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Optimum Design of Lightweight Mirrors

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ABSTRACT

Automated optimum design techniques based on nonlinear programming methods can be applied to the design of large, lightweight mirrors. Requirements of general purpose finite element programs with optimization capabilities, and specific design issues related to optics are presented. Examples lightweight mirror designs illustrate the benefit of optimization tools.

1. INTRODUCTION

High performance mirrors such as those used in orbiting telescopes or large, ground-based observatories require light-to-moderate weight, low stress, and small deflections under static and dynamic loads. The design approach in the past has been through parametric studies to achieve the 'best' design within the trade space studied. In this paper automated optimum design techniques based on nonlinear programming will be discussed as applied to large, lightweight mirrors.

2. OPTIMUM DESIGN OVERVIEW

Nonlinear programming techniques were first applied to structural design in 1960 [1]. Early work was limited to problems where the designer could write the analysis equations as a subroutine and embed them in a general purpose optimization program such as DOT [2]. This limitation prevented the technique from becoming a popular design tool for complex structures. When the theory became available for design sensitivity (Ref [3]) of general purpose structures through finite elements (Ref [4]), optimization gained favor quickly.

2.1 Design problem statement

Any design problem can be stated as a general nonlinear programming problem.

Minimize	$F(X)$
Subject to:	$g_j(X) \leq 0$
and	$XL_i < X_i < XU_i$
where	F = objective function
	g = inequality constraints on behavior
	X = vector of design variables
	XL, XU = lower & upper bounds on variables

If equality constraints are present, they may be treated as two inequality constraints.

$$h_j = 0 \;\Rightarrow\; g_j \leq 0 \text{ and } g_{j+1} \geq 0$$

Note that functions F and g are nonlinear functions of X. In a finite element code, constraints on displacement and stress are found numerically (not analytically). A constraint on displacement written as:

$$\delta \leq \delta_U$$

where δ_U is an upper limit on displacement, can be converted to the general form as:

$$g = (\delta - \delta_U) / \delta_U \leq 0$$

2.2 Design sensitivity

Nonlinear programming methods are iterative in nature, moving from one design to a better design. An efficient optimization code requires first derivatives of the responses to determine a proper move direction in the design space. Finite difference operations are too time consuming for most applications. The efficient alternative is the use of implicit derivatives for design sensitivity of constraints with respect to variables (Ref [3]). In a static analysis, the system equation

$$[K]\,\{\delta\} = \{f\}$$

is varied by implicit derivatives

$$[K]\,\{d\delta/dX\} + [dK/dX]\,\{\delta\} = \{df/dX\}$$

To find the response derivative, an additional 'load case' is applied to the system equation, where the right-hand load terms are easily calculated.

$$[K]\,\{d\delta/dX\} = \{df/dX\} - [dK/dX]\,\{\delta\}$$

Note that additional load vectors are just another column of multiplication, verses the alternative of of new decompositions of the stiffness matrix as required by a finite difference approach to the response derivatives. Finite element programs which provide these sensitivities internal to the code are efficient in a general design optimization program.

2.3 Design variables

Almost all FEA programs which offer design optimization offer <u>sizing</u> variables which include beam cross-sectional properties and plate thicknesses. These variables effect the 'property' cards (pbar or pshell), but not grid locations. A more general capability would include <u>shape</u> variables which change grid point locations. In a continuum structure such as a mirror, individual grid points should not normally be independent variables, but rather overall shape parameters are the variables. Shape optimization can be approached with a variety of techniques, but two methods are prevalent:

i) Basis Vector Technique
ii) Automesh Technique

In the basis vector method, a valid mesh of the nominal structure is created. This mesh is perturbed in various directions which represent candidate designs. The grid and element numbering are unchanged in each candidate vector. The optimizer then finds the scale factors for linear combination of all candidates which yields the 'best' design. This is a highly efficient, but is limited in the amount of variation possible before a remesh is required.

