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Active fatigue control of adhesive bonded joints

Craig T. Dempsey, Shi Li, Chen Liang, Craig A. Rogers

Center for Intelligent Material Systems and Structures,
Virginia Polytechnic Institute and State University,
Blacksburg, VA 24061-0261

ABSTRACT

This paper investigates the concept of active fatigue control of an adhesively bonded structure. Adhesive bonding has recently become very widely used, especially with the gaining popularity of composite materials. The adhesive bonds are usually the weakest link in the entire bonded structure. Additionally, one of the major causes of failure in the bond is fatigue. With increasing knowledge in the field of intelligent material systems, it is now possible to reduce the stress in the adhesive bond by using induced strain actuators, such as PZT. Experimental results show that the fatigue life of a vibrating, cantilever beam with an adhesive bond can be increased by nearly an order of magnitude with active fatigue control. This paper presents an analytical model which can be used to determine the correct control schemes of induced strain actuators on a cantilever beam vibrating over a range of frequencies. With proper control, the peeling stresses in the adhesive bond can be minimized for a broad frequency range.

1. INTRODUCTION

The concept of active damage control is a relatively new topic which has recently begun to draw considerable attention as more and more is learned about the behavior of smart material systems. With this new technology comes new applications. One such application is that of active fatigue control, which is what will be investigated in this paper. This type of control includes using the dynamic response from induced strain actuators to reduce internal stresses rather than displacement or sound radiation as have been widely studied.

In this paper, the behavior of an adhesively bonded cantilever beam is being investigated. The response of the beam due to actuator excitation will be superimposed with the response of the beam due to an external excitation. This superposition will result in a net response of the beam which will minimize the stress in the adhesive bond if done at the proper phase and with the correct actuation voltage. To perform the computations, a computer program has been written. This program performs the steps involved with a modal analysis to find the moment at the beam root due to the inertial forces of the beam. The reduction of this moment is the dominant factor in increasing the fatigue life of the adhesive bond. Using this program, a parametric study is performed using different actuator positions to examine the effect of their location on the net resultant moment.

2. BACKGROUND

There are two main approaches to active fatigue control. The first approach uses direct stress cancellation.[1] This basically involves using actuators to apply an induced strain out-of-phase with the applied load. This will directly reduce the stress in a specific area, such as a circular hole in a plate as seen in Figure 1.

The other approach to active fatigue control is known as indirect stress cancellation (ISC). This method is used when the dynamic response of the structure is the dominant factor in causing stress in a local area. It involves superimposing two or more out-of-phase dynamic responses to decrease stress in a specific area. By using the correct phase difference, the stress in a local area can be minimized for a broad frequency range. Figure 2 illustrates the difference between the static and dynamic moment along a cantilever beam.

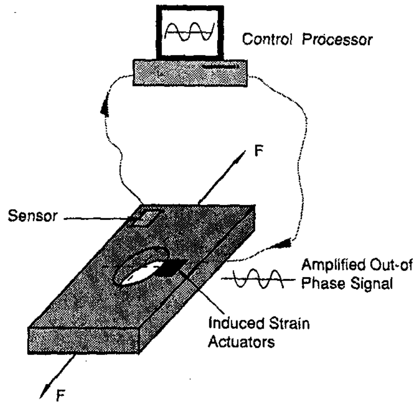

Figure 1: Direct stress cancellation.[1]

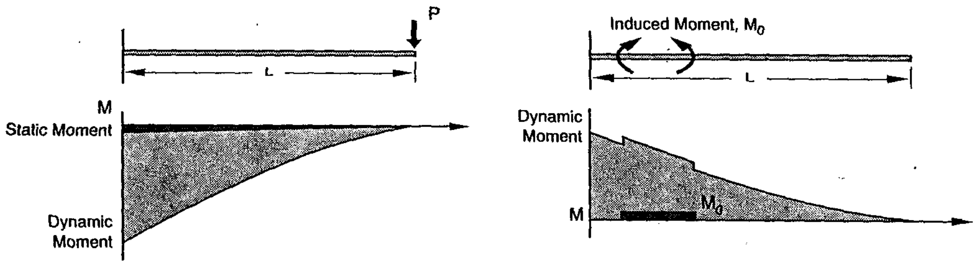

Figure 2: Schematic illustration for indirect stress cancellation.[2]

3. EXPERIMENTAL RESULTS

An experiment was initially performed to determine what kind of gains could be made in the fatigue life of an adhesive bond using ISC. This experiment consisted of an 185 x 25.4 x 1.5 mm³ graphite/epoxy beam glued to a steel frame in a cantilever configuration.[2] Then, two PZT patches were symmetrically bonded to the beam near the root. Finally, a shaker which exerted 13.3 N was placed at ¾ of the beam's length from the wall to provide harmonic vibration. The experimental set-up can be seen in Figure 3.

