

**Design and tolerance specification of a wide-field,
three-mirror, unobscured, high-resolution sensor**

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ABSTRACT

Santa Barbara Research Center has been exploring technology related to the design, tolerance, and alignment of wide-field, all-reflective sensors for multispectral earth observation. The goals of this study are to design an optical system with reduced fabrication risks, to develop a detailed tolerance budget and to demonstrate our ability to align the system to a tolerance of 0.05 waves rms at 0.6328 microns. The final optical system is a three-mirror unobscured telescope. It is telecentric and flat field over 15 degrees at F/4.5, and achieves diffraction-limited imagery at visible wavelengths.

This paper describes the results of the design effort, the tolerance exercise, and a metrology approach for the optics fabrication. The alignment results are discussed in a separate paper.¹ Three conclusions are made: 1) design of unobscured optical systems is still best approached using fundamental optical design principles, 2) time spent in careful modeling of fabrication, testing, and alignment interactions will result in more relaxed tolerances and a higher probability of success for the assembled system, 3) precise metrology of optical surfaces can be achieved with fairly simple techniques.

1. BACKGROUND

The Landsat Thematic Mapper imagers have provided moderate resolution, multispectral imagery of the Earth for the past 7 years. Yet, the need exists for higher spatial and spectral resolution and better radiometric accuracy. Santa Barbara Research Center has been exploring concepts for wide-field pushbroom sensors for meteorological and earth sensing applications. This has involved many complementary disciplines including optics, mechanics, detectors, and electronics.

During the past two years efforts have focused on demonstrating the feasibility of designing and aligning high-performance, low-risk optics housed in a structure that allows kinematic, stress-free mounting and adjustment. Work reported here is the result of the first phase of that project.

2. DESIGN PHILOSOPHY

Many designers have approached the problem of designing unobscured systems with varying degrees of success. The design effort often seems to be more guesswork and computer crunching than insightful optical design. It is important to realize that the basic precepts of optical design need not be abandoned because the solution appears to be immersed in aspheric coefficients and nonsymmetric forms. Three rules apply: begin with a good starting point, rely on the inherent strength of symmetry, and apply an enlightened understanding of aberration properties. This approach will almost always yield superior solutions.

A good starting point design should not be just a springboard to unconstrained optimization, it should also serve as a point of reference to keep the design within an acceptable envelop of first-order and third-order solutions. During optimization, it is easy to forget that many high-order aberrations are a direct consequence of the first-order properties of the system. Finding an appropriate starting point, however, is not always easy. Deitrich Korsh ^{2,3} has published articles defining analytic solutions for third-order correction of three-mirror systems. This approach offers the designer the opportunity to define specific solutions as well as parametrically explore related families of design forms. An unpublished work by C.W. Chen ⁴ treats the third-order solution to the three-mirror problem using the structural aberration coefficients as defined by R.V. Shack.⁵ What the designer is striving for at this time is a feel for the relative magnification between individual components, size of elements, and the magnitude of conic correction. Parametrically exploring a range of solutions gives the designer an understanding of how broad a solution space might exist for a given design form.

A good design will also have a sense of symmetry about it. Able and Hatch ⁶ showed the inter-connection between symmetry and performance for wide-angle reflective systems. Even though many wide-angle systems tend to be unobscured, their basis as symmetric forms is important. Some of the aspects of symmetry one strives for are: a common optical axis for all elements, symmetry about the stop, concentricity about a pupil or image, and a balance in the degree to which elements are used in an off-axis manner. Other authors have explored the design of tilted-component systems.^{7,8} This approach can yield well-corrected systems with moderate f-number and field, but is often quite limiting in its construction geometry.

