Principles of First Order Optics Applied to Components of the BioTek Absorbance Spectrophotometer Cuvette Assembly

Tutorial Presentation to the BioTek Mechanical Design Team

OPTI521

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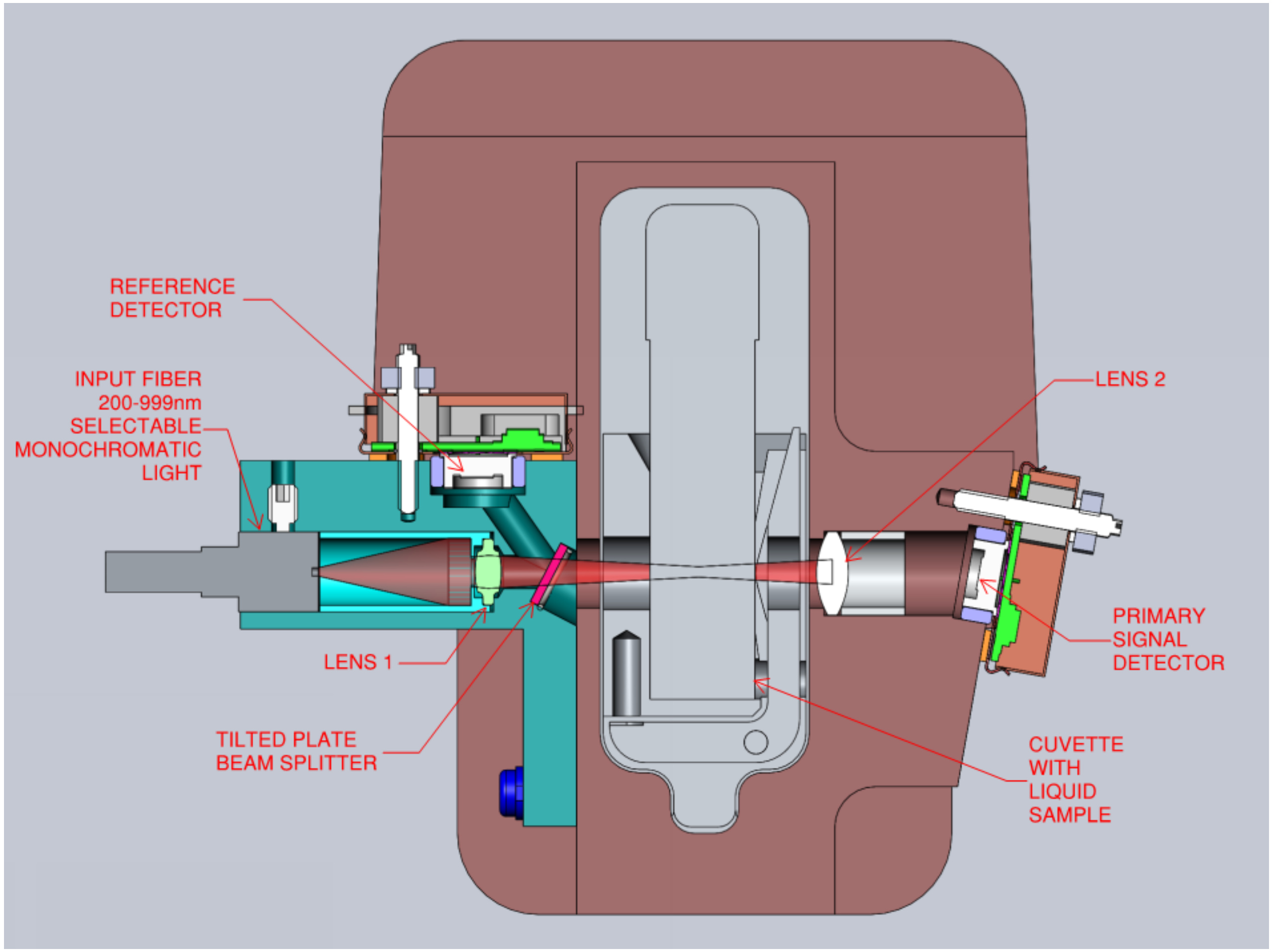
What is “First Order Optics”?

First Order Optics = Perfect Lenses (No Aberrations)

Analysis Methods:

- Gaussian Optics – Formulas referencing:
  - Principal Points
  - Focal Points
- Paraxial Optics
  - Rays near the axis (ignore curvature of lens)
  - Small angles ($\sin\theta \approx \theta$)

BioTek Cuvette Assembly



Not an imaging system. We put energy into the sample at a particular wavelength and collect what comes out, measuring absorbance. Wavelength range = 200-999nm.

