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Off-loading of a cylindrical optic to simulate zero-gravity

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Abstract

The problems have long been recognized in supporting a conventional, disc-shaped optical element in the presence of gravity, so as to minimize the gravity-induced deflections. With the increasing size and quality of optics that are being used in space come correspondingly difficult problems in simulating a zero-gravity situation while supporting the optic on Earth. These problems are made worse by the fact that no additional stresses and strains should be introduced into the optic as it slowly changes shape because of, for instance, temperature drifts. In other words, a zero-gravity support should be kinematic. In this paper, we present a scheme for a zero-gravity support for a cylindrical optic. The type of optic being considered is used in grazing incidence X-ray telescopes, and consists of a cylindrical shell with a reflecting surface on the inside of the cylinder. The zero-gravity support presented permits extremely small gravity-induced deflections, yet contacts the mirror in only four places, and is therefore rather easily made kinematic. Moreover, no counterbalances or calibrated forces are needed - the distribution of the weight of the mirror itself guarantees that the off-loading will be calibrated correctly. We discuss the application of this concept for the alignment of an X-ray telescope. Using some thin-member approximations, we give some analytical approximations of the deflections. Also, some results of a finite element method analysis of the support are given. These results are interpreted in terms of a previously published set of functions that are orthonormal over the surface of a cylinder.

Introduction

The requirement to support a spaceborne optical element in a simulated zero-gravity (zero-g) environment has long been recognized as a very challenging and demanding problem to solve. Put simply, the requirement is to be able to figure and mount the optic adequately under the influence of gravity (i.e., in a one-g environment) such that when gravity is removed, the optic is in the correct shape and free of all residual strains. (For example, for conventional, disc-shaped optical elements, zero-g simulation has been accomplished by floatation of the optic in a pool of mercury or on an air bag or plenum, and by supporting the mirror by a number of discrete point forces.) The optimum off-loading system is one which minimizes the surface error of the optic by compensating for the distortions caused by gravity, while introducing negligible residual forces.

In addition to the ability to simulate a zero-g environment, another critical requirement on the off-loading scheme is to provide a mount that is kinematic. A kinematic mount is one which constrains the optic against rigid body motions, but does not over-constrain the optic against non-rigid body deformations, either within the optic or within the mounting assembly. Such deformations could be caused, for example, by thermal fluctuations, or by extraneous constraint forces generated by outside vibrations, frictional forces, or unpredictable hardware misalignments. If a mount is not kinematic, then the introduction of any of these extraneous forces will cause unwanted stresses and strains within the optic. A kinematic mount will react to these extraneous forces so as to prevent stresses and strains within the optic, while still defining a unique position in space for the optic.

The preceding discussion has been general enough to apply to all cases of off-loading, whether they be applied to conventional, disc-shaped optics, or to cylindrical optics used in X-ray systems. However, obviously, the unique geometry of a cylinder poses unique constraints on the off-loading approach. Moreover, cylindrical optics tend to have a very large aspect ratio. That is, their size is much greater than their wall thickness, which makes cylindrical optics much flimsier than conventional optics of comparable size. The reason for the large aspect ratio is that in X-ray systems, individual telescopes, each consisting of a pair of cylindrical optics, can be nested one inside the other. The best analogy in conventional optical systems is perhaps the case of multiple mirror telescopes, where a single large primary mirror is made out of a mosaic of separate smaller mirrors. What makes the X-ray case unique is that each optic must be made thin simply in order not to obscure the aperture of the next nest out. In the conventional optics case, there is no a priori constraint on the thickness of each of the mirrors. Figure 1 illustrates the difference.

To summarize, we have discussed the requirement for a kinematic mount which simulates a zero-g condition for a high quality optical element. This mount, or off-loading device, may be required during metrology or system integration or both. We have shown that nested X-ray optics which are cylindrical in nature are inherently thin-walled, and therefore require particular attention in the area of off-loading. In this paper we discuss one particularly simple and effective off-loading approach recently developed for cylindrical optics. This approach is being applied to the Technology Mirror Assembly (TMA), which is an X-ray telescope intended to demonstrate the capability to fabricate the Advanced X-ray Astrophysical Observatory (AXAF).