The concept of symmetry is also fundamental to the interpretation and control of aberrations, even in nonsymmetric systems. R.V. Shack and K.P. Thompson ^{9,10} have developed the concept of aberration fields and the vector representation for them. Aberration fields allow the designer to think in terms of Seidel aberrations while designing nonsymmetric systems. An aberration field is the representation of the pupil and field dependence of each Seidel term projected to image space. Each optical surface is associated with an aberration field. This field is the same as the centered aberration but is symmetric about a point in the image plane that depends on the tilts, decenters, and field angles of the optics. It is the vector summation of these symmetric aberration fields that generates the complex geometries of image plane aberrations in nonsymmetric systems. This author has shown how the field symmetries of the dominant aberrations give clues on how to evaluate the effects of such fundamental first-order trades as field bias and aperture offset.¹¹

3. OPTICS DESIGN

The starting point for this design was an unobscured three-mirror anastigmat designed by L. Cook and R. Kebo.¹² This design was the result of a long effort to select the most promising form from a myriad of three- and four-mirror systems. The "reflecting triplet" form of the three-mirror anastigmat was chosen as a particularly compact configuration with good performance over wide fields of view and fast f-numbers.

The specific design goal of this current task, was to reduce fabrication risks to the optics, while maintaining system performance, field of view, and f-number. The final solution evolved as a consequence of following the design approach discussed above. Admittedly, the solution did not come from a few third-order equations, some fancy plots and good intentions; many hours were spent at the computer improving a basic design form and exploring promising inroads. However, as certain design iterations became more and more difficult, it was often found to be the result of straying too far from one of the important mileposts; a good starting design, symmetry, and aberration control.

The design and performance are shown in the following figures. Figure 1 shows an isometric plot of the system and shows how the strip field is imaged. Figure 2 is a photograph of a prototype optical system of the same design. This hardware was used to perform the alignment demonstration cited in reference 1. Figures 3 and 4 show the spot diagrams and MTF of a 1.0-meter focal length system. It is clear from these figures that the system is well-corrected with balanced performance across the field. Some of the notable design points are shown in Table 1.

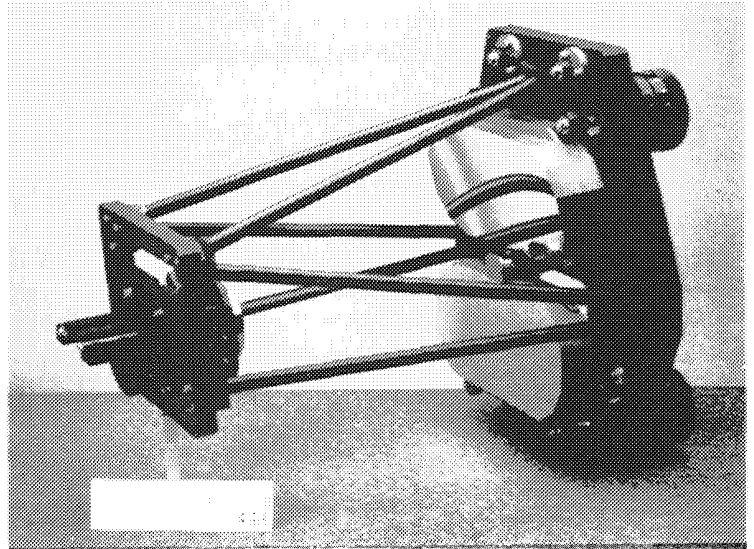
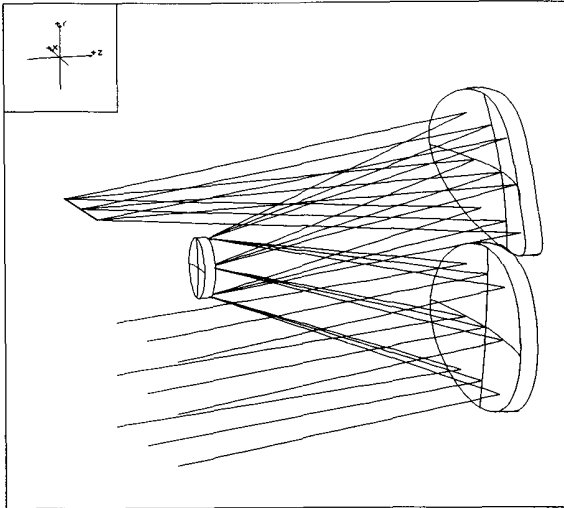


Figure 1. Optical raytrace for a 15 degree, system diffraction-limited, unobscured telescope.

