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Optical surface evaluation

Victor L. Genberg

Eastman Kodak Company
Rochester, NY 14650

Abstract

The finite element analysis of mirrors is now common practice. The results of such an analysis is the vector displacement of a great number of grid points within the mirror. Evaluation and interpretation of the raw data is difficult, even when represented as contour plots. It is more convenient from an optical engineer's viewpoint to describe the total deformation in terms of its components: tilt, defocus, and common aberrations. A NASTRAN post-processing program is described which performs a least squares fit of Zernike polynomials to a deformed surface.

Zernike polynomials

Over the past few years, the Zernike polynomial has had increased use in optics in representing the wavefront of optical systems. Although there are many different polynomials that could be used to represent a wavefront, the Zernike polynomial possesses two very desirable features: a) it forms an orthogonal set over an unobstructed unit circle and b) the lower order polynomial coefficients correspond to the Seidel aberrations.

The Zernike polynomials can be represented in the following forms:

(i) cosines (A) and sines (B)

$$Z(r,\theta) = A_{00} + \sum_{\substack{n=2 \\ n \text{ even}}}^{K} A_{n0} R_n^0(r) + \sum_{\substack{n=1 \\ (n-m)}}^{N} \sum_{\substack{m=1 \\ \text{even}}}^{n} \{A_{nm}\cos(m\theta) + B_{nm}\sin(m\theta\} R_n^m(r), \qquad (1)$$

where: $R_n^m(r)\cos(m\ )$ and $R_n^m(r)\sin(m\ )$ are the Zernike circle polynomials, the radially symmetric factors of which are given by:

$$R_n^m(r) = \sum_{k=0}^{\frac{1}{2}(n-m)} (-1)^k \frac{(n-k)!}{k!(\frac{1}{2}(n+m)-k)!\ (\frac{1}{2}(n-m)-k)!} r^{(n-2K)}. \qquad (2)$$

(ii) magnitude (c) and orientation (ϕ)

$$Z(r,\theta) = A_{00} + \sum_{\substack{n=2 \\ n \text{ even}}}^{K} A_{n0} R_n^0(r) + \sum_{\substack{n=1 \\ (n-m)}}^{N} \sum_{\substack{m=1 \\ \text{even}}}^{n} C_{nm}\cos(m(\theta-\phi_{nm}))\ R_n^m(r),$$

where: (3)

$$C_{nm}^2 = A_{nm}^2 + B_{nm}^2 \text{ and } \phi_{nm} = \frac{1}{m} \arctan\left(\frac{B_{nm}}{A_{nm}}\right).$$

(iii) x - y polynomials

$Z(X,Y) = A_1 +$	BIAS
$B_2Y + A_3X +$	TILT
$A_4(2r^2-1) +$	DEFOCUS

$2XYB_5 + A_6(X^2-Y^2) +$	PRIMARY ASTIGMATISM
$(3r^2-2)\ (B_7Y + A_8X) +$	PRIMARY COMA
$A_9(6r^4-6r^2+1) +$	PRIMARY SPHERICAL
$A_{10}(20r^6-30r^4+12r^2-1) +$	SECONDARY SPHERICAL
$(10r^4-12r^2+3)\ (A_{11}X+B_{12}Y) +$	SECONDARY COMA
$(4r^2-3)(A_{13}(X^2-y^2) + 2XYB_{14}) +$	SECONDARY ASTIGMATISM
$A_{15}X(X^2-3Y^2) + B_{16}Y\ (3X^2-Y^2) +$	TREFOIL
$A_{17}X(5r^2-4)(X^2-3Y^2)+B_{18}Y(5r^2-4)(3X^2-Y^2) +$	SECONDARY TREFOIL
$A_{22}(X^4-6X^2Y^2+Y^4)+4XYB_{23}(X^2-Y^2) +$	TETRAFOIL
$A_{19}(70r^8-140r^6+90r^4-20r^2+1) +$	TERTIARY SPHERICAL
$(35r^6-60r^4+30r^2-4)(A_{20}X+B_{21}Y) +$	TERTIARY COMA
$(15r^4-20r^2+6)(A_{24}(X^2-Y^2)+2XYB_{25})$	TERTIARY ASTIGMATISM
$+A_{26}(25r^{10}-630r^8+560r^6-210r^4+30r^2-1)$	QUATERNARY SPHERICAL

Where $r^2 = X^2 + Y^2$ (4)

These terms can be classified as:
(a) rigid body motion:
bias, x-tilt, y-tilt
(b) defocus or power change
(c) aberrations:
spherical, coma, astigmatism, trefoil, etc.
Examples of some of these terms are shown in Figure 1.