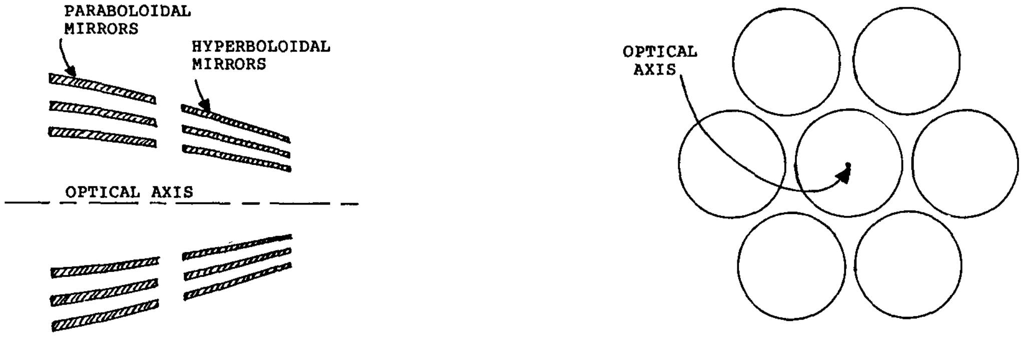

Figure 1. Cross section of nested X-ray telescope, and end view of segmented conventional mirror. Note that nesting requires thin cylindrical elements, while segmenting imposes no a priori constraints on mirror thickness.

Previous approaches

As an example of how cylindrical optics have been off-loaded in the past, we examine the High-Energy-Astrophysical-Observatory-B (HEAO-B), or Einstein Observatory. This X-ray telescope, which consisted of four nested pairs of cylindrical optics, revolutionized the field of X-ray astronomy by providing higher resolution and collecting area than any previous X-ray telescope. To fulfill the tight requirements on fabrication and alignment of the telescope, several off-loading approaches were used during different phases of the surface metrology and assembly operations. Thus, each of the approaches had widely varying requirements, and gave quite different accuracies in simulating zero-g.

The simplest off-loading mechanism consisted simply of a V-block support for holding the optic horizontally, as shown in Figure 2. The purpose of this support was to minimize the gravity-induced deflection of the optic along its lowest meridian, so that the profile of the optic could be measured along that meridian. Although the mount was not kinematic, it was able to reduce the deflections to the level of several micro-inches. This residual deflection was calculated, and removed analytically from the metrology data.

Another more elaborate off-loading mechanism consisted of a mercury floatation trough used in conjunction with three support points for holding the optic vertically, as shown in Figure 3. The purpose of this support was to minimize the gravity-induced deflections of the optic along a circumferential track, again so that the profile of the optic could be measured along that track. Like the V-block support, this mount was not kinematic, but was never-the-less able to reduce the deflections to the level of several micro-inches.

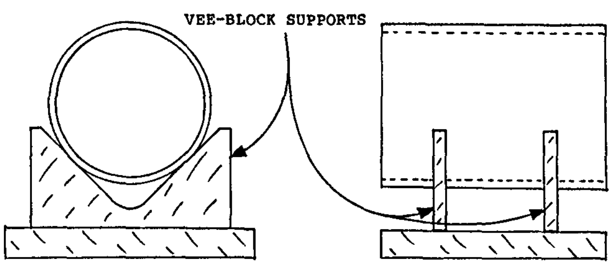

Figure 2. Vee-block support for horizontal off-loading of a cylindrical mirror.

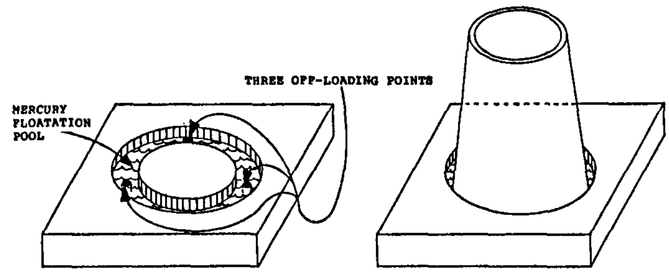

Figure 3. Mercury floatation support for vertical off-loading of a cylindrical mirror.

