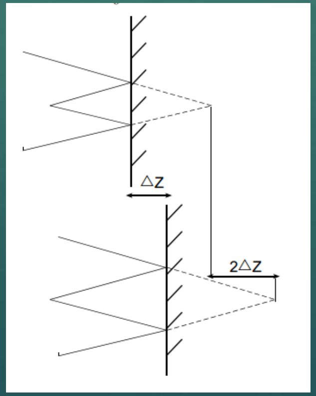


Object and Image Motion Reflective Elements: Mirrors

- The general rule of thumb for image rotation is that any rotation of a mirror will cause a image rotation of twice the mirror rotation.

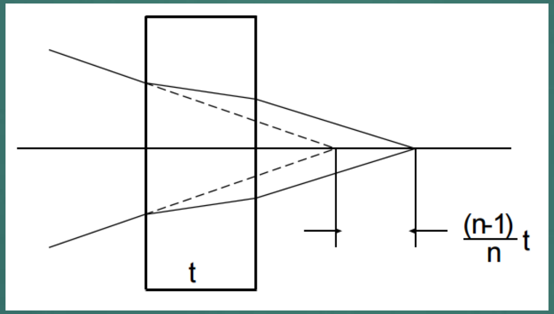


- Any translation of the mirror, which is not orthogonal to the direction of propagation, will increase the OPL by twice the mirror translation.

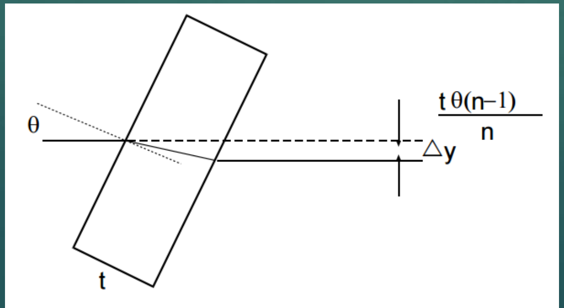


Object and Image Motion Refractive Elements: PPP I

- Plane parallel plates (PPP) can be used for a number of different applications. In this example, a PPP will be used to demonstrate the concept of the optical path difference (OPD).

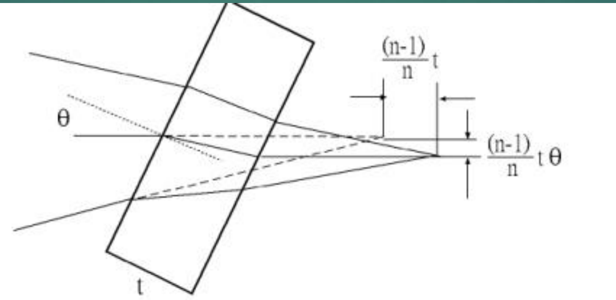


- A titled PPP in a collimated beam will cause a vertical deviation of the beam.



Object and Image Motion Refractive Elements: PPP II

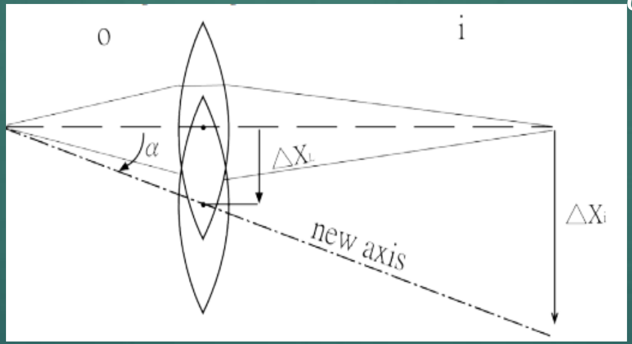
- A tilted PPP in a converging beam will cause an axial and vertical displacement which will also induce several types of fourth order aberrations within the optical system.



Aberration	Equation
Spherical	$-\frac{t(n^2-1)}{f/\#^4\,128n^3}$
Coma	$\frac{t\theta(n^2-1)}{f/\#^3\,16n^3}$
Astigmatism	$\frac{t\theta^2(n^2-1)}{f/\#^2\,8n^3}$
Transverse Color	$\frac{t\theta(n-1)}{n^2\,\nu}$
Longitudinal Color	$-\frac{t(n-1)}{n^2\,\nu}$

Object and Image Motion Refractive Elements: Lenses I

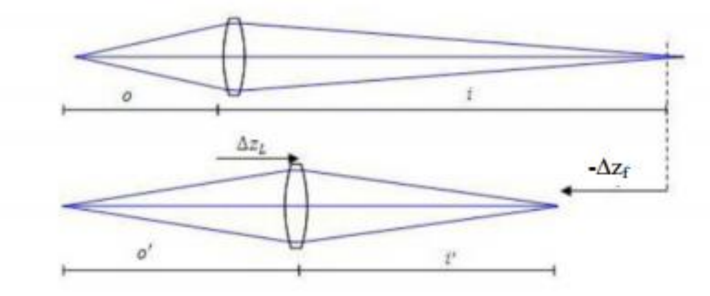
- When designing optical systems it is important to understand how the motion of a lens will affect the motion of the image.
- The lens can either tilt about the optical axis, move perpendicular to the optical axis (decenter) or move along the optical axis (axially).
- . The change in the optical axis angle is: $\alpha = \frac{\Delta X_L}{o}$



- The image motion is: $\Delta X_I = \Delta X_L \frac{(o+i)}{o} = \Delta X_L(1-m)$

Object and Image Motion
Refractive Elements: Lenses II

- Axial translation of a lens will result in an image motion along the optical axis.
- . In this example, as the lens is translated axially by a distance Δz_L the resulting image moves $-\Delta z_F$.



- The resulting image motion is: $\Delta z_F = \Delta z_L \ (1 - m^2)$