UV GRADE FUSED SILICA

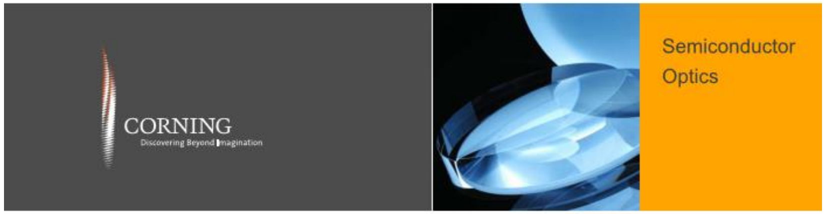
- Because of the 200 – 380 nm wavelengths, we used fused silica optics.

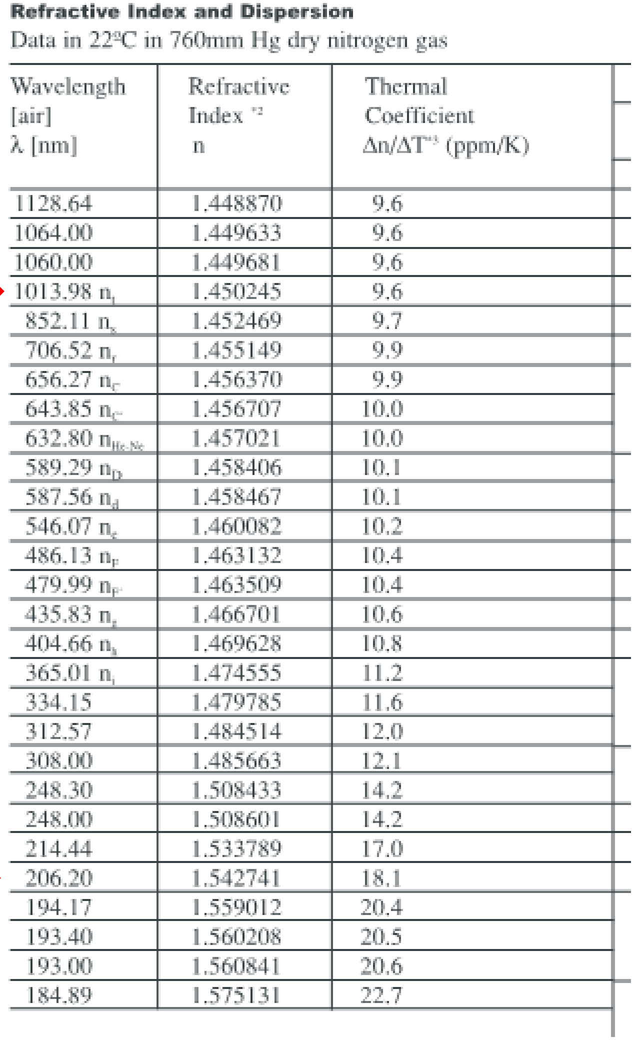
- Note the variation in Refractive Index

- Refractive Index:

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in the material}}$$

HPFS® Fused Silica Standard Grade



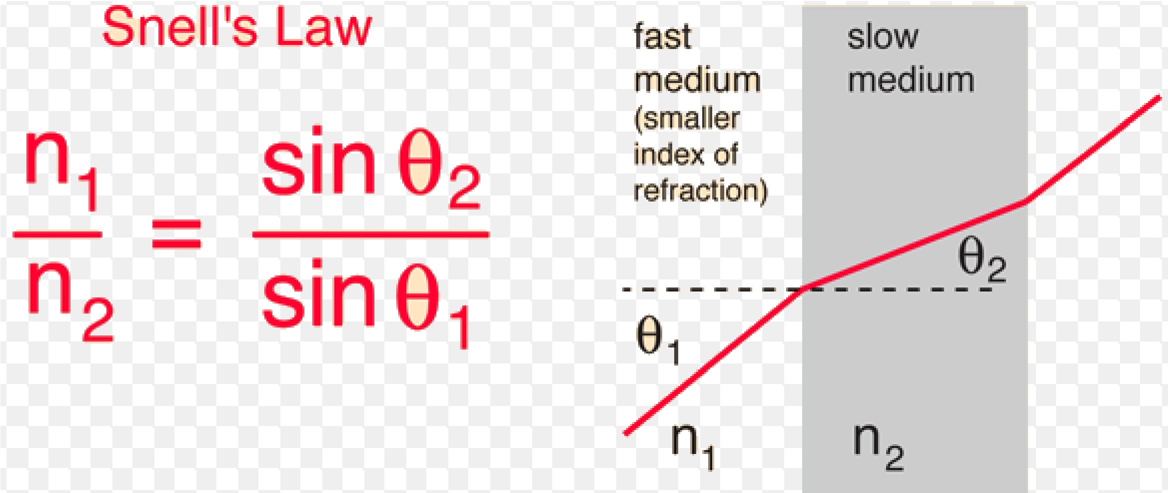


Refractive Index and Dispersion

Data in 22ºC in 760mm Hg dry nitrogen gas

Wavelength [air] λ [nm]	Refractive Index ^{*2} n	Thermal Coefficient Δn/ΔT^{*3} (ppm/K)
1128.64	1.448870	9.6
1064.00	1.449633	9.6
1060.00	1.449681	9.6
1013.98 n_t	1.450245	9.6
852.11 n_s	1.452469	9.7
706.52 n_r	1.455149	9.9
656.27 n_C	1.456370	9.9
643.85 $n_{C'}$	1.456707	10.0
632.80 $n_{He\text{-}Ne}$	1.457021	10.0
589.29 n_D	1.458406	10.1
587.56 n_d	1.458467	10.1
546.07 n_e	1.460082	10.2
486.13 n_F	1.463132	10.4
479.99 $n_{F'}$	1.463509	10.4
435.83 n_g	1.466701	10.6
404.66 n_h	1.469628	10.8
365.01 n_i	1.474555	11.2
334.15	1.479785	11.6
312.57	1.484514	12.0
308.00	1.485663	12.1
248.30	1.508433	14.2
248.00	1.508601	14.2
214.44	1.533789	17.0
206.20	1.542741	18.1
194.17	1.559012	20.4
193.40	1.560208	20.5
193.00	1.560841	20.6
184.89	1.575131	22.7

Snell's Law



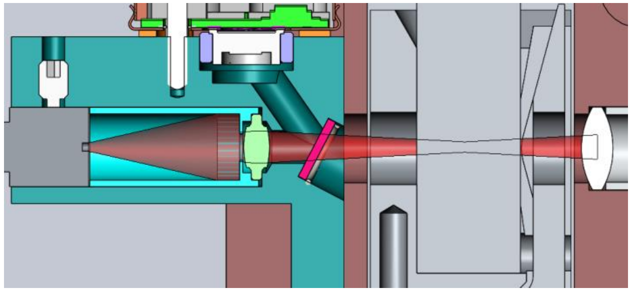
www. http://hyperphysics.phy-astr.gsu.edu/

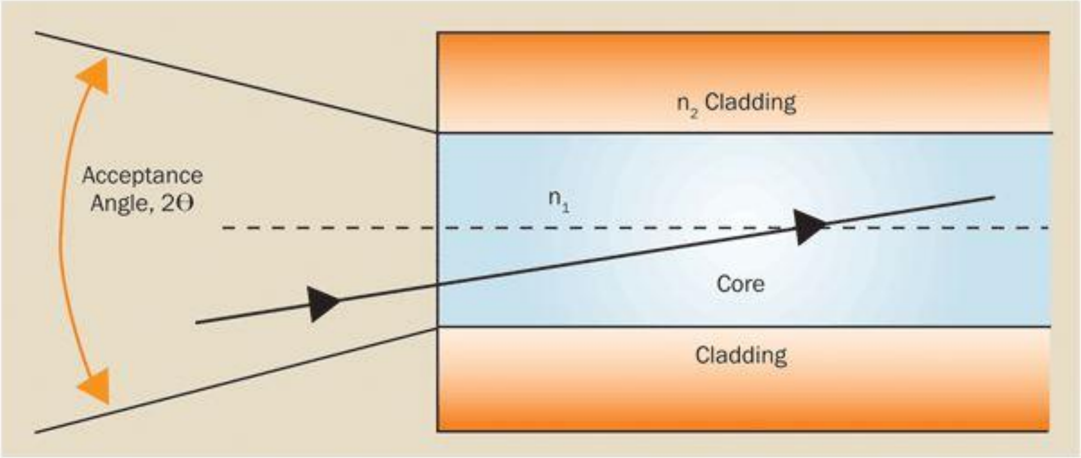
$n_{\text{air}} = 1$

$n_{\text{water}} = 1.333$

$n_{\text{fused silica}} = 1.45$

Optical Fiber Numerical Aperture (NA)