The automesh technique allows a greater amount of variation in design because an automatic remesh is redone at every design step. However, an automatic mesh requires a good error evaluation technique which tests the accuracy of the automesh and modifies the mesh for sufficient accuracy. This extra iteration loop, combined with the automeshing algorithm, can be quite time consuming when buried inside a shape optimization loop. Another bothersome feature of automeshing is that symmetric response is not maintained for symmetric structures such as optics. Any level of asymmetric response for symmetric checkout loads usually signals a modeling error.

2.4 Design constraints

Optical systems must survive and operate in a variety of environments. For example, during transportation and handling, the stresses must be less than the allowable stress, during launch the natural frequency must be greater than a minimum value, and during operation the surface deformations must be less than an allowable value. A design approach which optimizes for static stress by providing a soft mount will often violate dynamic response with low natural frequencies. To obtain a truly optimum mirror, both the static and dynamic constraints must be considered simultaneously. If the finite element code is to be useful, it must have the combined analysis capability. In fact, a very desirable feature is to include frequency response, transient responses and buckling as simultaneous analysis and constraint options.

Since the optical surface performance is often difficult to relate to raw finite element displacements, some user function capability is required. If a mirror which has a large tilt, but the surface remains perfectly smooth, the results will show large finite element displacements. However, if the optical system has pointing capability, the smooth surface will perform satisfactorily. What is needed is the ability to find relative motions by writing responses as equations, or by letting the user include subroutines to calculate response functions.

2.5 Algorithms

Many iterative algorithms have been created for the solution of general nonlinear programming problems. In the DOT optimizer (Ref [2]), the method of modified feasible directions and the method of sequential linear programming are chosen for their efficiency and robustness. The key issue when combined with a finite element program is efficiency, especially as related to the number of full FE analyses required per design optimization. In order to reduce the number of full FE analyses, the best procedure is to create an approximate problem [3] which is a first order Taylor series expansion of the design responses:

$$R(X) = R(X_0) + dR/dX\ (X - X_0)$$

where R is any response quantity, X_0 is the current design, and dR/dX is found from design sensitivity. This approximate problem is optimized to get a new design. A full FE analysis is then run on the new design, along with design sensitivity, to create a second approximate problem. At each cycle, the constraints are checked, sorted, and inactive ones temporarily dropped. Using this approximation technique, the design examples given in section 4 required 5 to 10 full FE analyses to reach an optimum design.

3. LIGHTWEIGHT MIRROR DESIGN ISSUES

3.1 Mirror design parameters

In a solid mirror, the only structural design variable is the thickness. Conventional lightweight mirrors can be described structurally by a few parameters as shown in Figure 1 where

H = overall height
Tp = faceplate thickness
Tc = cell wall thickness
B = effective cell spacing

For a mirror with a regular core spacing, whether square, triangular, or hexagonal in cross-section, the key parameter is the effective cell spacing (B) which is the diameter of the inscribed circle. The core density ratio (a) and core height (Hc) are defined by

$$a = Tc / B \qquad Hc = H - 2*Tp$$

The effective 2d plate properties for such a mirror are given by (Ref [6])

$$Tm = 2*Tp + a*Hc = \text{membrane thickness}$$

$$Ib = [H^3 - (1-a)*Hc^3] / 12 = \text{bending inertia}$$

$$Ts = (2/3)*a*[H^3 - (1-a)*Hc^3] / [H^2 - (1-a)*Hc^2] = \text{shear thickness}$$

$$\rho' = \rho * [2*Tp + Hc*(2a-a2)] / Tm = \text{effective density}$$

Thus a conventional mirror's structural behavior can be described to first order accuracy by the four design parameters. These terms can be placed in standard plate equations to determine stress, displacements, and dynamic behavior.

