Synopsis of "Design of Quasi-Kinematic Couplings" by Martin L. Culpepper[1]

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Introduction:
Culpepper wrote "Design of Quasi-Kinematic Couplings" to detail a design method for quasi-kinematic couplings (QKC's). Culpepper's goal was to achieve a precision coupling that has good manufacturability, cost effectiveness, and sealability. At the time, methods existed for high precision couplings that did not meet the other three goals and other methods existed for manufacturable, inexpensive, sealable couplings that do not meet precision requirements.

Culpepper's work for practical couplings that achieve high levels of precision was funded by the automotive industry, but the results are generally applicable to any field that requires precision coupling. Culpepper has also written other papers, including one in 2005[2] concerning alignment correction that is general in nature but funded by the automotive industry. Barraja[3] wrote a paper on tolerancing kinematic couplings that was directed toward reducing costs while maintaining performance. Barraja's work featured a statistical analysis. Goodman[4,5] has written a number of papers on optical cylinders in V's. Goodman's work is concerned with practical considerations in optomechanical mounting, but it is primarily applicable to optics, as V-blocks are not generally useful elsewhere. Hale[6,7] wrote on the general techniques on designing kinematic couplings. Hale's work is research oriented in nature where the performance greatly outweighs any affordability or manufacturability concerns.

Numerous papers have been written on precision coupling, and a wealth of information is readily available to the avid researcher. Culpepper's work on QKC's provides excellent insight into design considerations with a practical thrust. The ability to utilize inexpensive and readily manufacturable couplings is critical for many optomechanical designs, which is why this paper was chosen for review.

Paper Synopsis:
Introduction:
Automotive systems, precision optics, and photonics require high precision alignment tolerances on the nano/micrometer level, which is currently beyond current low-cost capabilities. This has resulted in the development of the QKC to achieve these cost and performance requirements.

Existing, commonly manufactured couplings include the pinned joint, rail-slots, tapers, and dove tails. Micron-level precision is not easily achievable with these since such precision would require very tight tolerances on pins, holes, surface finishes, etc.

Kinematic coupling, typically balls and grooves, provides exact constraint with better than 1-µm precision, but these couplings are used infrequently in manufacturing since they do not meet the following low-cost coupling requirements. 1. Low-cost generation of fine surface finish: inexpensive fine surface finish balls are readily available, but fine surface finish grooves are not. 2. Low-cost generation of alignment feature shape: balls and grooves are more complex than pin joints, and the surface hardness processing required that allow joints to survive Hertzian contact stresses drives the costs higher. 3. Low-cost means to form sealed interfaces: adding sealing flexures to kinematic couplings increases costs.

There is a significant gap in cost and performance between kinematic coupling and commonly manufactured couplings. The QKC is designed to address this gap, as is shown in Culpepper's Figure 2.

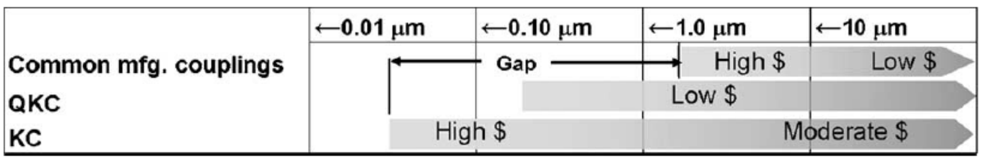

Fig. 2. Cost and precision of common couplings.

Culpepper's Figure 2. Shown because it succinctly defines the gap in cost and performance that QKC's are to fill

Quasi-Kinematic Coupling:
Culpepper's Figure 1 shows the basic concepts of the kinematic coupling and the QKC. Kinematic couplings (KC's) feature a ball-in-groove joint where three balls on one component mate with three grooves on the second component with small area contacts. In QKC's, axisymmetric balls of the first component mate with axismmetric grooves of the second component, forming arc contacts. The contact relief may be provided on either the ball or the groove. For both types of couplings, joints are oriented with symmetrically positioned ball-groove contacts with respect to bisectors of the coupling triangle.

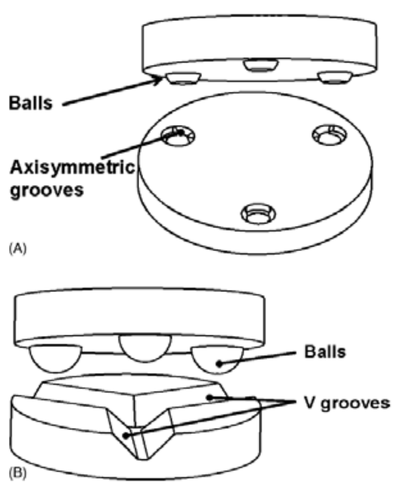

Fig. 1. Quasi-kinematic (A) and kinematic (B) couplings.

Culpepper's Figure 1. Shown because it succinctly describes KC's and QKC's.