If the deflections are steady state then x-tilt and y-tilt can be corrected by realignment whereas bias and power change can be corrected by refocusing. The aberrations on the other hand cause degradation of the image.

Computer programs exist which fit polynomials over a regular array of data points. In typical finite element models, the grid points are not uniformly spaced throughout the mirror. In lightweight mirrors for example, it is common to have more densely spaced grid points near the boundaries to account for edge rings and core ends. (See Figure 2). Thus, the grid points should not be weighted equally in a least squares fit. A logical choice for a weighting factor is the optical surface area associated with each grid. A finite element program such as NASTRAN will determine the weighting factor by applying a uniform pressure to the optical surface and using the resulting load vector as a measure of area per grid point.

$$P_i^e = \int pN_i^e dA$$

$$p = 1 \tag{5}$$

$$P_i^e = \int N_i^e dA = A_i$$

For a 3 node triangle, A_i is 1/3 of the triangle's surface area. Upon matrix assembly, the system load vector will contain the amount of area associated with each grid point from all connected elements.

Let U(x,y) be the finite element displacement vector and Z(x,y) be the approximating Zernike polynomial. A weighted least squares error E can be represented as:

$$E = \sum_{i=1}^{n} W_i [U_i - Z_i]^2 \tag{6}$$

where n is the number of grid points and $W_i = P_i$ = grid point weight. If Z is written symbolically as

$$Z = \sum_{j=1}^{m} c_j \phi_j$$

then

$$Z_i = \sum_{j=1}^{m} c_j \phi_j (x_i, y_i) = \sum_{j=1}^{m} c_j \phi_j^i \qquad (7)$$

where c_j are undeterminded coefficients of the m terms in the Zernike polynomial.

$$E = \sum_i W_i [U_i - c_j \phi_j^i]^2 \qquad (8)$$

Minimizing the error with respect to the coefficients

$$\frac{\partial E}{\partial c_k} = 2\sum_i W_i [U_i - \sum_j c_j \phi_j^i] \phi_k^i = 2\sum_i w_i U_i \phi_k^i - 2\sum_{ij}\sum W_i c_j \phi_j^i \phi_k^i = 0 \qquad (9)$$

In matrix form

$$[H]\ \{C\} = \{F\}$$

where

$$F_k = 2\sum_i W_i U_i \phi_k^i \qquad (10)$$

$$H_{jk} = 2\sum_i W_i \phi_j^i \phi_k^i$$

This matrix can be solved by Gauss elimination to find the best fit coefficients C_j. The choice of which terms to include in the polynomial fit should be left to the user for maximum efficiency and flexibility.

NASTRAN post-processor

A NASTRAN post-processing program called NASTRACT was written to fit Zernike polynomials to the deformed optical surface. The following data files were passed from NASTRAN to NASTRACT.

(i) BGPDT = a table of all grid point locations in a common basic rectangular coordinate system, eliminating the need for coordinate transformations.
(ii) GPL = a table listing the external grid number versus the internal sort location.
(iii) PG = the system load vector, of which one column contains the area weighting factors for the optical surface grids.
(iv) UGV = the system displacement vector for all grid points in all sectors under all load conditions.

When reading the NASTRAN output files, NASTRACT must account for null vector, output precision, packing, scalar points, and data record lengths.

Not all structural grid points in the model lie in the optical surface. In a lightweight mirror for example, all grids on the back plate do not enter into the wavefront calculation. Data storage and the number of calculations can be greatly reduced by immediately discarding those points which do not represent image motion. This is accomplished by sorting on grid point identification numbers and grid point location. The list is then further reduced by eliminating points which have a weighting factor of zero, or which fall outside the optical window or inside an optical obstruction.

Symmetry, inherent in most mirrors, is often utilized to reduce the cost of a finite element analysis. For example, a circular mirror with a rectangular core arrangement resting on a ring support can be modelled as a 45° segment. If the load is axisymmetric, conventional symmetry conditions can be used in which a single displacement vector can be repeated for the other 7 segments. If an asymmetric load is applied, the NASTRAN cyclic symmetry is used which provides 8 unique displacement vectors to totally describe the mirror motion. The structural symmetry may or may not be the same as the optical symmetry represented by the windows and obstructions. In general it is necessary to generate the complete optical surface from the response of a symmetric sector before the surface fit.