The most demanding off-loading task for HEAO-B came during the alignment operation, when each cylinder had to be off-loaded in a horizontal position, and adjusted for final bonding into the nested assembly. The requirements for this operation were significantly tighter than for the operations discussed above, in that deflections had to be held to the micro-inch level over the entire surface of the optic. To accomplish this, a complicated system of discrete supports was designed and implemented. On the order of thirty wires were used to push and pull on the optic in various locations, with well calibrated forces.

Even though this final off-loading system performed well, it was quite difficult and cumbersome to implement operationally. The implementation difficulties, coupled with the need to reduce the gravity-induced deflections still further for systems such as TMA and AXAF, led to the development of the simple, new approach discussed in this paper.

System configuration and requirements

The basic approach usually assumed for mounting systems such as HEAO-B, TMA, or AXAF is to attach each cylindrical optic to an outer cylindrical shell, or sleeve, via one or more steel rings. The optic is off-loaded and aligned, and the outer sleeve is finally bonded at its ends to the telescope assembly. Such an arrangement is shown schematically in Figure 4. A successful off-loading system for this configuration must satisfy two criteria. First, the system must minimize the deflections in the optic while it is being aligned, so that very small misalignments can be detected and corrected. And second, the system must impose minimal forces on the sleeve itself, so that, when the sleeve is finally bonded and the off-loading removed, no undue deflections are transmitted to the optic by the springback of the sleeve.

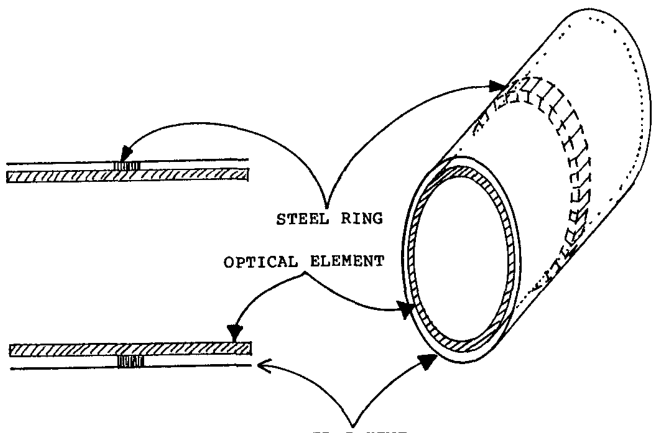

Figure 4. Generalized mirror mounting approach for cylindrical mirrors. Mirror is attached by one (or more) steel rings to an outer sleeve.

In order to characterize the requirements on an off-loading system for a cylindrical optic, it is necessary first to define the critical performance parameters in a system using such optics. Conventional optical systems work in the infra-red to ultra-violet regions of the spectrum (where wavelengths range from perhaps 1,000 Angstroms to 100,000 Angstroms), and are often diffraction-limited in their performance. In contrast, the cylindrical optical systems considered in this paper work in the X-ray region (where wavelengths range from perhaps 1 Angstrom to 100 Angstroms), and are far from being diffraction-limited in their performance. (A system is considered to be diffraction-limited if the wavefront errors are a small fraction of the radiation wavelength.) This difference in performance between conventional and X-ray systems implies a difference in the critical performance parameters. In particular, for conventional, diffraction-limited systems, the rules of physical (or diffraction) optics apply, and the most significant single parameter of the optical surfaces is their Root Mean Square (RMS) figure quality. For X-ray systems, the rules of ray (or geometrical) optics apply, and the most significant single parameter of the optical surfaces is their RMS slope. (Moreover, it can be shown that, because of the grazing incidence nature of the reflection on the surface of the cylinder, axial slope is significantly more important than circumferential slope in determining performance.) Not surprisingly, the characterization of conventional systems by their RMS surface quality and X-ray systems by their RMS axial slope is a bit of an over-simplification. In both cases, some consideration must be made of such issues as the spatial frequency distribution of the surface errors. None-the-less, axial slope error is a reasonable parameter for characterizing the performance of an X-ray system, and therefore, for characterizing the effectiveness of an off-loading system.