3.2 Fabrication issues

Previous fabrication and assembly techniques limited the mirror to regular, uniform spacing, with uniform wall thickness. Therefore mirror cores were restricted to square, triangular, or hexagonal cells of constant size (B) and constant wall thickness (Tc). Recent advances in waterjet cutting has allowed a very general core structure to be a possibility [7]. Now the core can be created with irregular geometry (spacing, shape, and thickness) over the whole mirror. This new freedom adds extensive design freedom as seen in from the mirror design in Section 4.2.

3.3 Processing issues

In the past, mirrors were polished to a high figure by polishing laps rubbing on the surface. The pressure forced the center of the cell to deflect relative to the cell edge, causing a non-uniform pressure with associated non-uniform material removal. The core print-through effect on the finished surface was labeled quilting. The cell spacing (B) was determined by the polishing quilting displacement (q) which is a function of cell geometry and faceplate thickness.

$$q = \text{function}(B^4 / Tp^3)$$

New procedures using ion figuring [7], can place a finished surface of very high quality on a mirror without the use of surface pressure. This allows greater freedom in core geometry with larger cell sizes.

3.4 Design equations

In most applications, the mirror diameter and curvature are specified by the optical requirements. The structural design of conventional uniform lightweight mirrors could be described by 4 design variables: Tp, Tc, B, H, as given by the equations in Section 3.1. Since the usual goal is the lightest weight mirror which satisfied all performance criteria, the design problem could be stated:

Find the design = X ={Tp,Tc,B,H}
which will minimize W = weight
subject to:

- $q <$ quilting limit (polishing)
- $\sigma <$ stress limit (handling, transportation, launch)
- $d_{pv} <$ peak-to-valley displacement limit (test,use)
- $d_{rms} <$ rms displacement limit (test, use)
- $f_n >$ natural frequency limit (transportation, launch)

In the conventional design approach for uniform mirrors, the starting design was often obtained by using the thinnest cell walls obtainable. The face plates were picked based on experience with handling and fragility. The largest regular cell size was picked based on quilting. Originally all cells were square, but were changed to triangular as fabrication methods allowed. Then hand analysis predicted the frequency. First order stress and displacements could be estimated by hand using plate equations with effective properties, but usually required a finite element analysis. Based on the analysis results, experience, and intuition, the design was altered until all requirements were met. There was no test to show that the resulting design was near an optimum. Usually only 1 or 2 constraints were active; but with only 4 variables, the final design was considered adequate

As the use of finite elements became more prevalent, the design process was tightly coupled to the computer. Quick analyses could be accomplished using 2d plate models (Figure 2) with the effective properties in Section 3.1. Changes in the primary design variables modified only the property cards (pshell), not the model geometry (grid points). The 2d model's limited accuracy eventually requires the jump to a 3d shell model.

A 3d plate shell model of the mirror would use the geometry directly. Grid points would be located at all core/plate and core/core intersections with additional grid points as desired for modeling accuracy. Changes in Tp and Tc involve only the respective pshell cards, but changes in B and H involve moving grid points, a very time consuming process.

4. OPTIMUM DESIGN OF MIRRORS

Two general approaches to design optimization of lightweight mirrors are possible with todays capabilities in finite elements. Depending on the mirror complexity and the program's capability, either sizing or shape design may be used.

4.1 Sizing optimization of mirrors

Sizing optimization is limited to changes in effective plate thickness (pshell entries). Thus any 3d model can use as many independent design variables as desired to change core thickness or faceplate thickness. Mirror height (H) and cell size (B) cannot be changed.

If a mirror is regular enough that a 2d equivalent stiffness plate model can be used accurately, the the equations in Section 3.1 show that mirror height (H) and cell spacing (B) can be treated as sizing variables. In the following example, a 2d equivalent stiffness mirror model is shown in Figure 2. This represents a conventional lightweight mirror with uniform core structure. The design problem can be summarized as

Design Variables:
- Tp = faceplate thickness (front & back are same)
- Tc = core cell wall thickness (uniform over full mirror)
- H = mirror overall thickness
- B = effective cell spacing

Objective Function:
- Min Wt = minimize total weight on mirror

Design Constraints:
- $d < d^*$ => max sag under 1g less than allowable
- $f > f^*$ => 1st natural frequency greater than min limit

The data deck for MSC/NASTRAN using sizing optimization in SOL 200 version 67 is given in the Figure 3.