Figure 2. Photograph of the prototype optical of this design used for an alignment demonstration.

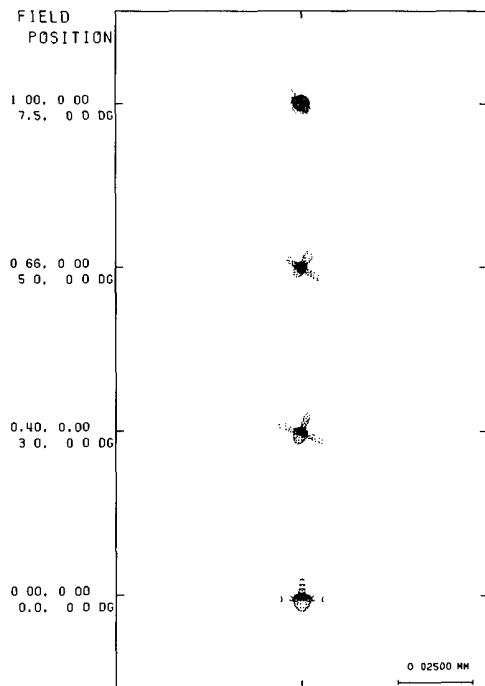


Figure 3. Spot diagrams of the geometric image show a good balance across the field.

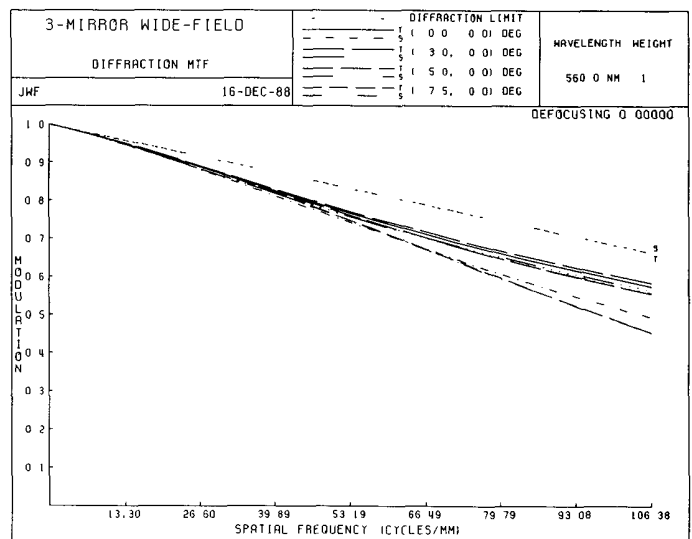


Figure 4. The MTF confirms the uniformity across the field.

Table 1.
Design Characteristics of the Three-Mirror Telescope

EFL	1.0 meter
F-number	4.5
Field of view	0.6 x 15 degrees
Flat focal plane, telecentric	
Average rms spot size	0.007 mm
Average wavefront error	0.09 waves rms @0.6328 um
Average MTF @20 lp/mm	0.92 (0.94 is diff. lim.)
Average MTF @40 lp/mm	0.83 (0.87 is diff. lim.)
<u>Aspherics:</u>	
Primary	Conic, 6th order, 8th order
Secondary	Conic, 6th order
Tertiary	Conic, 6th order
<u>Max. Aspheric Departure:</u>	
Primary	0.040 mm
Secondary	<0.001 mm
Tertiary	0.044 mm
<u>Max. Aspheric Slope:</u>	
Primary (@ r=335mm)	13.6 waves/cm
Secondary (@ r=65mm)	0.59 waves/cm
Tertiary (@ r=335mm)	15.9 waves/cm
Parabola, comparable to the Primary	5.7 waves/cm

4. THE ERROR BUDGET

Having a design in hand is an important step, but it is far from having demonstrated a viable system. The next milestone is the development of a detailed error budget. When building precision optical systems, total system performance is typically close to the diffraction limit. Every tolerance is squeezed. The error budget must keep track of all factors that affect performance and attain a balance among tolerances so that each one can be achieved.