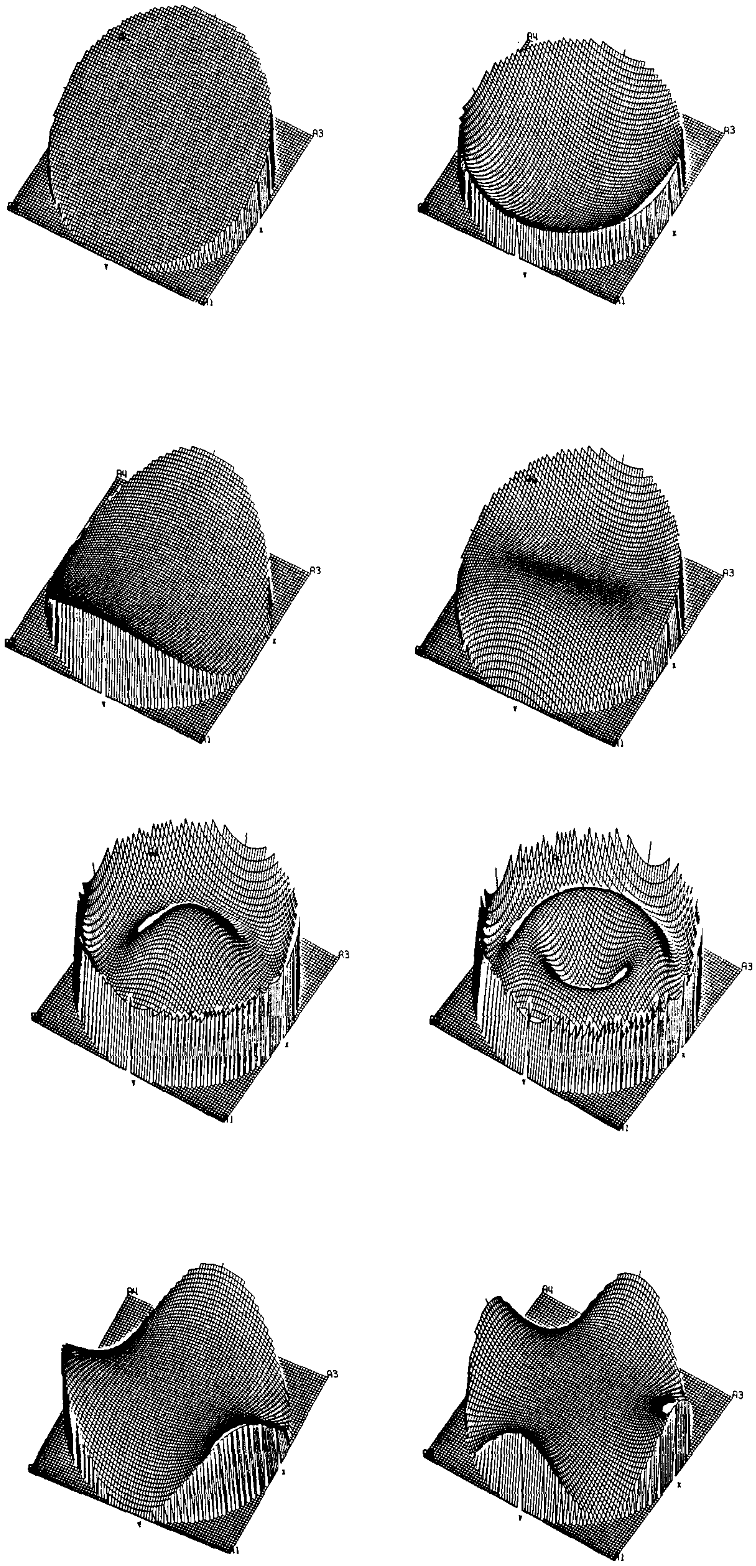
NASTRACT calculates a full surface from a segment model using either conventional or cyclic symmetry (dihedral or rotational). On the other hand, if the windows and obstructions offer the same symmetry as the structure, some computer cost savings can be realized by fitting only a selected subset of aberrations to a segment model. For example, a solid, axisymmetric mirror with axisymmetric loads and boundary conditions can be represented as a radial model with only bias, power and spherical aberrations. A 360° surface need not be generated nor analyzed. The type of symmetry and the choice of aberrations in the post processor is under user control.