For highly irregular geometry of the core, a 2d equivalent stiffness 'plate' model is very difficult to create and has questionable accuracy. For these irregular mirrors a 3d shell model is required for design/analysis.

4.2 Shape optimization of mirrors

A more general, and more accurate capability for design optimization is the combination of sizing and shape optimization. With this capability, a full 3d shell model of the mirror is used. Design variables include faceplate and individual core wall thicknesses as sizing variables, and cell strut intersections and mount locations as shape variables. See the following example.

An elliptic mirror (27" x 14") is to be mounted at 3 points on the back surface with gravity acting normal to the face. Due to space requirements, the mirror thickness is limited to 2". The design problem can be summarized as

Objective Function:
- Min Wt = minimize total weight on mirror

Design Constraints:
- $d_{pv} < 4\ \mu$-in => max peak-valley under 1g less than allowable

As a reference, 4 design solutions are presented.

1) A regular square core mirror with a hexagonal outline
2) A solid elliptic mirror
3) An unconventional lightweight mirror resulting from parametric studies
4) An unconventional mirror using optimization techniques

The square core mirror was presented as a design option by an unknown source, so the amount of design effort is unknown. The solid mirror represented the cheapest solution from both a fabrication and a design effort measure. The 'parametric' mirror was the 'best' design available after a large amount of design effort from experienced engineers supported by several finite element analyses. The optimized mirror was the result of 2 trial runs with the new GENESIS program combining sizing and shape optimization. (Key portions of the data deck for VMA/GENESIS is given in Figure 4). A plot of each of the finite element models appear in Figure 5. Symmetric half models were used for efficiency. Comparing the unconventional designs, the optimizer moved the mount locations (shape variables) and changed many of the core strut thicknesses (size variables). It was the combination of these changes that was successful, and could not be found from the parametric studies. A summary of the resulting designs is:

Design	Displacement (μ-in)	Weight (Lb)
Square Core	8.8	31.6
Solid	8.0	53.6
Parametric	6.0	38.1
Optimized	3.8	30.1

The significant result is that the optimized design was the only design to meet the design requirement, but it did so at the lightest weight. From a design cost viewpoint, the parametric design required about 3 times as much labor and cpu time as the optimized design. The conclusion to be drawn is that optimization techniques can produce better designs with less effort.

Another conclusion that can be drawn from the above study is that the new design freedom available from new fabrication (waterjet cutting) and processing (ion figuring) techniques have provided more design variables than an experienced design engineer can handle. Only automated optimization techniques can utilize the many new variables successfully.

5. ON-GOING WORK

Both GENESIS and NASTRAN will continue to be used for the optimization of mirrors. Both programs have some unique features that the other lacks. Currently MSC/NASTRAN version 67 is limited to sizing optimization, but includes a wider variety of elements, allows superelements, and includes buckling constraints. Version 68 of MSC/NASTRAN will incorporate shape variables, frequency response and transient response constraints, and variables which span superelements. The current version of VMA/GENESIS has both sizing and shape variables, a family of convenient beam and plate cross-sections, and allows a user supplied subroutine. Upcoming versions will soon have frequency response and transient response constraints.

The user supplied subroutine feature in GENESIS will allow the use of customized optical surface post-processing programs to control the response of any Zernike aberration term, the rms surface, or the peak-to-valley response. This subroutine will be incorporated in GENESIS soon.

6. CONCLUSIONS

With the new design freedom provided by advances in waterjet cutting and ion figuring, highly irregular mirrors are possible. Figure 6 shows an actual prototype elliptic mirror produced with the unconventional core. This was a parametric design of different size (14" x 20") than discussed in Section 4.2. These new mirrors, which challenge intuition, cannot be designed using conventional 2d plate equations. Optimum design techniques using shape variables provide a useful tool for achieving creative, highly efficient, lightweight mirrors.