An error budget should be a dynamic document. This is not to say that the system performance requirement is flexible (on the contrary, a well-defined, firmly fixed performance specification is a necessity), but rather that the entries in an error budget are fluid so that successes in one area can help balance difficulties in other areas.

As more and more components are built, the nature of the error budget also changes from a statistical model to a more deterministic one. Consider the important interaction between fabrication and assembly. Once the optical components are made and measured, deviations from nominal can be entered into the optical designer's model. If the model is detailed enough, the designer can reoptimize the performance using rigid-body motions of the elements to compensate for fabrication errors. This step removes the statistical uncertainty regarding fabrication errors and defines a new "nominal"

system to which the assembly tolerances should apply. At this point there is a deterministic value for the best possible performance. The statistical distribution of fabrication tolerances can be replaced with measured data and its much smaller measurement error. Any margin in the budget can then be applied to relieve assembly tolerances. It is in this way that exceedingly tight tolerances for precision systems are continually massaged to take advantage of every piece of measured data.

Organization of the error budget is important. It should start with as specific a performance criterion as possible, then follow the actual flow of the hardware: from design and fabrication, to assembly and operation. Under each major heading, subassemblies should be broken down until every critical dimension has been identified and assigned a tolerance. The tolerances should be distributed so that all are physically obtainable and no single error contribution dominates. This concept is depicted in Figure 5.