Thus far we have emphasized the importance of axial slope in determining the performance of cylindrical optics. Stated simply, the goals for residual fabrication errors in systems such as TMA and AXAF is to achieve RMS slope errors that are on the order of one quarter of a micro-radian. The goals for the residual deformations left by an off-loading system must be in the same range, lest the operation of mounting the optic negate the achievements of fabrication. A requirement on the residual RMS axial slope is essentially all that need be placed on an off-loading system. However, we additionally need some insight into the nature of the residual deformations so that we can understand what aspects of the off-loading system might be changed to improve the performance. To that end, we use in this paper a previously published set of functions which are orthonormal over the surface of a cylinder[1]. These functions have proven to be extremely useful in the present analysis, for decomposing a given deformed surface shape into its set of component functions, each of which has relatively simple interpretations. Analogies of this approach include the use of a Fourier series to characterize the frequency components of an arbitrary time signal, and the use of Zernike polynomials to characterize the aberration components of an arbitrary deformation on the surface of a conventional, disc-shaped optic. Before proceeding with discussions of the off-loading approach and its analysis, we briefly summarize the orthonormal cylinder functions, and mention three subgroups of the functions which are particularly useful in analyzing the off-loading approach.

The orthonormal cylinder functions are normalized products of Legendre polynomials in the axial direction, and sine or cosine functions in the azimuthal directions. In particular, the functions are defined as

$$G_{n0} = \sqrt{2n+1}\ P_n(z) \tag{1}$$

$$G_{nm}{}^{(C)} = \sqrt{2(2n+1)}\ P_n(z)\ \cos(m\theta) \tag{2}$$

$$G_{nm}{}^{(S)} = \sqrt{2(2n+1)}\ P_n(z)\ \sin(m\theta) \tag{3}$$

where z is a normalized axial coordinate such that z = -1 and z = +1 correspond to the endpoints of the cylinder. The three groups of functions that are most useful in characterizing surface deformation for this off-loading analysis are Garden-Hose, Roundness, and Delta-Delta-Radius. These particular functions are defined as

$$n^{th} \text{ order Garden-Hose} = G_{n1}{}^{(C)\text{-or-}(S)} \tag{4}$$

$$m^{th} \text{ order Roundness} = G_{0m}{}^{(C)\text{-or-}(S)} \tag{5}$$

$$\text{and } m^{th} \text{ order Delta-Delta-Radius} = G_{1m}{}^{(C)\text{-or-}(S)} \tag{6}$$

where, in Equation 4, n is greater than or equal to two, and where, in Equations 5 and 6, m is greater than or equal to two.

Garden-Hose errors are so named because they are characterized by an axially varying, lateral displacement of the cylinder axis, with no change in the circular shape of the cross section. (This is exactly the behavior of a garden hose as it curves along the ground.) Roundness errors are so named because they correspond to out-of-roundness in the cylinder. Delta-Delta-Radius errors are so named because they correspond to a change (or delta) in cylinder radius from end to end (thus the first delta), which varies azimuthally (thus the second delta). The usefulness of these three subgroups of functions in characterizing the residual surface deformations from the off-loading will be discussed in the Analysis section of this report.

To summarize, then, we have discussed a basic mounting approach for cylindrical optics, and discussed axial slope error as the most important parameter in characterizing the performance of an off-loading approach. Finally, we have reviewed a set of functions on the surface of a cylinder which will ease the interpretation of the residual surface deformations from the off-loading, and we have singled out three subgroups of functions which will be particularly useful for this purpose.

Development and summary of current approach

We have already discussed the approach used for HEAO-B to off-load the cylindrical optic during alignment. The reason for developing a new approach was to try to avoid the considerable complexity of the HEAO-B system, while actually reducing the residual deformations even further.

The first consideration in selecting an off-loading approach was whether the off-loading and alignment would be accomplished with the cylinder axis vertical or horizontal. There are advantages to either approach, as summarized in Table 1. The conclusion we drew from Table 1 was that the horizontal approach would be preferable, *if* a suitable off-loading approach could be devised. Naturally, since HEAO-B was successfully off-loaded horizontally (though with great difficulty and with less accuracy than required for TMA), we looked for modifications to the HEAO-B approach.

