Principles of First Order Optics Applied to Components of the BioTek Absorbance Spectrophotometer Cuvette Assembly

Tutorial Presentation to the BioTek Mechanical Design Team

OPTI521

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What is “First Order Optics”?  

First Order Optics = Perfect Lenses (No Aberrations)  

Analysis Methods:  

• Gaussian Optics – Formulas referencing:  
  – Principal Points  
  – Focal Points  

• Paraxial Optics  
  – Rays near the axis (ignore curvature of lens)  
  – Small angles ($\sin \theta \approx \theta$)
BioTek Cuvette Assembly

Not an imaging system. We put energy into the sample at a particular wavelength and collect what comes out, measuring absorbance. Wavelength range = 200-999nm.
UV GRADE FUSED SILICA

• Because of the 200 – 380 nm wavelengths, we used fused silica optics.

• Note the variation in Refractive Index

• Refractive Index:
  \[ n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in the material}} \]

HPFS® Fused Silica Standard Grade
Snell’s Law

\[
\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1}
\]

\(n_{\text{air}} = 1\)
\(n_{\text{water}} = 1.333\)
\(n_{\text{fused silica}} = 1.45\)

www. http://hyperphysics.phy-astr.gsu.edu/
Optical Fiber Numerical Aperture (NA)

• **NA** = \( n_{\text{air}} \sin \theta = (n_{\text{core}}^2 - n_{\text{cladding}}^2)^{1/2} \)

• \( \theta \) = half angle

• Our fiber is .040 inch diameter and has **NA** = 0.22. This is a common fiber **NA**.
Lens 1 images the fiber face into the cuvette sample
Where is the image? How big is it?

- Thin lens imaging equation: \( \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \) (primed coordinates refer to the image)
- Magnification = \( \frac{z'}{z} = \frac{h'}{h} \)
- So what is \( f \)?
Focal Length of a Surface

• Curved surfaces separating materials of different index have optical power.
• Power of a surface = $\Phi = (n' - n)/R$
• Curvature = $C = 1/R$
• $f = 1/\Phi$

• But the lens has two surfaces...

Reference 1
Gaussian Optics – Cardinal Points

- Front and Rear Focal Points
- Front and Rear Principal Planes
Focal Length of a Lens

\[ \Phi = (n-1)[C_1 - C_2 + (n-1)(C_1)(C_2)\tau] \]

- \( \tau = t/n = \text{Reduced Thickness} \)
- \( d \) and \( d' \) locate the principal planes from the Vertices \( V \) and \( V' \)
- Some lenses are so thin that the separation between \( P \) and \( P' \) approaches 0.
Focal Length of a Lens

• For a thin lens in air, \( t \to 0 \).
• Power = \( \Phi = (n-1)(C_1-C_2) \)
• \( d = d' = 0 \), so the principal planes are on the surfaces and the surfaces coincide as \( t \to 0 \).

• So what about our lens?
Thin and Thick Lens focal length for 7092199

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Angle Φ = (n' - n)/R</th>
<th>Radius of surface</th>
<th>Power of Lens</th>
<th>Power of Thick Lens</th>
<th>Power of Thin Lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>206nm</td>
<td>9.9</td>
<td>206nm</td>
<td>-9.9</td>
<td>0.054848485</td>
<td>0.103652952</td>
</tr>
<tr>
<td>248nm</td>
<td>9.9</td>
<td>248nm</td>
<td>-9.9</td>
<td>0.051313131</td>
<td>0.09721352</td>
</tr>
<tr>
<td>546nm</td>
<td>9.9</td>
<td>546nm</td>
<td>-9.9</td>
<td>0.046464646</td>
<td>0.088345193</td>
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<tr>
<td>1013nm</td>
<td>9.9</td>
<td>1013nm</td>
<td>-9.9</td>
<td>0.045454545</td>
<td>0.086491878</td>
</tr>
</tbody>
</table>

Notes:
- Note the 5% error.
- Note the wavelength dependence of the focal length. Blue focuses closer than red.
So where is the image?

• Now that we know the focal length, we can locate the image: \( \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \) and \( m = \frac{z'}{z} \)

  \( z, z', f \) all measured from the principal planes.

• Let’s go with the 5% error thin lens result for \( f \).

  • Note that the magnification is \( \sim 1:1 \). (Negative means inverted.)
  • Note that these values are in air. (The cuvette will hold a liquid.)
Next the beam passes thru a tilted plate beam splitter

- $D \approx t\theta(n-1)/n$
- This results in an offset of $\sim 0.007$ inch.
Next, we enter the cuvette

\[ d = \frac{(n-1)t}{n} \]

- The cuvette acts as a plane parallel plate of index 1.33 if it is full of water.
- The image displacement due to \( \frac{1}{2} \) the cuvette is \(~.062\) inch.
- The beam splitter on the previous page also has this effect \(~.014\) inch.
- The two plates together displace the image \(~ .076\) axially.

Reference 1
Influences on the Image location

• The system is used with many wavelengths, each with its own focal length.
• The cuvette sample is usually aqueous, but its index may vary.
• The sample may be turbid.
• The system is used in air when running its startup routine.

So the location of the image in the cuvette can vary around the general location we determined. The second lens could be analyzed in a similar way using the image in the cuvette as its object.
Conclusion

The name of the game for this system is:

• To use inexpensive optics that can
• Put energy at a wide variety of wavelengths into the sample chamber
• And collect all that comes out the other side regardless of what is in the chamber.

First order optics can help with the initial layout of a system, or to understand in general how it works. To design it to be robust and accurate requires more advanced methods and lots of testing.
References


2. Prof. John Grievenkamp, class notes and lectures from “Optical Design and Instrumentation I, Fall 2013.”