Since these lightweight mirrors must survive a variety of handling, transportation, launch, and in-use load conditions, all effects must considered in the design process. A general design capability embedded in a finite element program must include the following tools as a minimum:

a) Sizing and shape variables
b) Static analysis with multiple load and boundary conditions
    and constraints on displacements and stress
c) Natural frequency analysis with constraints on frequency

Additional tools which are highly desirable include

d) Frequency response with constraints on displacement and stress
e) Transient response with constraints on displacement and stress
g) Buckling analysis with constraints on critical load
f) User defined equations for response functions
g) User defines subroutines/programs for response functions

The above analyses must be available as simultaneous solutions, so that the design is not optimized to static loads alone, then separately to natural frequency constraints. The design algorithm must work on all design constraints simultaneously.

For lightweight mirrors, the basis vector approach to design variables is efficient and sufficiently general for most mirror designs. The use of automesh is not a viable tool unless there is also an error estimator to revise the mesh for sufficient accuracy. This automesh capability allows wider design variation within a given run, but is more time consuming than the basis vector approach.

7. REFERENCES

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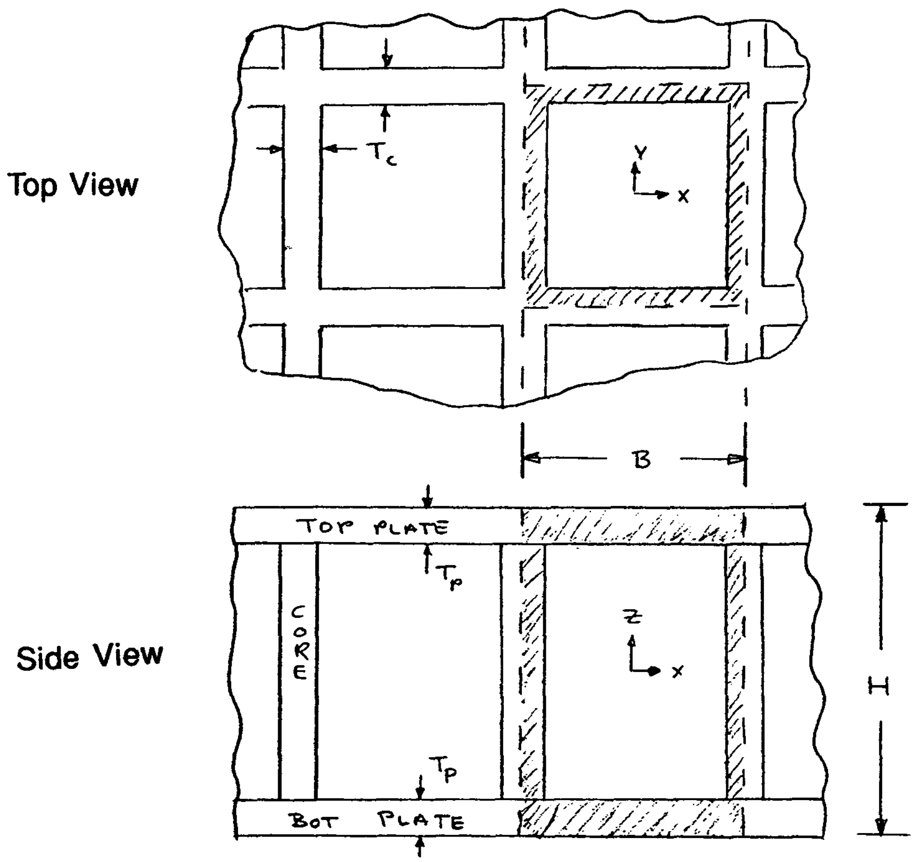