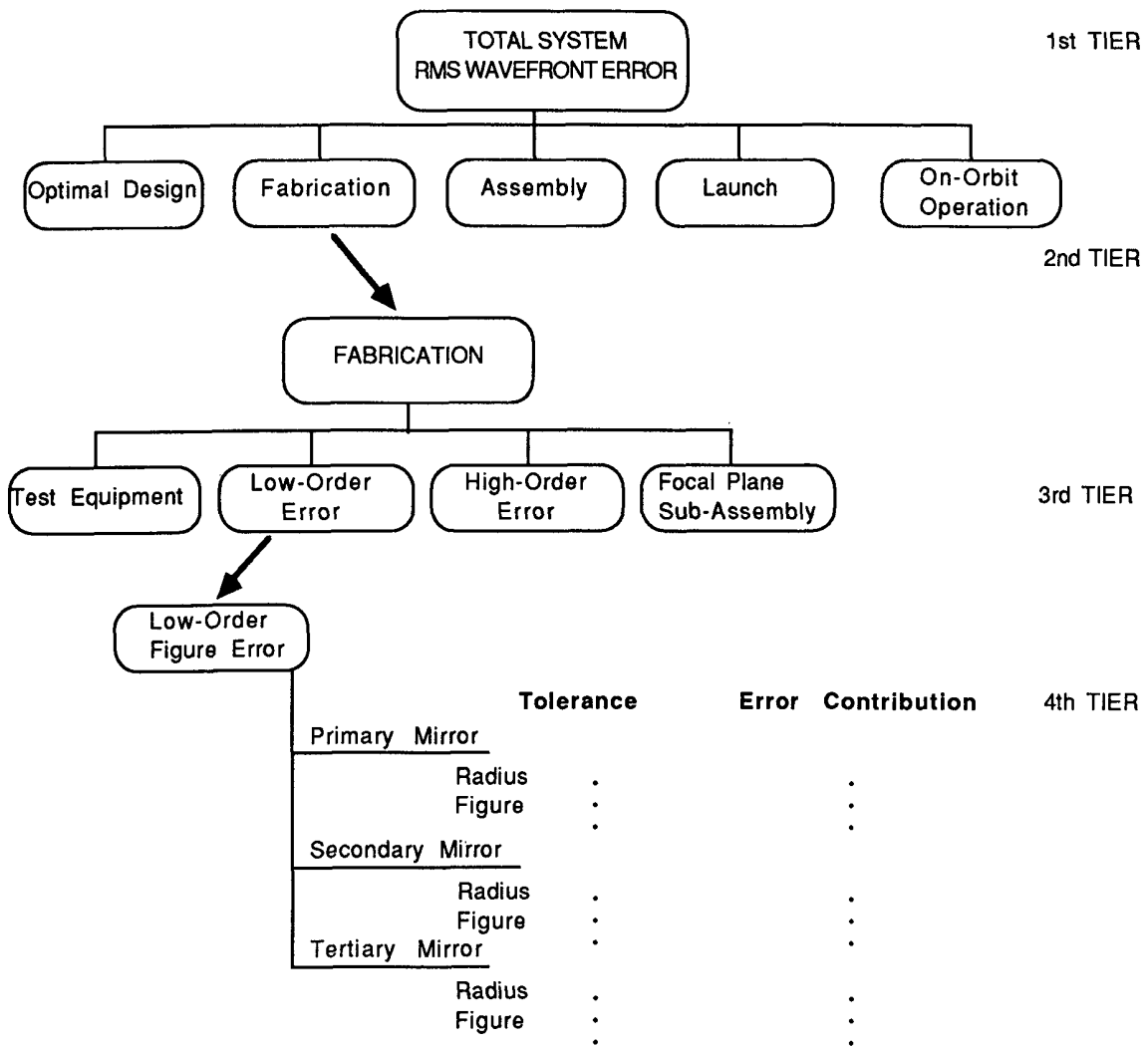


Figure 5. The error budget should follow the flow of the hardware and attempt to balance the distribution of tolerances.

Every tier lower in the error budget should also be more amenable to change. On the lowest end, an individual tolerance on a single parameter should not be taken as an absolute. At this level, there is cushion resulting from the statistical strength of all the combined parameters. At the higher end, however, changes should not occur without a total system reconciliation. Much time and money can be saved if it is realized that if a few individual tolerances are exceeded, the total system performance will not be jeopardized. Indeed, the entire approach assumes that some values will exceed their tolerance while others will come in under tolerance.

Of course, there are as many ways to design an error budget as there are authors. But there is a common technical challenge that has to be addressed. It is the job of the optical designer to take vague, redundant, and conflicting performance requirements, distill them down to the essential physical phenomenon they were meant to describe, identify all contributing technologies that could influence achieving the goal, translate these esoteric concepts to quantifiable measurements, and then convert all millimeter units to inches so that it can be built!

5. MODELING AND METROLOGY OF FABRICATION ERRORS

The importance of computer modeling to simulate actual hardware as well as "what-if" situations cannot be over-stressed. It is a lot easier, and cheaper, to learn and make mistakes in a computer environment, than to do it in hardware over and over again. An important example is the specification of the fabrication tolerance. Too tight a tolerance can result in excess time and dollars spent by the optic's fabricator, while too loose a tolerance might result in the final performance not meeting the specification.

The error budget allocation for fabrication can be divided into mirror surface error, null lens error, and other test equipment uncertainty. The most important tolerance is on mirror surface error. The following definitions are important:

Low-Order Error - Spatial frequency errors across the mirror whose effect can be partially or completely compensated by similar errors in other mirrors or by alignment (another way of thinking about this is as statistically correlated errors).

High-Order Error - Spatial frequency errors that contribute to wavefront degradation but cannot be compensated; i.e. statistically uncorrelated.

These definitions appear qualitative, but are intuitively satisfying and can be assigned a quantitative value after the appropriate computer modeling exercises.

The most effective modeling approach is to directly simulate the anticipated errors. For low-order figure errors this is a very enlightening exercise. To determine how much astigmatism our design could tolerate, for example, an astigmatism error was applied to each optical surface with a Zernike surface deformation. The system was reoptimized using rigid-body motions of tilts, decenters, and despace to regain the best performance. More and more aberration was applied until the performance could no longer be improved better than a maximum tolerable error. This procedure was then repeated for coma and spherical aberration terms.

