OPTI-521 Graduate Report 2
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Tutorial: Introduction to imaging, and estimate of image quality degradation from optical surfaces

Abstract

The purpose of this tutorial is to introduce the concept of image quality, how it might be quantified and determined for both coherent and incoherent imaging, and how these concepts relate to the more familiar (and importantly, measurable at the manufacturing stage) quantities of wavefront error and/or surface figure error. Because various rules of thumb exist for relating these specifiable and measurable quantities both to one another and to the increase in cost and difficulty in the manufacturing process, the bulk of this tutorial serves as a primer to imaging, with appropriately cited references for the reader who wishes to learn more.

Introduction – what is image quality, and how can it be quantified?

Before discussing how the quality and characteristics of optical surfaces may affect image quality, it is necessary to develop an understanding of what is meant by the term "image quality." A complete definition of image quality requires answering the following questions: What information is desired from the image? How will that information be extracted? What objects will be imaged? What measure of performance will be used? [1] With these questions in mind, the authors present the concept of task-based imaging, in which the information you desire to extract is the *task*, and the means by which it is extracted is the *observer*.

For the purposes of this tutorial, we consider a basic imaging system, represented by the imaging equation: $g = Hf$. Here, f describes a vector in object space, H is a matrix representing the imaging system, and g describes the measured data vector. For a simple photographic camera, f represents discrete samples of an infinite series of points within object space, and g represents the digital value for each pixel in the output image.

There are several ways to quantify the performance of a photographic imaging system, and thus several ways to consider performance degradation (for an overview, see [2]). Our definition of an imaging system lends itself naturally to the point spread function (PSF) interpretation, and to the consideration of image quality degradation from diffraction blur, rather than from geometrical aberration. This is a reasonable assumption to make, as optical surface imperfections will rarely be so large as to create any geometric aberration.

To quantify degradation of performance, we therefore first define "perfect quality imaging" to be diffraction limited (the mathematics of which are not considered in this tutorial). In this case, a point object is imaged to a radially symmetric distribution with width (peak to zero) of

$$r = \frac{.61\lambda}{NA'}, \tag{1}$$

where λ is the wavelength of light and NA' is the numerical aperture of the imaging system in image space.

We assume that the imaging system and detector are well matched, such that the smallest resolvable point object is imaged entirely onto a single detector element (pixel); any "blurring" in the image is caused by an increase in the width of the system point response, and results in a spread of energy across multiple pixels. This is a good rule of thumb in imaging system design, and is easily approximated in the visible light regime by $D \approx f / \#_W$, where D is the width of the system point response in [μm] and $f/\#_W$ is the working f/# of the imaging system.

Diffraction limited imaging and the Rayleigh Criterion

The conditions for diffraction limited imaging are defined by a perfect thin lens, which images a plane wave into a perfect spherical wave that converges to the optical axis after propagating a given distance (what we call the focal length of the lens) along the +z axis. A perfect lens therefore has a transfer function:

$$t_{lens}(r) = e^{-ikR_f} t_{ap}(r) \tag{2}$$

where k is wavenumber, $R_f = \sqrt{r^2 + f^2}$, f now represents the focal length of the ideal lens, and $t_{ap}(r)$ represents the amplitude transmission of the aperture (typically described as a cylinder function) [1].

However, any real lens does not transmit this perfect spherical wave, but instead propagates a wave that deviates in phase from the ideal spherical wave (i.e. the Gaussian reference sphere) by some amount in the plane of the exit pupil. We call this phase deviation the wavefront error, and it can be expressed as $kW(r)$, where k is wavenumber and $W(r)$ is a spatial distance measured along a line parallel to the z-axis (i.e. an optical path difference, OPD) between the ideal and actual wavefronts. The transfer function for the aberrated wave is therefore

$$t_{lens}(r) = e^{-ik(R_f - W(r))} t_{ap}(r), \tag{3}$$

and it is this OPD function, $W(r)$ (i.e. the generalized pupil function [3]), which is expanded assuming rotational symmetry into the well known Seidel aberrations. For convenience, we consider a thin lens with the aperture stop at the lens, and define the pupil transmission by

$$t_{pupil}(r) = t_{ap}(r) e^{ikW(r)}. \tag{4}$$

In this way, making the Fresnel approximation and disregarding the constant phase factor allows us to relate the pupil function to the aberration and aperture of the lens, while ignoring the inherent power of the lens. To account for the dependence of wavefront error on the incident wave itself, $W(r)$ is modified to $W(r;r_0)$, where r_0 is the vector describing the object point location.

The Rayleigh Criterion (a generally useful relative measure of optical performance) states that if this OPD is less than or equal to a quarter wave $\left(W(r) \le \frac{\lambda}{4}\right)$, then the performance of the imaging system will be nearly indistinguishable from perfect [4]. Additionally, the Rayleigh Criterion leads to another useful rule of thumb for depth of focus. If we define the maximum blurred spot size to be just within this limit, and considering that our imaging system is otherwise diffraction limited, the depth of focus is approximated as $\delta \approx (f/\#)^2$ in [μm] [4].