Off-loading configuration	Residual deformations	Multiple nest Considerations	Safety considerations	Facility requirements	Environmental considerations
Vertical	Minimized by symmetry of configuration	None	High risk	High vertical test chamber	Thermal stratification
Horizontal	Asymmetry of configuration requires high quality off-loading	Images from inner nests droop when off-loading is removed	Low risk*	Standard metrology area*	No stratification problem

* Critical advantage

Table 1. Comparison of vertical and horizontal off-loading configurations. Conclusion is that horizontal is preferable, _if_ a satisfactory off-loading approach exists.

The first thought on how to improve on the HEAO-B approach was to avoid using so many discrete supports by supporting the cylinder more continuously with one or more "chin straps," or bands that would run along the bottom half of a circumference of the cylinder, and then be pulled on each end from above. This led to the observation that, under such a configuration, the top half of the cylinder was entirely supported tangentially at the three o'clock and nine o'clock positions by the chin straps. This led to the conclusion that, if a chin strap left negligible deformations in the top half of the cylinder, then a simple tangential support at the three o'clock and nine o'clock positions would leave negligible deformations in the entire cylinder. This, then, formed the basis for the current off-loading approach. The optic could be supported tangentially at four discrete positions, equidistant from the centerline axially, and placed at the three o'clock and nine o'clock positions azimuthally. The final improvement on this basic approach came from thinking of the problem as being similar to supporting a ring tangentially at two points. In that case, the self-weight deflection of the ring would be as shown in Figure 5a. Note the valentine shape (or third order Roundness error, as discussed previously) of the deflection. Note in particular the circumferential slope error of the ring at the support points. We observed that the application of a moment at the support points could counteract the slope error, and therefore, to some extent, reduce the deflections throughout the ring as shown in Figure 5b.

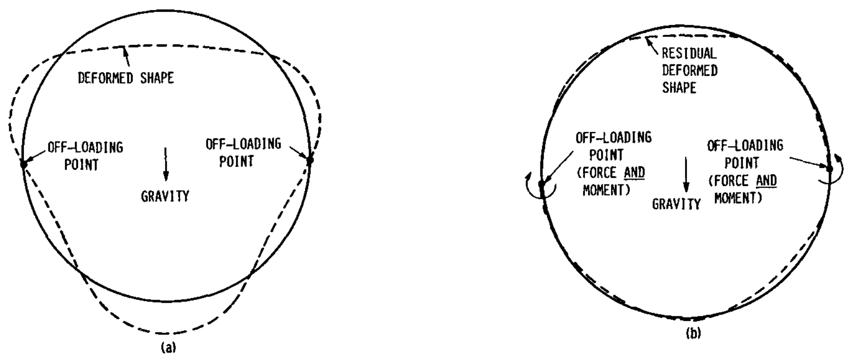

Figure 5. Qualitative comparison of cross section of cylinder deformations, with and without the application of moment. Application of moment reduces the deformations.

Figure 6, then, shows the current off-loading approach. It consists of four stubs mounted to the glass, each of which is supported at some distance out from the edge of the optic. The simple lever arm effect of the stub is what imparts the required moment. One of the major advantages of this approach is that no calibrated forces are required. If the optic is supported uniformly and kinematically at the four points, then the off-loading force at each point is defined completely by the weight of the optic. If the length of the stub is correctly chosen, then the magnitude of the applied moment is also defined completely. In short, the optic can be off-loaded with four forces and moments without the use of any means of calibration.

Analysis

In this section, we present an analysis of the current off-loading approach. We begin with some over-simplified thin-member approximations to get some insight into the nature of the residual deformations. Then, we discuss a Finite Element Method (FEM) computer analysis of the approach. We interpret the results of the FEM analysis using the cylinder functions previously discussed.