Figure 1: Lightweight Mirror with Square Core

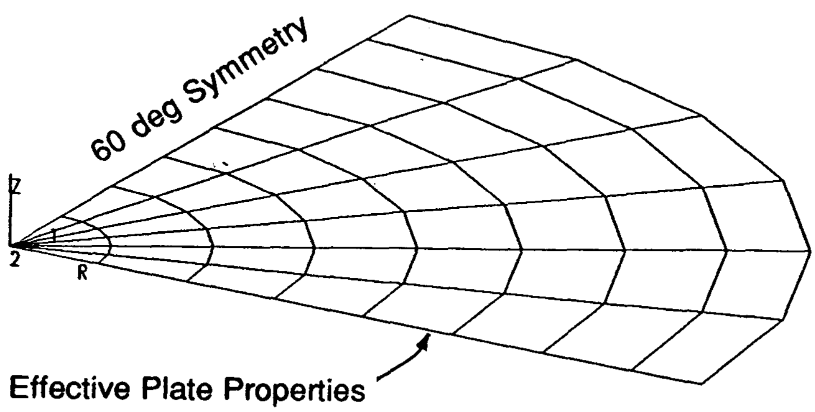

Figure 2: 2D Equivalent Stiffness Mirror

```
SOL 200
CEND
TITLE= OPTIMIZE LW MIRROR / RIGIDITY,FREQ
SUBTITLE= 2D EQUIVALENT STIFFNESS
SUBCASE 1
LABEL= 1G SAG
PARAM,APPC,STATICS
LOAD=100
SUBCASE 2
LABEL=NATURAL FREQ
PARAM,APPC,MODES
METHOD=99
BEGIN BULK
$ DESIGN VARIABLES ARE MIRROR PARAMETERS
DESVAR,1,H,12.0,6.0,15.0
DESVAR,2,B,7.0,6.5,7.5
DESVAR,3,TC,0.06,0.01,0.3
DESVAR,4,TP,0.175,0.03,0.6
$ RELATE DES VAR TO ANALYSIS VAR
DVPREL2,11,PSHELL,100,4,.06,101
,DESVAR,1,2,3,4
DVPREL2,12,PSHELL,100,6,500.0,102
,DESVAR,1,2,3,4
DVPREL2,13,PSHELL,100,8,.05,103
,DESVAR,1,2,3,4
DVPREL2,14,PSHELL,100,9,1.-7,104
,DESVAR,1,2,3,4,5
$ EQUIV 2D SECTION PROPERTIES
DEQATN  101     A(H,B,TC,TP) = TC/B;
+1A             HC = H-2.0*TP;
+1B             TM = 2.0*TP+A*HC
DEQATN  102     A(H,B,TC,TP) = TC/B;
+2A             HC = H-2.0*TP;
+2B             TM = 2.0*TP+A*HC;
+2C             RI = (H**3-(1.0-A)*HC**3)/12.0;
+2D             RB = 12.0*RI/TM**3
DEQATN  103     A(H,B,TC,TP) = TC/B;
+3A             HC = H-2.0*TP;
+3B             TM = 2.0*TP+A*HC;
+3C             RI = (H**3-(1.0-A)*HC**3)/12.0;
+3D             S  = (H**2-(1.0-A)*HC**2)/A;
+3E             RS = 8.0*RI/(S*TM)
DEQATN  104     A(H,B,TC,TP,HB) = TC/B;
+4A             HC = H-2.0*TP;
+4B             RM = HC*(A-A*A)*2.047E-04+.0214*HB*2.047E-04
$ OBJECTIVE FUNCTION = MINIMIZE WEIGHT
DRESP1,500,WT,WEIGHT
DESOBJ,500,WT,,MIN
$ DISPLACEMENT CONSTRAINTS (SAG < .002)
DRESP1,151,SAG,DISP,,,3,,101
DCONSTR,151,ALL,-.002,.002
$ NATURAL FREQ CONSTRAINT (FREQ > 100 HZ)
DRESP1,191,F1,EIGN,,,1
DCONSTR,191,ALL,394800.
...  REMAINDER OF MODEL (GRIDS,ELEMENTS,ETC)
ENDDATA
```