Example

A simple circular flat mirror sitting on a 3 point support subjected to a gravitational load was analyzed. The finite element model is shown in Figure 3. Deformed shapes and contour plots are shown in Figure 4. When sitting on a real support, the support will move, adding rigid body motion to the displacement state. Motion of one support point is shown in Figure 5. Evaluation of the raw data is difficult even with contour plots.

A NASTRACT surface fit was performed on both cases with the results shown in Figure 6. From the tables it is obvious that there is a great deal of tilt in the second case. However the remaining terms are unchanged. From this table one can deduce that realignment will correct the tilt, and refocusing will correct the bias and power change (defocus). The RMS remaining after bias, tilt, and power have been removed will degrade the image. The magnitude and orientation of the aberration terms may offer the engineer insight into the cause of resulting image degradation.

The NASTRACT program can display graphically the individual aberration terms or the resulting wavefront after each term is removed. This is available as density maps (Figure 7) or contour plots.



Tilt

Defocus

Primary Astigmatism

Primary Coma

Primary Spherical

Secondary Spherical

Primary Trefoil

Primary Tetrafoil

Figure 1 Aberrations

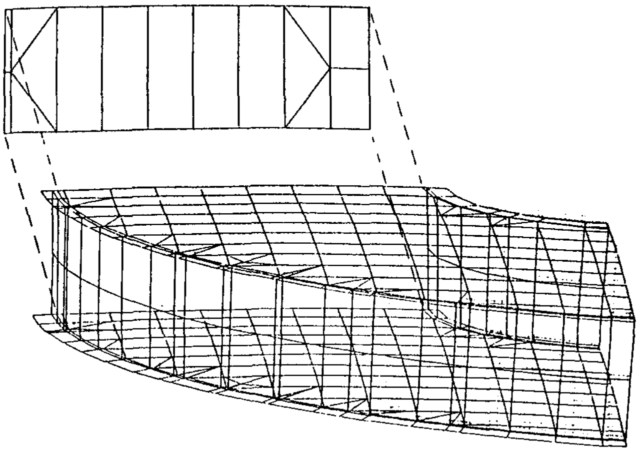


Figure 2 Lightweight Mirror

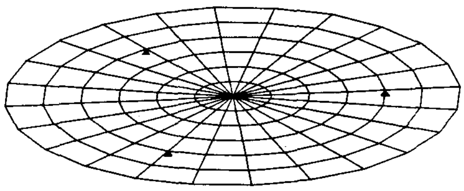


Figure 3 Flat Mirror

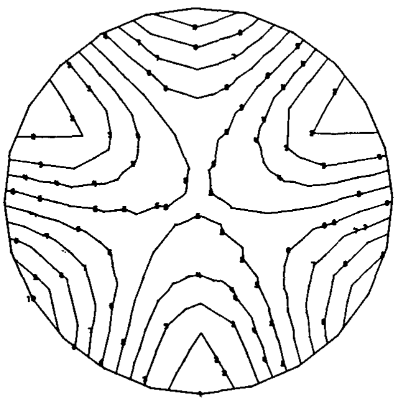


Figure 4

Contours with no tilt

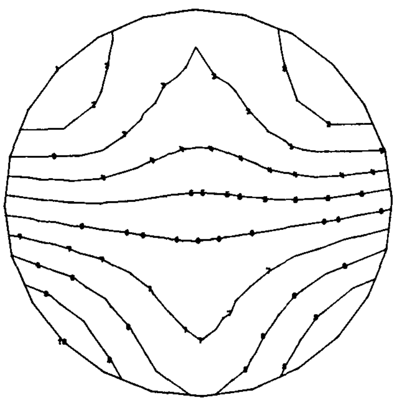


Figure 5

Countours with tilt

ZERNIKE ABERRATION ANALYSIS

Figure 6

With no tilt

ABERRATION	FUNCTIONAL (P-V)* (WAVES)	ANGLE (DEGREES)	RESIDUAL RMS** (WAVES)	RESIDUAL (P-V)** (WAVES)***
INPUT ARRAY			2.4045	10.7295
BIAS			2.4045	10.7295
Y TILT			2.4045	10.7295
X TILT			2.4045	10.7295
POWER	1.1299	0.0	2.3791	10.7295