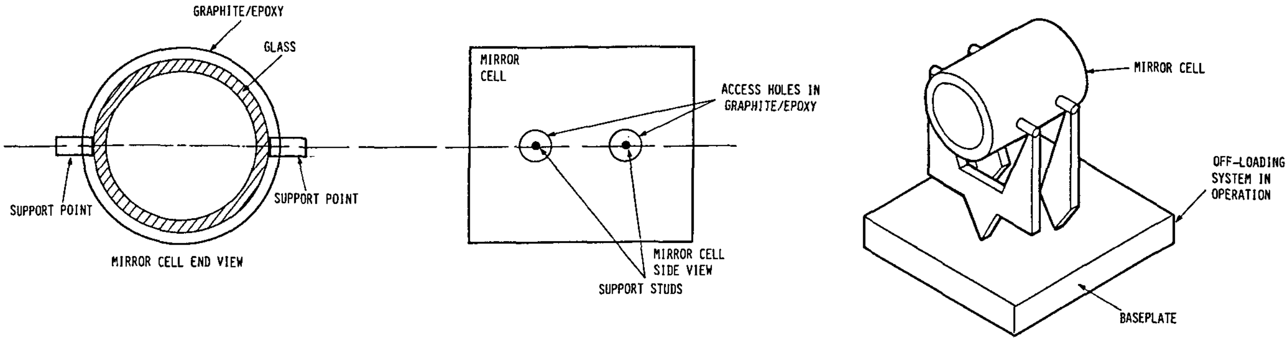

Figure 6. Conceptual schematic of horizontal off-loading approach. (Actual support of the four stubs is by a multi-point flexural support.)

The first step in gaining insight into the nature of the residual deformations was to estimate the Garden-Hose errors. This was done by considering the cylinder to be a beam supported at its midpoint. We took two approaches. First, we considered the beam to be thin, which meant that all the deformations arose from bending. Second, we considered the beam to be very thick, which meant we were examining the effect of shear strain on the deflection. Although neither of these approaches was really appropriate for a thin-walled cylinder, the approaches served to give a rough estimate of the Garden-Hose errors to be expected. The results of each approach are summarized below.

For the case of a thin beam, the deflection y as a function of distance x from the end of the cylinder can be written[2] as

$$y = (q/(24EI)) * (x^4 - 4L^3x + 3L^4) \tag{7}$$

where q is the weight per unit length, E is the modulus of elasticity, I is the cross sectional moment of inertia, and L is the half-length of the cylinder.

For the case of deflection due to shear deformations only, the deflection can be written[2] as

$$y = (qs/(2AG)) * (x^2 - 2Lx + L^2) \tag{8}$$

where s is a shape factor whose value approaches two for a thin annular cross section, A is the cross sectional area, and G is the shear modulus.

Equations 7 and 8 give a very rough estimate of the deflection of the axis of the cylinder (i.e., the Garden-Hose errors). Using the appropriate values for the TMA cylindrical optics, Equation 7 indicates a maximum axial slope error of 0.016 micro-radians, while Equation 8 indicates a maximum axial slope error of 0.034 micro-radians. Thus, the equations indicate that axial slope errors due to Garden-Hose errors are probably not too high, even for the case of the cylinder being supported at its midpoint. When the cylinder is supported away from its center, as proposed, the equations give some confidence that the axial slope errors will be acceptable.

The next step in gaining insight into the nature of the residual deformations was to estimate the Roundness errors. This was done by considering the cylinder to be a thin ring supported tangentially at the three o'clock and nine o'clock points. (Again, this approach was not really appropriate for a cylinder, but the results gave a rough idea of the deflections to be expected.) Blake[3] has tabulated several results for loaded rings, and has given formulae which can be combined to give our result as

$$u = (PR^3/(EI)) * (.6366\theta\sin\theta + 1.0063\cos\theta - .1592\theta^2\cos\theta - 1) \tag{9}$$

where u is the radial deflection, P is the half-weight of the ring, R is its radius, and θ is the angle away from the bottom (six o'clock position) of the cylinder. (In Equation 9, $0 \leq \theta \leq (\pi/2)$. To obtain results for $(\pi/2) \leq \theta \leq (\pi)$, one can use the relationship $u(\pi-\theta) = -u(\theta)$.) Using the appropriate parameters for the TMA cylindrical optics, Equation 9 predicts a peak radial deflection of 13 micro-inches. (The shape of the deflection agrees with the general shape shown previously in Figure 5a for the case of no applied moment.) Since this ring deflection corresponds to Roundness errors in the cylinder, we compared the deflection of 13 micro-inches to the fabrication error budget for Roundness, and found a good margin of safety. Again, this gave confidence that the off-loading approach would give acceptably small residual deformations.