Coherent vs. Incoherent Imaging

When an object amplitude distribution is decomposed into a series of delta functions, the field in the image plane can be considered as the field in the object plane convolved with the system point-spread function (PSF). In the case of coherent imaging, this PSF is proportional to the Fourier transform of the scaled pupil function

$$p_{coh}(r) \propto T_{pupil}\left(m\frac{r}{\lambda z'}\right). \tag{5}$$

Here, m is the magnification of the imaging system, r is the coordinate vector, and z' is the distance along the +z-axis that the diffracting wave propagates (typically, this is evaluated for the case of z' = f). For a full description of this calculation, see Section 9.2 of [1]. This is related to the incoherent transfer function by

$$p_{incoh}(r) \propto \left|p_{coh}(r)\right|^2. \tag{6}$$

For a full description of this relationship, see Section 9.7.6 of [1]. If coherence is of particular interest to the object being imaged, then it should be noted here that this proportionality involves the coherence area of the source. However, partial coherence and the concept of coherence area are outside the scope of this tutorial, and any interested readers are encourage to review Ch. 9 of [1].

The intensity in the image space, scaled for magnification, is now related to the intensity in the object space by a convolution (Eq. 9.287 in [1]):

$$I_{im}^{(s)}(r) = I_{obj}(r) * p_{incoh}(r). \tag{7}$$

In the presence of aberrations, the aberrated image is determined by calculating an aberrated PSF, using the pupil function in Eqn. (4). This convolution may also be expressed as a multiplication in Fourier space, which will be discussed in the following section.

Relation to OTF/MTF

As linear, shift-invariant systems, ideal imaging cameras are described by transfer functions in addition to PSFs. The optical transfer function, another useful performance metric, is the normalized transfer function of the system. It describes the translation and contrast reduction of an imaging system observing a periodic sine pattern at various frequencies (0, increasing to some maximum), and is the Fourier transform of the incoherent PSF. The result of this relationship is that the OTF is described by the normalized auto-correlation of the amplitude transfer function, which is itself a scaled version of the pupil function:

$$\mathcal{H}(\xi,\eta) = \frac{P_{incoh}(\rho)}{P_{incoh}(0)} = \frac{\mathcal{F}_2\{p_{incoh}(r)\}}{\mathcal{F}_2\{p_{incoh}(0)\}} \propto \frac{\left[t_{pupil} \star t_{pupil}\right](\lambda z'\rho)}{\left[t_{pupil} \star t_{pupil}\right](0)} \qquad (8)$$

Where $\mathcal{H}(\xi,\eta)$ is the OTF, P_{incoh} is the 2D Fourier Transform (represented by $\mathcal{F}_2$), and $\star$ represents the auto-correlation integral. A full description of this relationship and the associated calculations may be found on Section 6.3 of [3] and in wk12-b-12 of [7]. The MTF, which represents the ratio of output modulation to input modulation and is often the specified transfer function of the system, is the modulus of the OTF.

Aberrated Transfer Functions

As discussed above, a lens with aberration contains an additional phase term. In the case of coherent imaging, the transfer function is the Fourier transform of the PSF, and the coherent PSF is the Fourier transform of the pupil. Therefore, the coherent transfer function is a scaled version of the pupil, and this same scaling may be applied to the erroneous phase term that aberrations cause in the pupil plane (see Eqn. (4)):

$$p_{coh,ab}(\rho) = t_{pupil}\left(\frac{\lambda z'}{m}\rho\right)e^{ikW\left(\frac{\lambda z'}{m}\rho\right)}. \qquad (9)$$

For an incoherent system, this additional phase term must be included. This can be done in the auto-correlation by as follows

$$\mathcal{H}(\rho) = \frac{\int_{-\infty}^{\infty} t_{pupil}\left(r+\frac{\lambda z'}{2m}\rho\right) t_{pupil}\left(r-\frac{\lambda z'}{2m}\rho\right) e^{ik\left[W\left(r+\frac{\lambda z'}{2m}\rho\right)-W\left(r-\frac{\lambda z'}{2m}\rho\right)\right]} dr}{\int_{-\infty}^{\infty} t_{pupil}(r)\,dr}. \qquad (10)$$

Though complex, this is a direct relationship between wavefront error and OTF for an incoherent imaging system.

Optical Surfaces

With an understanding of the transmission of an aberrated lens, and how wavefront error reduces image quality by affecting the two most common performance metrics for an imaging system, it is necessary to examine how optical surfaces may affect $W(r)$. In the general case, wavefront error may be directly related to surface error by

$$W(r) = \Delta S(r)(n-1)\cos(\theta_i) \tag{11}$$

where $\Delta S(r)$ is the surface deviation from ideal, n is the index of refraction, and θ_i is the angle of incidence from the aberrated surface normal. In determining lens requirements, a useful extension and approximation to this relationship is that the RMS wavefront error (RMSWFE) is one quarter the peak-to-valley (PV) surface error [5]. It should be noted that this ratio is different for various types of surface error (i.e. corresponding to different Seidel aberrations), and if high frequency surface error components (e.g. from diamond turning) exist. A more complete analysis of these variations can be found on Pg. 30 of [6]. Scatter from optical surfaces may also increase stray light, reducing the signal to noise ratio (SNR) of the imaging task at hand. However, a discussion of this is outside the scope of this tutorial.

Conclusion

The material discussed above may be used both in understanding the consequences of a particular surface error on either the point spread function or transfer function of a system, or to determine an appropriate wavefront error tolerance (which may then be related to surface error tolerance) given some maximum allowable spot size (usually determined by detector element size), or minimum required OTF/MTF value (usually determined by application, and convention).

References

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