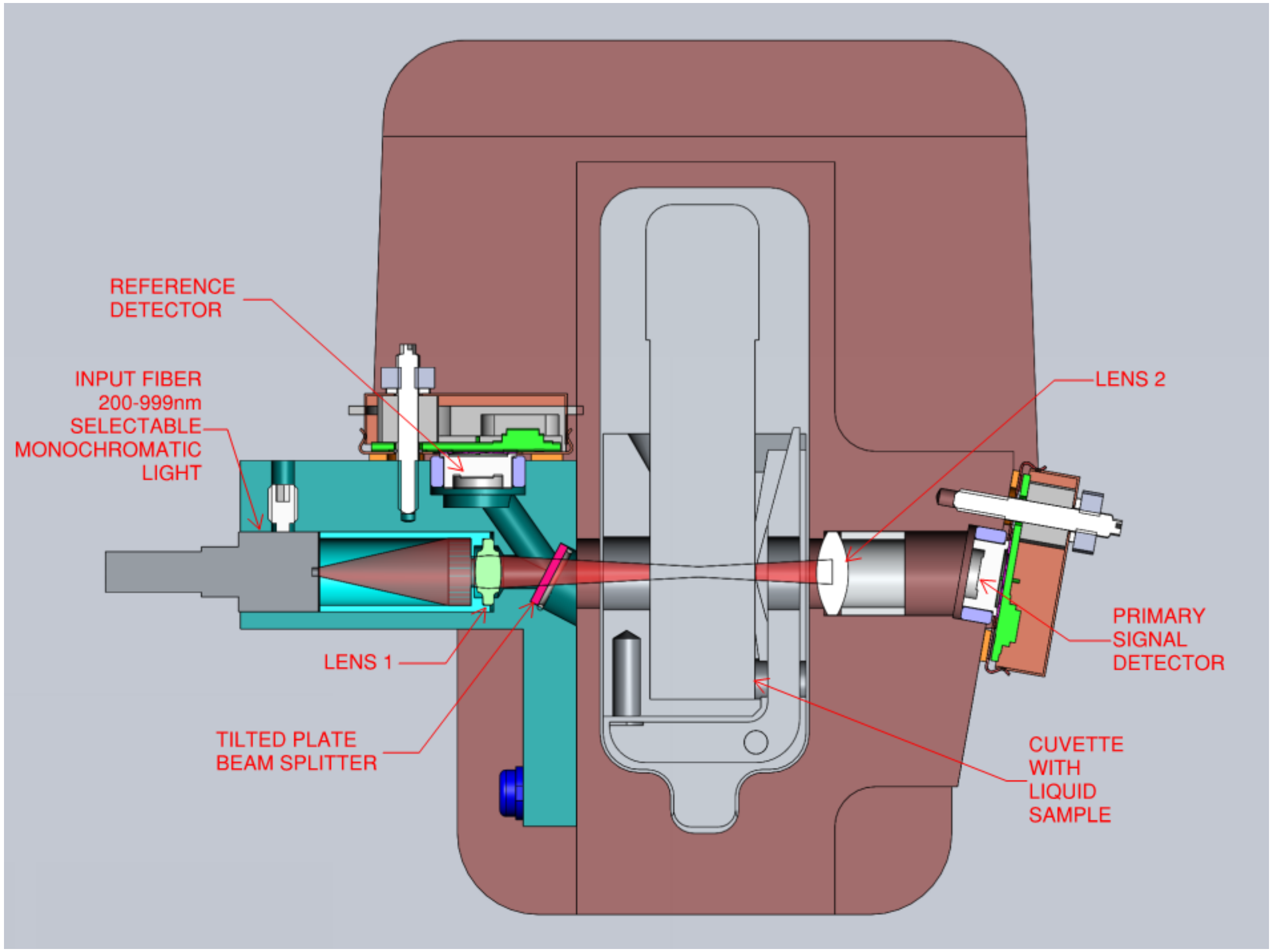




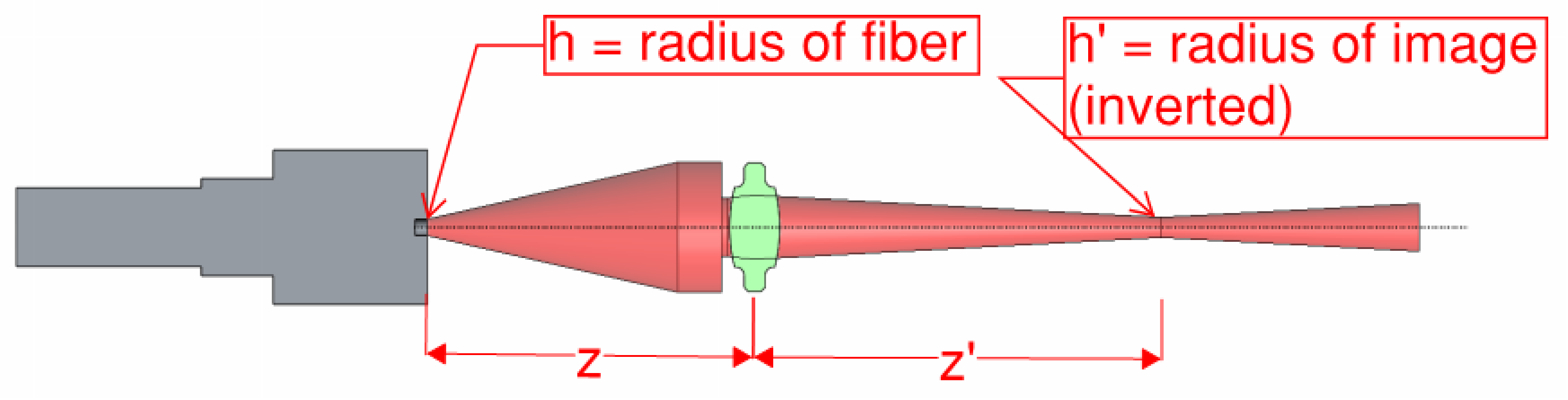
http://www.photonics.com/EDU/Handbook.aspx?AID=25151

- NA = $n_{air}\sin\theta = (n_{core}{}^2 – n_{cladding}{}^2)^{1/2}$
- θ = half angle
- Our fiber is .040 inch diameter and has NA = 0.22. This is a common fiber NA.

Lens 1 images the fiber face into the cuvette sample



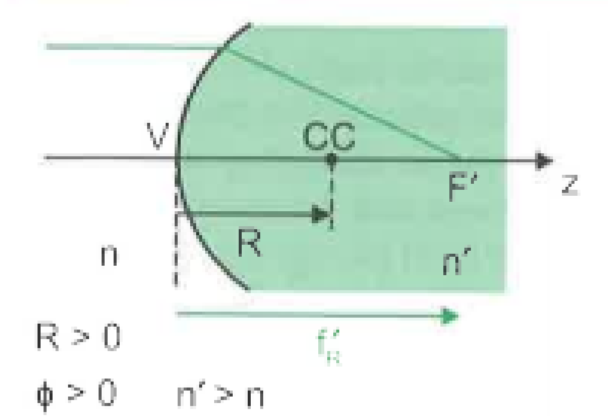
Where is the image? How big is it?



- Thin lens imaging equation: $1/z' = 1/z + 1/f$ (primed coordinates refer to the image)
- Magnification = $z'/z = h'/h$
- So what is f?

Focal Length of a Surface

- Curved surfaces separating materials of different index have optical power.
- Power of a surface = Φ = (n’-n)/R
- Curvature = C = 1/R
- f = 1/ Φ