Given the confidence that came from the rough calculations above, the next step was to analyze the residual deflections in detail. This was done by using an FEM computer model. The FEM model was generated using FEMGEN, a graphics processor for generating input data for a number of finite element codes, including MSC/NASTRAN, the program used by us. Finally, the predicted deformations were least-squares fit to the orthonormalized cylinder functions previously described by the Optical Surface Analysis Code (OSAC)[4], a multi-purpose package for analyzing the effects of surface deformations on optical performance.

First, an FEM model was generated for the entire telescope assembly, as shown in Figure 7. The figure shows all of the aperture plates and support cylinders necessary to complete the telescope assembly. This model eventually served to complete our investigation, some of which is beyond the scope of this paper. For the current analysis, the optic was stripped out for detailed investigations. The first step after the model was generated was to verify it against closed form solutions. This was accomplished by applying an internal pressure over a given width, to the cylinder at its axial centerline. It can be shown[5] that the deflection for this case is given (at the centerline) by

$$u = (qR^2/(Et))\ (1 - e^{-\beta b}\cos(\beta b)) \tag{10}$$

where u = radial deflection
q = internal pressure
t = cylinder wall thickness
b = half-width of the applied pressure
$\beta = (3(1-\nu^2)/(R^2t^2))^{1/4}$
ν = Poisson's ratio

The FEM model was subjected to this same loading condition, and the peak deflection at the cylinder axial centerline was within 2.8 percent of the closed form solution. The deflected shape of the optic was in the correct form of an exponentially damped sine wave, with the peak deflection at the axial centerline and an opposite sign radial deflection occurring at approximately the quarter points along the length. This correlation gave us a high level of confidence that the model was behaving properly. Therefore our attention was turned to optimizing the offloading positions and moments to minimize the Garden-Hose, Roundness, and Delta-Delta-Radius errors, as previously described.

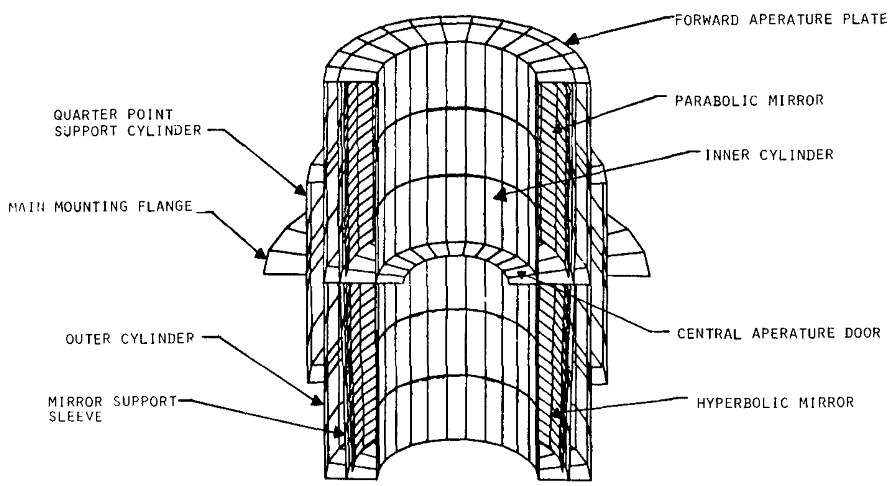

Figure 7. Full-up Finite Element Method (FEM) model of X-ray telescope.

The first step of the analysis was to minimize the Garden-Hose errors by applying the tangential forces at a series of axial locations, using no moment. The output of each FEM computer analysis run was fed into OSAC, which characterized the types of deformations which were present for each of the offloading locations. A typical deformed plot is shown in Figure 8. This shows the general shape of the optic in a one-g force field when the offloaders are utilized. It is of interest to note that the radial displacements are relatively constant as a function of axial position. In other words, the predominant error is Roundness error. (This was verified by the OSAC analysis, in that the coefficients of the Roundness functions as described in Equation 5 were significantly higher than the coefficients of the Delta-Delta-Radius functions as described in Equation 6.) The dominance of the Roundness errors is fortunate, since Roundness errors do not contribute to axial slope errors which, as discussed previously, are the most significant errors in terms of optical performance. None-the-less, these Roundness errors must be minimized, lest the deflections that are transmitted to the ring and the sleeve should spring back to deflect the optic, once the bonding is complete and the off-loading released.