PRIMARY SPHERICAL	0.7888	0.0	2.3684	10.7295
PRIMARY COMA	0.0000	72.6	2.3684	10.7295
PRIMARY ASTIGMTISM	0.0000	17.6	2.3684	10.7295
PRIMARY TREFOIL	12.8946	-30.0	0.5237	2.2246
PRIMARY TETRAFOIL	0.0000	-19.0	0.5237	2.2246
PRIMARY PENTAFOIL	0.0000	32.9	0.5237	2.2246
PRIMARY HEXAFOIL	0.4759	-0.0	0.5174	2.1651
SECONDARY SPHERICAL	-0.4202	0.0	0.5040	2.2612
SECONDARY COMA	0.0000	161.2	0.5040	2.2612
SECONDARY ASTIGMATISM	0.0000	-79.9	0.5040	2.2612
SECONDARY TREFOIL	3.0394	30.0	0.1702	0.8743
SECONDARY TETRAFOIL	0.0000	25.1	0.1702	0.8743
SECONDARY PENTAFOIL	0.0000	15.0	0.1702	0.8743
SECONDARY HEXAFOIL	0.5617	30.0	0.1672	0.9522
TERTIARY SPHERICAL	-0.0481	0.0	0.1665	0.9522
TERTIARY COMA	0.0000	6.2	0.1665	0.9522
TERTIARY ASTIGMTISM	0.0000	63.5	0.1665	0.9522
TERTIARY TREFOIL	0.8193	-30.0	0.0756	0.4667

ZERNIKE ABERRATION ANALYSIS

With tilt

ABERRATION	FUNCTIONAL (P-V)* (WAVES)	ANGLE (DEGREES)	RESIDUAL RMS** (WAVES)	RESIDUAL (P-V)** (WAVES)***
INPUT ARRAY			6.5091	24.5439
BIAS			6.5091	24.5439
Y TILT			2.4045	10.7295
X TILT			2.4045	10.7295
POWER	1.1299	0.0	2.3791	10.7295
PRIMARY SPHERICAL	0.7888	0.0	2.3684	10.7295
PRIMARY COMA	0.0000	77.8	2.3684	10.7295
PRIMARY ASTIGMTISM	0.0000	17.6	2.3684	10.7295
PRIMARY TREFOIL	12.8946	-30.0	0.5237	2.2246
PRIMARY TETRAFOIL	0.0000	-19.0	0.5237	2.2246
PRIMARY PENTAFOIL	0.0000	33.8	0.5237	2.2246
PRIMARY HEXAFOIL	0.4759	-0.0	0.5174	2.1651
SECONDARY SPHERICAL	-0.4202	0.0	0.5040	2.2612
SECONDARY COMA	0.0000	-145.3	0.5040	2.2612
SECONDARY ASTIGMATISM	0.0000	-79.9	0.5040	2.2612
SECONDARY TREFOIL	3.0394	30.0	0.1702	0.8743
SECONDARY TETRAFOIL	0.0000	25.1	0.1702	0.8743
SECONDARY PENTAFOIL	0.0000	15.7	0.1702	0.8743
SECONDARY HEXAFOIL	0.5617	30.0	0.1672	0.9522
TERTIARY SPHERICAL	-0.0481	0.0	0.1665	0.9522
TERTIARY COMA	0.0000	50.7	0.1665	0.9522
TERTIARY ASTIGMTISM	0.0000	63.5	0.1665	0.9522
TERTIARY TREFOIL	0.8193	-30.0	0.0756	0.4667

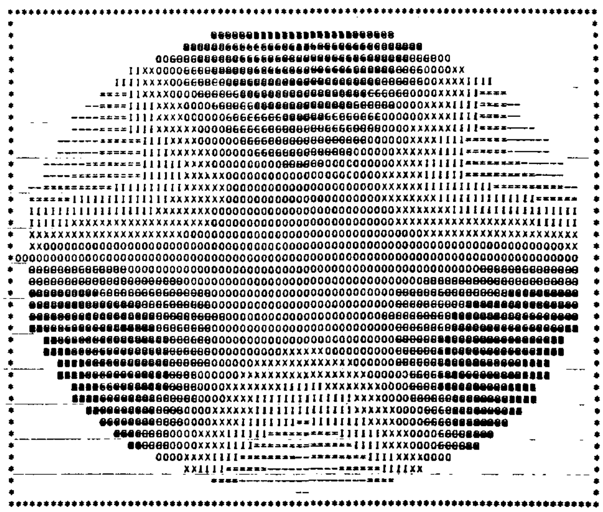


Figure 7

Density Plot