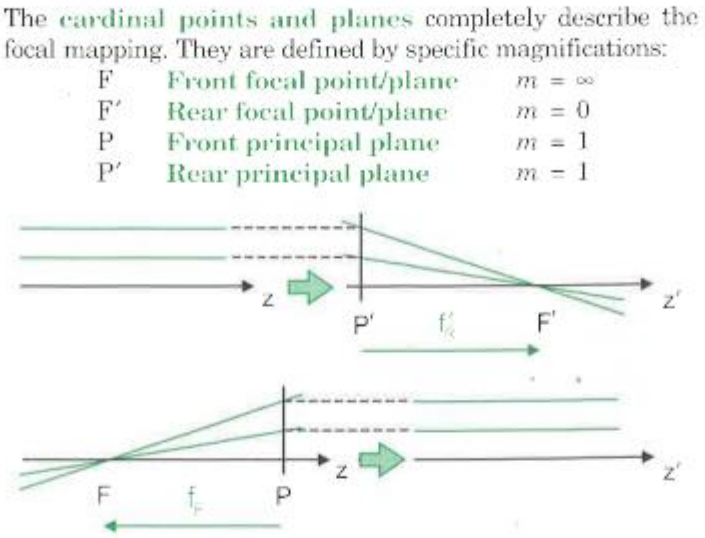


Reference 1

- But the lens has two surfaces…

Gaussian Optics – Cardinal Points

- Front and Rear Focal Points
- Front and Rear Principal Planes

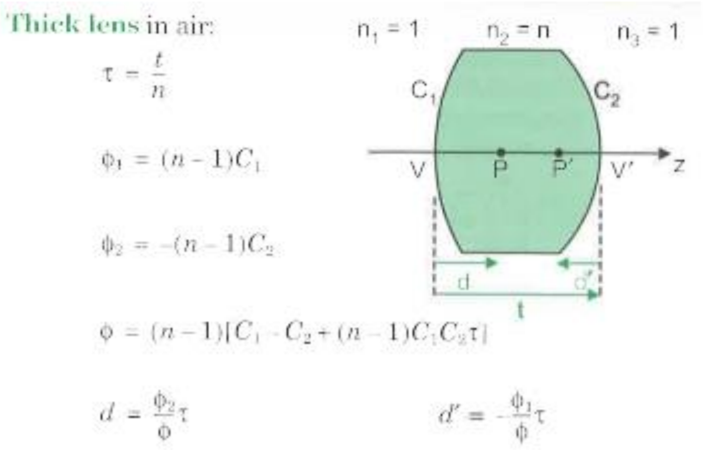


The **cardinal points and planes** completely describe the focal mapping. They are defined by specific magnifications:

F	**Front focal point/plane**	$m = \infty$
F′	**Rear focal point/plane**	$m = 0$
P	**Front principal plane**	$m = 1$
P′	**Rear principal plane**	$m = 1$

Reference 1

Focal Length of a Lens



Thick lens in air:

$\tau = \frac{t}{n}$

$\phi_1 = (n-1)C_1$

$\phi_2 = -(n-1)C_2$

$\phi = (n-1)[C_1 - C_2 + (n-1)C_1 C_2 \tau]$

$d = \frac{\phi_2}{\phi}\tau$

$d' = -\frac{\phi_1}{\phi}\tau$

$$\Phi = (n-1)[C_1 - C_2 + (n-1)(C_1)(C_2)\tau]$$

- $\tau = t/n$ = Reduced Thickness
- d and d' locate the principal planes from the Vertices V and V'
- Some lenses are so thin that the separation between P and P' approaches 0.

Focal Length of a Lens

- For a thin lens in air, t → 0.
- Power = $\Phi = (n-1)(C_1-C_2)$
- d = d' = 0, so the principal planes are on the surfaces and the surfaces coincide as t → 0.

- So what about our lens?

Thin and Thick Lens focal length for 7092199

7092199	[mm]	wavelength	index
Thickness t =	3.1	206nm	1.543
		248nm	1.508
		546nm	1.460
		1013nm	1.450

ENTER DATA IN THE GREEN CELLS

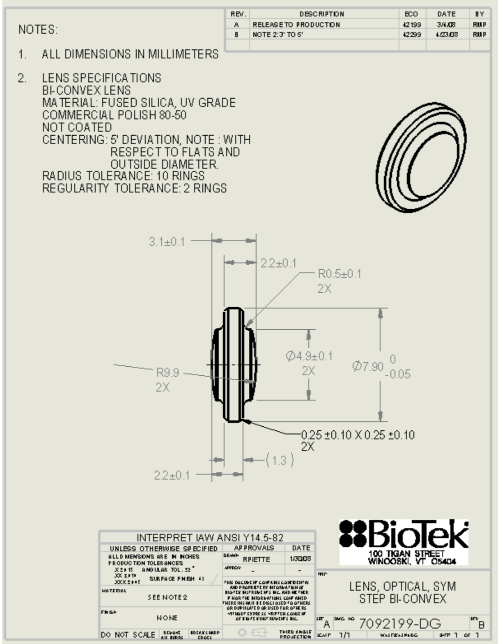
Power of first surface	Radius		Power of second surface			Power of Lens	THICK LENS EQ'N	THICK LENS	
	of surface	Φ = (n' - n)/R	wave	Radius of surface	Φ = (n' - n)/R	wave	Φ = (n-1)[1/R1 - 1/R2 + (n-1)(1/R1)(1/R2)τ]	Focal Length	Focal Length
wavelength	[mm]	[1/mm]	length	[mm]	[1/mm]	length	[1/mm]	[mm]	[inch]
206nm	9.9	0.054848485	206nm	-9.9	0.054848485	206nm	0.103652962	9.65	0.380
248nm	9.9	0.051313131	248nm	-9.9	0.051313131	248nm	0.09721352	10.29	0.405
546nm	9.9	0.046464646	546nm	-9.9	0.046464646	546nm	0.088345193	11.32	0.446
1013nm	9.9	0.045454545	1013nm	-9.9	0.045454545	1013nm	0.086491878	11.56	0.455

@206nm:

Front Principal Plane: $d = (\Phi_2/\Phi)\tau =$ 1.063111185

Rear Principal Plane: $d' = (-\Phi_1/\Phi)\tau =$ -1.063111185

	THIN LENS EQ'N	THIN LENS		
wave	Φ = (n-1)[1/R1 - 1/R2]	Focal Length	Focal Length	Error
length	[1/mm]	[mm]	[inch]	[%]
206nm	0.10969697	9.12	0.359	5.5
248nm	0.102626263	9.74	0.384	5.3
546nm	0.092929293	10.76	0.424	4.9
1013nm	0.090909091	11.00	0.433	4.9



NOTES:

1. ALL DIMENSIONS IN MILLIMETERS
2. LENS SPECIFICATIONS
   BI-CONVEX LENS
   MATERIAL: FUSED SILICA, UV GRADE
   COMMERCIAL POLISH 80-50
   NOT COATED
   CENTERING: 5' DEVIATION, NOTE : WITH
   RESPECT TO FLATS AND
   OUTSIDE DIAMETER.
   RADIUS TOLERANCE: 10 RINGS
   REGULARITY TOLERANCE: 2 RINGS

REV.	DESCRIPTION	ECO	DATE	BY
A	RELEASE TO PRODUCTION	42199	3/4/08	RMP
B	NOTE 2: 3' TO 5'	42299	4/23/08	RMP

INTERPRET IAW ANSI Y14.5-82

UNLESS OTHERWISE SPECIFIED
ALL DIMENSIONS ARE IN INCHES
PRODUCTION TOLERANCES
ANGULAR TOL.
SURFACE FINISH

MATERIAL
SEE NOTE 2

FINISH
NONE

DO NOT SCALE

REMOVE ALL BURRS

BREAK SHARP EDGES

APPROVALS | DATE
DRAWN RPIETTE | 1/30/08
APPROV - | -

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THIRD ANGLE PROJECTION

BioTek
100 TIGAN STREET
WINOOSKI, VT 05404

LENS, OPTICAL, SYM
STEP BI-CONVEX

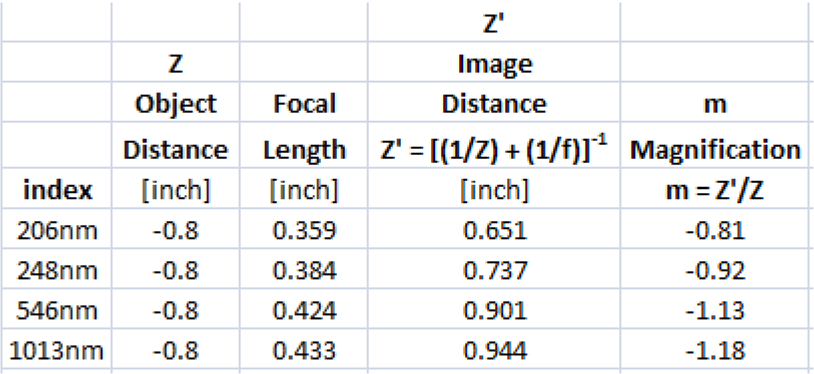
SIZE A
DWG. NO. 7092199-DG
REV B
SCALE 1/1
SHEET 1 OF 1

- Note the 5% error.
- Note the wavelength dependence of the focal length. Blue focuses closer than red.

So where is the image?

- Now that we know the focal length, we can locate the image: $1/z' = 1/z + 1/f$ and $m = z'/z$

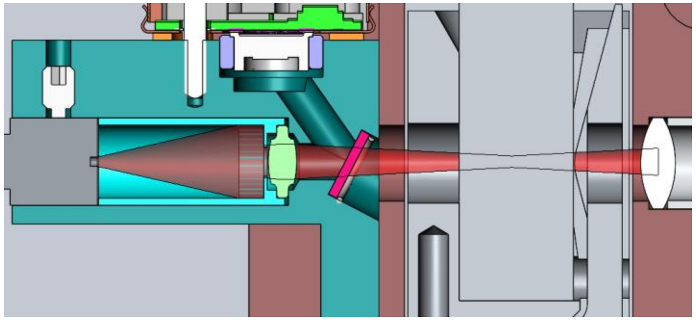
  z, z', f all measured from the principal planes.
- Let's go with the 5% error thin lens result for f.