The main difference between the two designs is the arc contact for the QKC rather than the small-area contact of KC's that provide exact constraint in the six degrees of freedom (DOF's). QKC's groove geometries are symmetric and are easier to manufacture, though the arc contacts give a certain amount of over-constraint. However, with careful design, QKC's can approach KC performance.

The over-constraint of the QKC arcs can be minimized by reducing the size of the contact arc. Smaller contact arcs result in smaller constraint forces parallel to the coupling triangle bisector, but it also reduces the coupling stiffness. A quantitative metric for this trade-off is detailed later in the article/synopsis.

The low-cost coupling requirements detailed earlier are met by properly designed QKC's. 1. Low-cost generation of fine surface finish: low cost polished spheres (bearings) are readily available. High quality grooves can be achieved by burnishing the groove's surfaces by pressing the harder, finer ball into it. This requires a ball with a polished surface finish and 3 to 4 times the modulus of elasticity of the groove. It also requires tangential sliding between the ball and groove surfaces to remove asperities. 2. Low-cost generation of alignment feature shape: the QKC groove can be made in simple drill operations using countersinks or form tools since the grooves are axisymmetric. Groove reliefs can be made in place by drilling, forming, milling, or casting with comparable costs to pinned joints. 3. Low-cost means to form sealed interfaces: by making the ball contact feature hollow, by adding an undercut, and by providing a sufficient nesting force, the gap between the two components may be closed if the ball-groove materials plastically deform during the first mate. Elastic recovery will allow a portion of the gap to return, which is necessary in maintaining its kinematic nature.

Theory of Quasi-Kinematic Coupling:
QKC's are not like KC's with point contacts where the displacements and contact forces are assumed to be normal to the contact. The direction of the forces may not be assumed and contacts must not be modeled as point contacts. The analysis method may be broken into the following steps: preload a coupling, impose displacement error on this mated state, calculate ball-groove contact forces, and calculate coupling stiffness. The derivation of the kinematic and mechanics theories used in the model is discussed in detail in Culpepper's Appendix A.

A constraint metric defined as $CM_i = \dfrac{\text{stiffness parallel to bisector}}{\text{stiffness perpendicular to bisector}} = \dfrac{k_{i\|bisector}}{k_{i\perp bisector}}$ is used to determine how the ball-groove arc contacts can be designed to optimize performance. The ratio serves as a useful metric in reducing the likelihood of over-constraint based on the joint stiffness and material characteristics, where low CM's indicate low over-constraint.

When $\theta_{contact} = 180°$, CM = 1, and as $\theta_{contact} \to 0$, CM approaches 0. $\theta_{contact} \sim 0°$ is not reasonable because of the substantial loss in coupling stiffness. Thus the coupling stiffness and CM need to be considered simultaneously, as is shown in Culpepper's Figure 10.

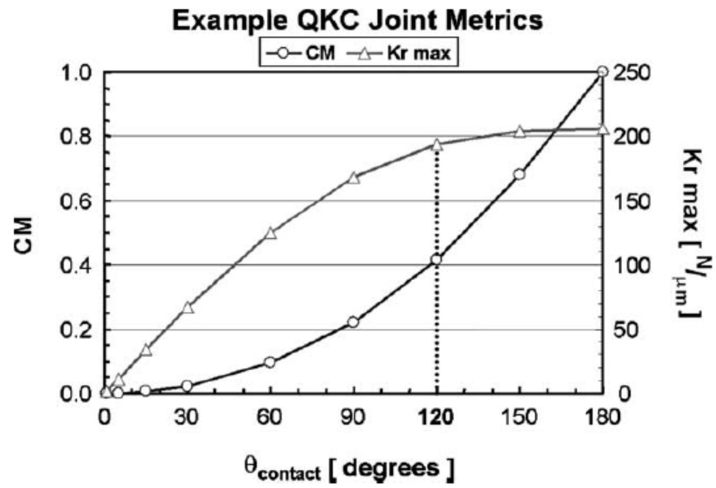

Fig. 10. Comparing performance metrics of QKCs.

Culpepper's Figure 10. Shown because it succinctly shows the trade-off between stiffness and constraint.

In a detailed example where $\theta_{contact} = 120°$ (with other parameters defined and values calculated using the theory derived in Appendix A), the CM was found to be 0.41 and the stiffness Kr was found to be 195N/m. If he design calls for only 125N/m stiffness, the contact angle can be reduced to 60°, and the resulting CM is 0.1. The trade-off between stiffness and constraint is favorable at large contact angles.

The full estimate of $\delta_{over\text{-}constraint}$ (error due to over-constraint) requires consideration of the post-plastically deformed mismatch between the ball and groove. This mismatch between QKC joints depends on elastic contact deformation, plastic deformation, and multiple ball-groove mismatch tolerances. The theory on describing this final mismatch is still undeveloped, so the CM along with joint stiffness will continue to be the factors examined for determining the QKC performance in this article.