The result of this exercise provided two important insights. One was that only astigmatism, coma, three-theta coma, and spherical aberration could be effectively compensated with post alignment. The second was that a surprisingly large magnitude of astigmatism could be allowed as a surface error, and

good image performance could still be regained during alignment. In attempting to compensate a single astigmatism term with rigid-body alignment motions, it was found that the primary mirror could tolerate a surface error of 4 waves peak-to-valley and be realigned with a performance degradation of only 0.12 waves rms!

This, of course is an unrealistic, isolated example, but it serves to "realign" (excuse the pun) one's thinking about fabrication errors in diffraction-limited systems. The designer, the fabricator, and the customer should be aware of such extremes in the fabrication/alignment interaction. It is not an unusual situation that one mirror out of a set might be out of specification. It can save considerable time, dollars, and handling risks if it is known, through a paper exercise, that such an error might be acceptable.

However, for a more realistic determination of the surface error tolerance the modeling exercise should be continued. This extension should combine errors of different type and different magnitude on all the surfaces. This can be done in a Monte-Carlo fashion or, without such capability, it is not too difficult to learn which combinations of sign and magnitude constitute a worse case set of errors.

One important caveat applies to this modeling approach. When compensating fabrication errors through alignment, it is important that the line of sight of the system, as well as the beam footprint on the individual surfaces, does not change significantly. It is very embarrassing to have the beam vignetted by structural supports, baffling, or simply walking off the mirror. Always constrain an alignment model to limit the amount of beam walk and pointing shift.

5.1. High-frequency surface errors

One of the more difficult fabrication errors to characterize, and yet one of the most damaging to MTF performance, is high-frequency surface error. By our definition, high-frequency errors are all those spatial frequencies higher than the particular Zernike terms that had been modeled as "compensatable" low-order errors. These features are more prevalent when sub-diameter polishing laps are used in aspherizing a mirror. They indeed can be extreme when very small laps are used in conjunction with computer-controlled polishing machines. Our definition of high spatial frequency, however, is a bit open-ended at this point; where does it stop? Equally important is how does one measure the error? An approach is described here that balances mathematical rigor with a judgmental decision whose uncertainty can be numerically bound.

A number of authors have treated, in depth, the modeling and measuring of high-spatial-frequency errors. The essence of their analyses boils down to a statistical treatment of surface errors that relate to an MTF degradation. One of the more rigorous treatments is the approach using correlation length.¹³ Another is based on slope error.¹⁴ A third approach uses rms surface error as its free parameter.¹⁵ We have taken this latter approach and have interpreted it in an intuitive fashion that minimizes the measurement burden but still addresses the technical heart of the problem.

An expression developed by E.L. O'Neill and modified by R.E. Hufnagel¹⁶ relates an MTF degradation factor to an rms wavefront (or surface) error through a term called the Hufnagel constant that defines the spatial frequency content of the error. This equation is:

$$T(v) = \exp[-(2\pi w)^2 (1 - \exp(-2 H^2 v^2))]$$

- T(v) is the MTF degradation factor at spatial frequency v
- w is the rms wavefront error
- H is the Hufnagel constant describing the frequency of the error in cycles/diameter of the test piece.

Figure 6 plots the MTF degradation, $T(v)$, as a function of rms OPD for a range of values of H from 5 to 20.