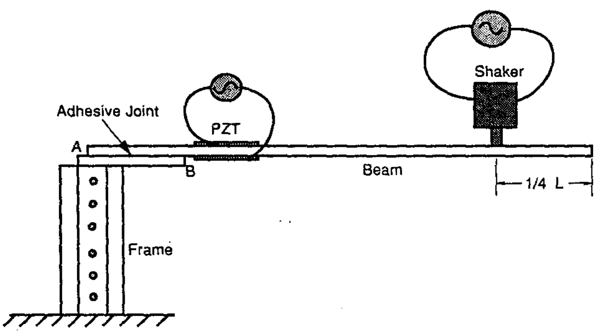

Figure 3: Active Fatigue Control Experimental Set-up.[2]

The material properties of the beam and the PZT actuators can be found in Table 1.

$E_{G/E}$(GPa)	$\rho_{G/E}$(kg/m³)	E_{PZT}(GPa)	ρ_{PZT}(kg/m³)	d_{31}(m/V)
130	1500	63	7650	-180×10^{-12}

Table 1: Material Properties of the Graphite/Epoxy Beam and PZT Actuators

The beam was then excited at its first natural frequency, and the cycles to failure in the adhesive bond with and without exciting the PZT patches were measured. Results indicated that the beam without active fatigue control lasted around 52,000 cycles, while the one with it lasted approximately 500,000 cycles. This is nearly an order of magnitude improvement. From this experiment, it can be seen that significant gains in fatigue life can be achieved by using this type of active fatigue control.

4. MODEL DEVELOPMENT

From the active fatigue control experiment, it was seen physically that great gains in the fatigue life of adhesive bonds can be made by utilizing intelligent material systems to reduce the stress level. This experiment illustrated just one set-up and only at one specific frequency. What will be developed here are the equations required to mathematically represent what is occurring in the beam over a broad frequency range and for various actuator and external load locations. An analytical model will be developed to predict the stress in the adhesive bond of the cantilever beam. This model is incorporated into a computer program to help make the calculations simpler. The results from the program will be presented for three different actuator locations. Then these results will be analyzed to see which actuator position provides the best stress reduction at different frequencies. It should be noted here that what is being performed is dynamic superposition, which minimizes the stress. By minimizing the stress, the vibration amplitude may or may not be minimized. Likewise, when the vibration amplitudes are minimized, the stresses may or may not be. This is an important item to remember as the model is derived.

The basic approach of this paper is to use modal analysis to determine the displacements all along the beam for each load case, i.e., those due to an external load and those due to actuator excitation. Once the displacements are found, the total moment due to inertia can be found. This moment is the dominant determining factor of the fatigue life expectancy of the adhesive bond. Then, scaling factors found from finite element analysis using unit moment loading can be used to obtain the stress level in the adhesive bond.

To begin, the critical parameters concerning this problem must be identified for simplicity's sake. First, adhesives behave well in shear and very poorly in peeling.[3] For this reason and because the beam vibrates transversely, which causes mainly peeling, it is assumed that the peeling stress level will be the likely cause of failure. Furthermore, from using the computer program BEAM VI[4] along with the stress scaling factors, it can be found that 97.4% of the peeling stress is due to the inertial bending moment of the beam[2]. Therefore, in this paper, the inertial moment of the vibrating beam will be considered the only cause of stress in the adhesive bond. Finally, through finite element modeling, it was found that the actuation of the PZT patches has no direct effect on the stress level in the adhesive bond, even if they were very close to the bond.[2]

There are also a few assumptions made concerning this problem. First, the actuators are excited 180° out-of-phase with each other so as to cause pure bending. Also, the actuators are considered to be perfectly bonded. Finally, the maximum moment induced by the actuators is found from the Pin-Force Model.[5] This is just

$$M_{eq} = (EI)_B K, \tag{1}$$

where

$$K = \frac{12\Lambda}{t_b(6+\Psi)} \, . \tag{2}$$

In this equation, Λ is the free induced strain which depends on the applied electric field, and $\Psi = (E_b t_b)/(E_a t_a)$. Then the dynamic responses caused by both this PZT actuation and the externally applied load will be used to calculate the total dynamic moment at the front edge of the adhesive.