Testing MathCAD Model:
The theory developed was implemented in MathCAD, shown in Culpepper's Appendix B. The MathCAD model was checked for consistency by the following five tests: 1. translation errors in z (vertical) direction resulted only in net z forces, 2. rotation errors about z-axis of coupling centroid resulted only in z moments, 3. displacements along one bisector of a 120° coupling resulted in no net y or z moments, 4. no x and y reaction forces resulted when the grooves are flat, 5. the model properly detected loose contacts as violations of "constant contact" constraint.

Experimental Results:
A QKC has been used in precision automotive assemblies, providing 0.67-μm repeatability, but unusual stiffness requirements resulted in large contact angles and

atypical orientations. Thus an experiment was run with a QKC more comparable to the angles and orientations of an ideal KC.

A QKC with $\theta_{contact} = 60°$ (CM = 0.1) with low-cost attributes of typical quasi-kinematic joints was used. It was manufactured with <25μm mismatch between the axis of symmetry of any ball and mated groove and it included lubricated joints. A repeatability of 0.25-μm was measured after an initial wear-in of 5 mates. In the event that this wear-in time is impractical, preloads have been shown to eliminate the wear-in period and the mismatch between ball and groove patterns.

Coupling Costs:
Ball-groove sets cost ~$1 in volumes >100k per year or ~$60 in volumes <500 per year. This is much less than the several hundred dollar price tag on high performance KC's. When the whole-life cycle cost is considered, not only are the initial savings included, but so are the replacement savings/costs.

Appendices (Steps to Model the Performance Characteristics of QKC's):
Culpepper provides two appendices in support the article. Appendix B, which is a MathCAD model, will not be summarized here, but it can be seen at the end of the paper. Appendix A, which are the steps to model the performance characteristics of QKC's, will be briefly summarized, but the full version is available in Culpepper's article.

Step 1: Material and geometry characteristics. Material data, such as Young's modulus and Poisson's ratio, are needed for the ball and groove materials. The geometry is defined by creating a coupling coordinate system (CCS), a displaced coordinate system (DCS), and a joint coordinate system (JCS). Each of the three joints (i = 1–3) and each of the six contact arcs (j = 1–6) are identified. Each JCS_i measures position in r_i, θ_{ri}, and z_i, and the contact half-cone angle is defined as θ_i.

Step 2: Imposed error motions. The coupling stiffness depends upon the ball-groove reaction force, which is a function of the compression of materials. This depends upon the error in the ball's far field displacement from its preloaded position. This displacement can be expressed as a combination of translation, $\vec{\delta}_c$, and rotation, $\vec{\varepsilon}$.

Step 3: Distance of approach between far field points in ball-groove joints. With multiple couplings of a ball to a cone, the compression forces about the arc will begin to vary about this contact. A common metric to describe material compression is δ_n, the distance of approach between two far field points. It is a function of the axial and radial displacements.

Step 4: Modeling interfaces as a function of δ_n. Relating δ_n to the force per unit length, $\vec{f}_n$, though simple with elastic-contact-only joints, requires more consideration for joints with some integral compliance or plastic deformation. FEA or other analytic methods can relate $\vec{f}_n$ to δ_n in the following form: $\vec{f}_n(\theta_{ri}) = K[\delta_n(\theta_{ri})]^b \hat{n}$, where K is a stiffness constant and b reflects the rate of change in contact stiffness with changes in δ_n. K and b are functions of ball-groove geometry.

Step 5: Reactive force on an arc contact. The reaction forces of all six contact arcs are summed to get a resultant reaction force. Each force for the contact arcs is determined by integrating $\vec{f_n}(\theta_{ri}) \cdot R_c$ along the arc of contact where R_c is the radius of the cone on the contact line. The sum of torques is determined from each ball-groove reaction force and moment arm between the CCS and the ball's far field displacement.

Step 6: Stiffness calculation. The resulting coupling stiffness is determined as $k = \dfrac{d(\text{Reaction})}{d(\text{Imposed Error Displacement})}$. When linear displacements are imposed, the reaction force is given by $\vec{F}_{\text{Reaction}} = \sum_{j=1}^{6} \vec{F_j}$ (from Step 5). When rotation displacements are imposed, the reaction torque is given by $\vec{T}_{\text{Reaction}} = \sum_{i=1}^{3} \vec{r}_{SI_i} \times \vec{F_i}$ (from Step 5).

Conclusions:
Culpepper has developed a method to design QKC's with minimal over-constraint while optimizing performance. Experimental results show comparable performance to KC's can be achieved with much lower cost QKC's. The ease of manufacturing, low cost, and ability to form sealed joints make this an enabling technology particularly important for high precision, high volume assemblies in automotive, photonics, and optical applications.

References:

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