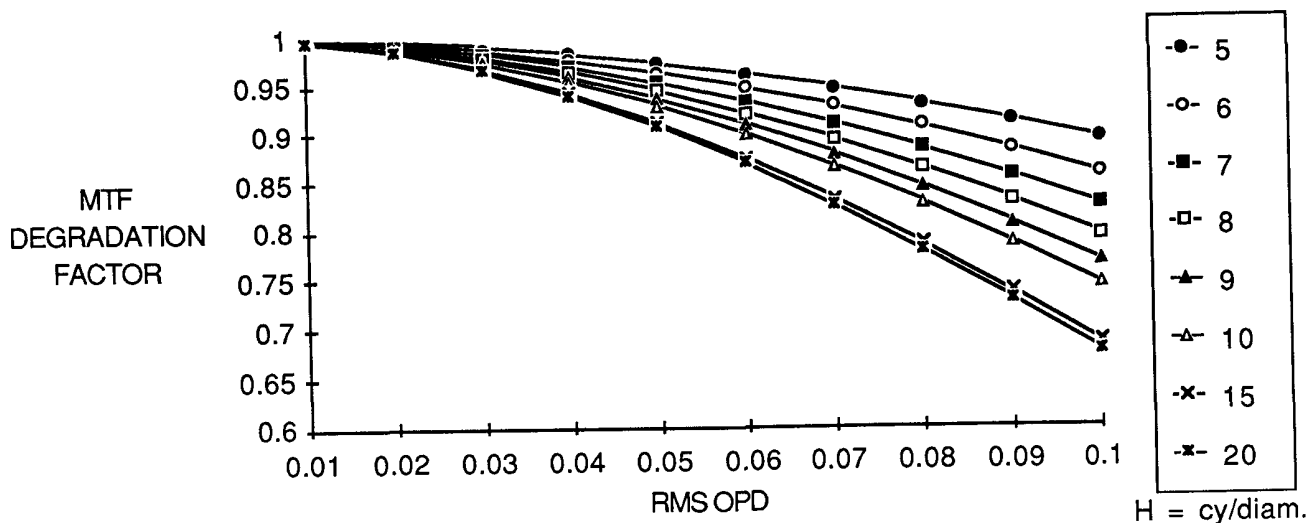


Figure 6: The MTF degradation reaches a stationary value, for a given rms OPD error, at surface spatial frequencies greater than 15 cycles/diameter.

The information in Figure 6 suggests that, for a given magnitude of rms OPD, the MTF degradation is worse for higher spatial frequency figure errors. This trend continues until, at a surface spatial frequency between 15 and 20 cycles per diameter, the MTF degrades no further. This provides a clue to what sampling density to use when digitizing an interferogram. An error of 15 cycles would be adequately sampled with a 30 x 30 grid across the interferogram. But what about the unsampled errors at still higher spatial frequencies? If one concedes that a 0.10 wave P-V error can be detected on a simple interferogram, and that such an error would be digitized regardless of its position in a sampling grid, then, depending on the statistical distribution of the remaining high-frequency error, the rms OPD error would be no more than 0.02 to 0.01 waves rms. This approach, combined with the low-order modeling described earlier, can be used to establish a simple but meaningful metrology plan.

- An interferogram is taken of the surface and is sampled using a grid of at least 30 x 30 points over the instantaneous beam diameter (this can be a small portion of the physical piece for wide-angle systems).
- A Zernike polynomial fit to the data is made that includes those low-order errors that have been modeled and can be compensated by alignment. The resultant error must satisfy the tolerance specified for low-order error.
- The rms OPD of the data points, after these low-order terms have been removed, must satisfy the tolerance defined for the high-order error.
- The additional error beyond this will be less than 0.02 waves rms.

For most imaging applications, the assumptions underlying this analysis are valid. However, if excellent imagery at short wavelengths or maximum encircled energy is the desired goal, then other more rigorous, metrology techniques should be used.¹⁷

7. SUMMARY

A strip-field optical system was designed that achieves diffraction-limited performance over a 15-degree field of view. It is a three-mirror unobscured telescope that is telecentric and flat field. The design uses a minimum number of aspheric terms on the surfaces to reduce fabrication risks. The success of the design was due in part to applying the design philosophy of using a good starting point, maintaining symmetry conditions, and controlling aberrations.

A detailed error budget was also developed for the system and the approach for achieving a balanced tolerance distribution was discussed. Finally the role of computer modeling of low-order fabrication errors was described and a simple metrology approach for quantifying low- and high-order surface errors was presented.

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