Once the optimum axial locations of the off-loading points were determined by minimizing the Garden-Hose errors, the next step was to vary the applied moment to minimize the Roundness errors, and particularly the Delta-Delta-Radius errors. (Note that Delta-Delta-Radius errors can exist, even though they are asymmetric about the axial centerline of the optic. This is because the optics are not true cylinders, but have a slight amount of taper, which leads to the asymmetry.) During this optimization process, we found that we could vary the average applied moment to minimize the Roundness errors, and that we could then apply a slight differential moment (i.e., apply a slightly different moment at the forward off-loading points than at the rear off-loading points) to minimize the Delta-Delta-Radius errors. (Again, the Roundness errors had to be minimized simply to prevent excessive deformations from springing back into the optic once the bonding is complete and the off-loading removed. The Delta-Delta-Radius errors had to be minimized, because they contribute directly to axial slope errors.)

Figure 9 shows a plot of the cross section of the FEM model, once the appropriate moment and differential moment were selected. Note the expanded scale, as compared to Figure 8. In other words, the application of moment has significantly reduced the Roundness and Delta-Delta-Radius errors. In terms of axial slope, OSAC showed that the requirements were met with a large margin of safety, with RMS residual axial slope errors on the order of 0.05 micro-radians.

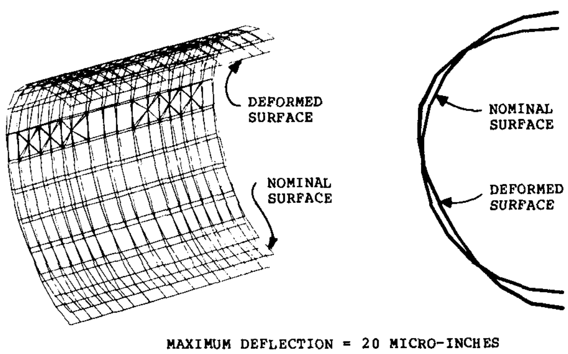

Figure 8. Isometric and end views of nominal and deformed mirror surface, using no applied moment. (Scale of deformation much larger than in Figure 9.)

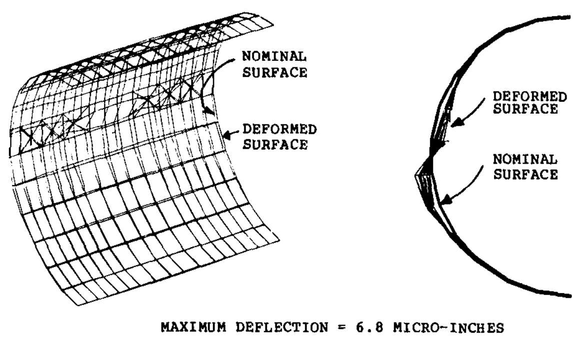

Figure 9. Isometric and end views of nominal and deformed mirror surface, using applied moment. (Scale of deformation much smaller than in Figure 8.)

Summary

In conclusion, we have discussed the need for kinematic off-loading of high quality optical elements during metrology, alignment, and system integration, and we have discussed the aspects of the off-loading problem that are unique to cylindrical optics which are used for X-ray systems. Using a set of functions that are orthonormal over the surface of a cylinder, we have characterized the residual surface deformations which are important in determining system performance. We summarized some previous approaches for minimizing these deflections, and discussed the limitations in using these approaches for a demanding system such as TMA. We then traced the development of a new off-loading approach. The approach has the advantages of being very simple (only four support points are required), being rather easily made kinematic, and requiring no calibration of any off-loading forces. Finally, we summarized the analysis of the proposed approach. The first stage of the analysis consisted of rough analytical estimates. The second stage consisted of using an FEM computer model to predict surface deformations, and then using an optical analysis software package to interpret the deflections in terms of orthonormal cylinder functions. The final result was that the requirements on residual surface deformations were met with a wide margin of safety.

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