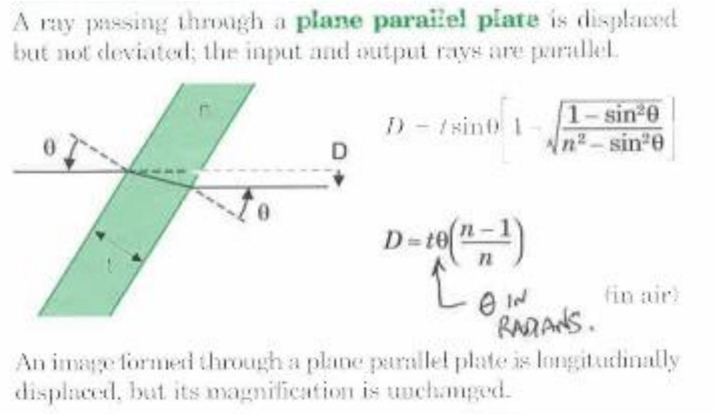


index	Z Object Distance [inch]	Focal Length [inch]	Z' Image Distance $Z' = [(1/Z) + (1/f)]^{-1}$ [inch]	m Magnification $m = Z'/Z$
206nm	-0.8	0.359	0.651	-0.81
248nm	-0.8	0.384	0.737	-0.92
546nm	-0.8	0.424	0.901	-1.13
1013nm	-0.8	0.433	0.944	-1.18

- Note that the magnification is ~ 1:1. (Negative means inverted.)
- Note that these values are in air. (The cuvette will hold a liquid.)

Next the beam passes thru a tilted plate beam splitter





A ray passing through a **plane parallel plate** is displaced but not deviated; the input and output rays are parallel.

$$D = t\sin\theta\left[1 - \sqrt{\frac{1-\sin^2\theta}{n^2-\sin^2\theta}}\right]$$

$$D = t\theta\left(\frac{n-1}{n}\right)$$

θ IN RADIANS.

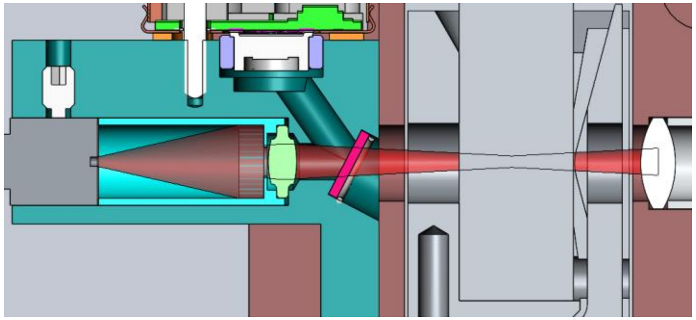
(in air)

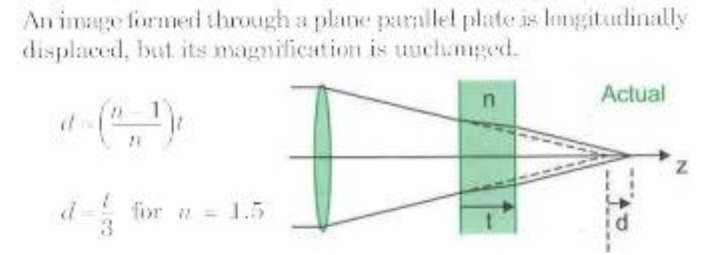
An image formed through a plane parallel plate is longitudinally displaced, but its magnification is unchanged.

Reference 1

- D ≈ tθ(n-1)/n
- This results in an offset of ~.007 inch.

Next, we enter the cuvette





An image formed through a plane parallel plate is longitudinally displaced, but its magnification is unchanged.

$$d = \left(\frac{n-1}{n}\right)t$$

$$d = \frac{t}{3} \text{ for } n = 1.5$$

Reference 1

d = (n-1)t/n

- The cuvette acts as a plane parallel plate of index 1.33 if it is full of water.
- The image displacement due to ½ the cuvette is ~.062 inch.
- The beam splitter on the previous page also has this effect ~.014 inch.
- The two plates together displace the image ~ .076 axially.

Influences on the Image location

- The system is used with many wavelengths, each with its own focal length.
- The cuvette sample is usually aqueous, but its index may vary.
- The sample may be turbid.
- The system is used in air when running its startup routine.

So the location of the image in the cuvette can vary around the general location we determined. The second lens could be analyzed in a similar way using the image in the cuvette as its object.

Conclusion

The name of the game for this system is:

- To use inexpensive optics that can
- Put energy at a wide variety of wavelengths into the sample chamber
- And collect all that comes out the other side regardless of what is in the chamber.

First order optics can help with the initial layout of a system, or to understand in general how it works. To design it to be robust and accurate requires more advanced methods and lots of testing.

References

1. Grievenkamp, John E., “Field Guide to Geometrical Optics” SPIE Press, Bellingham WA, 2004
2. Prof. John Grievenkamp, class notes and lectures from “Optical Design and Instrumentation I, Fall 2013.