Figure 3: MSC/NASTRAN Sizing Optimization Deck

```
ID
CEND
TITLE = Optimum Mirror Design - Elliptic on 3 pt
LOADCASE 1
        LABEL = 1G -Z BASIC
        GRAV = 1
        DISP = ALL
BEGIN BULK
$ OBJECTIVE = MIN WEIGHT
DRESP1  1       WT      MASS
DOBJ    1       WEIGHT          MIN
$ CONSTRAINT ON 1G DISPLACEMENT (4 MICRO-IN)
DRESP1  2       D12     DISP                            3       12
DCONS   2       1       -4.-6   4.-6
$ DSEIGN VAR = CORE THICKNESSES
DVAR    4       P4      .10     .01     .5
DVPROP3 4       4       SOLID
+       4
PSHELL  4       1       .1      1
$ DESIGN VAR = MOUNT LOCATION (RADIAL POS OF GRID 4 + OTHERS)
DVAR    11      M1-RAD  .1      -1.     1.
...
DVGRID  11      40      1       6.8     60.     0.
...
$ DESIGN VAR = MOUNT LOCATION (CIRCUMF POS OF GRID 4 + OTHERS)
DVAR    12      M1-CIR  .1      -1.     1.
...
DVGRID  12      40      1       6.7     61.     0.
...
$ REMAINDER OF MODEL (ALL GRIDS,ELEMENTS,MATERIALS,BC,ETC)
GRID    40      1       6.7     60.     0.      1
ENDDATA
```

Figure 4: VMA/GENESIS Shape Optimization Deck

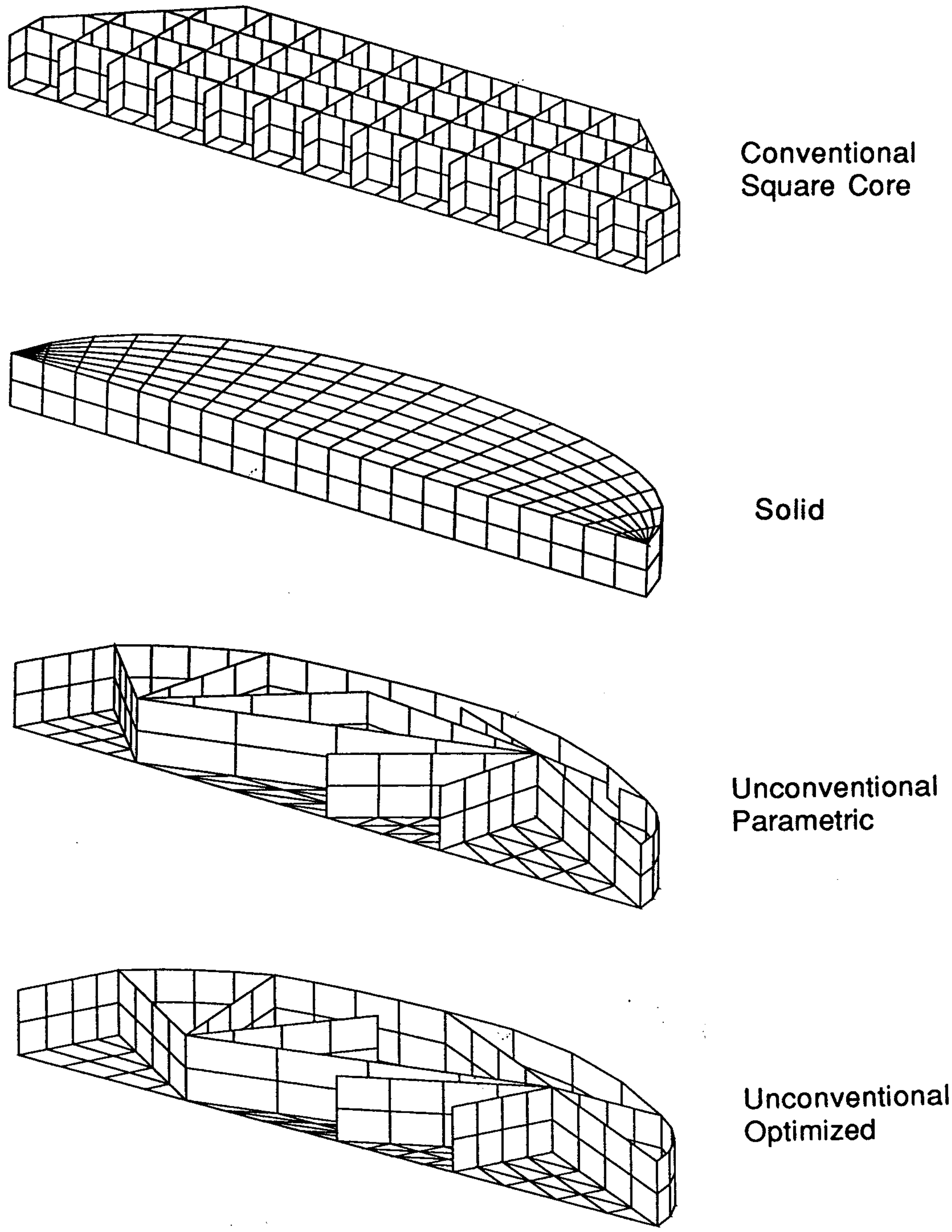

Figure 5: Elliptic Mirror Designs (Top Plates Removed)

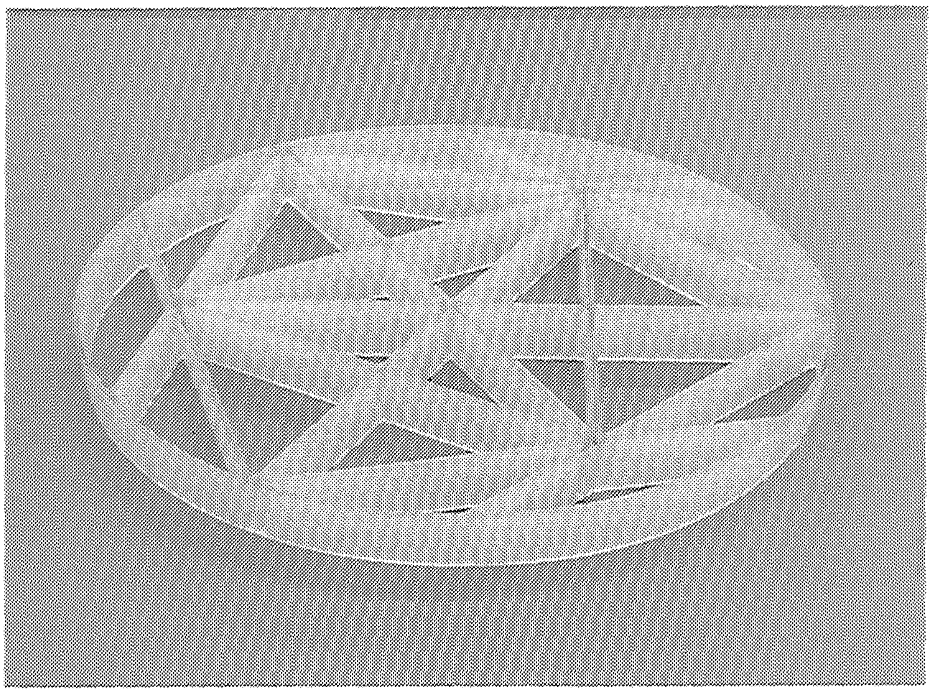

Figure 6: Unconventional Waterjet-cut Core for Elliptic Mirror