The total moment caused by the inertial forces of the beam can be found by using differential elements and Newton's Second Law, F=ma. So, using the assumed displacement field of

$$y(x,t)=\sum_{m=1}^{\infty} W_m X_m(x)\exp(i\omega t), \quad (3)$$

the total moment at steady state can be expressed as

$$M=-a\rho\omega^2\int_0^L y(x)\,x\,dx, \quad (4)$$

at each value of ω. In this equation, 'a' is the cross-sectional area of the beam, and x is the distance from the clamped end of the beam. The displacements, y, then all need to be found as functions of x so this integration can be performed. Then, superposition will be used to find the total dynamic moment,

$$M_T^{Net}=M_T^E+M_T^A, \quad (5)$$

where the superscripts E and A refer to the moments due to the external load and actuators, respectively. Finally, this total moment due to inertia is used with a scaling factor to obtain the peeling stress

$$\sigma_y^{Net}=M_T^{Net}\overline{\sigma}_n, \quad (6)$$

where the $\overline{\sigma}_n$ term is the scaling factor. It was found from finite element analysis for this beam to be 0.88 MPa per 1 N-mm of moment.[2] To minimize the peeling stress level, the dynamic moment due to the PZT actuators needs to be 180° out-of-phase with the dynamic moment from the external load.

Now that the basic equations relating the beam vibration to the stress level have been established, the equations required to find the displacements need to be developed.[6] First, start with the equation of motion of a continuous beam

$$\frac{\partial^2 y}{\partial t^2}=-\overline{c}^2K^2\frac{\partial^4 y}{\partial x^4}+\frac{p(x,t)}{\rho_B a}. \quad (7)$$

The term $\overline{c}$ is the complex wave speed of the beam given by

$$\overline{c}^2=\frac{\overline{Y}_B}{\rho_B}. \quad (8)$$

This term includes a complex Young's modulus, $\overline{Y}_B$, which incorporates a loss factor. Also, the K^2 term is the moment of inertia over area and ρ_B is the mass density of the beam.

All excitations and displacements are considered to be harmonic. The assumed displacement field was given in Eq. (3). In this equation, the W_m's are the modal amplitudes and the $X_m(x)$'s are the mode shapes. For a cantilever beam, the mode shapes are represented by

$$X_m(x)=\cosh\left(\frac{\lambda_m x}{L}\right)-\cos\left(\frac{\lambda_m x}{L}\right)-\sigma_m\left(\sinh\left(\frac{\lambda_m x}{L}\right)-\sin\left(\frac{\lambda_m x}{L}\right)\right), \quad (9)$$

where the λ_m's and σ_m's have values as given in many references.[7] Also, the assumed pressure from the forcing functions is

$$p(x,t)=\sum_{m=1}^{\infty} P_m X_m(x)\exp(i\omega t)\ . \quad (10)$$

Then, from plugging the assumed displacement and its derivatives back into the equation of motion, an expression for the modal amplitudes can be found.

$$W_m=\frac{P_m/\rho_B a}{\overline{c}^2 K^2 (\frac{\lambda_m}{L})^4-\omega^2}\ . \quad (11)$$

Next, the pressure caused by the actuators and the external load must be developed. The actuators can be modeled with Heaviside functions, which are just step functions. From this, the pressure function can be obtained. These steps are given below:

$$M(x)=M_{eq}[H(x-\zeta_1)-H(x-\zeta_2)] \quad (12)$$

$$p(x)=\frac{d^2M(x)}{dx^2}=M_{eq}[\delta'(x-\zeta_1)-\delta'(x-\zeta_2)]\ . \quad (13)$$

In these equations, ζ_1 is where the actuators start and ζ_2 is where they end. Then, the external load can be modeled as just a point load using the Dirac delta function,[8]

$$P(x)=P\delta(x-X_L)\ , \quad (14)$$

where X_L is the location and P is the magnitude of the externally applied load.

The next step is to determine the modal amplitudes for the pressure expression, P_m. By using the property of orthogonality of mode shapes, this can be written as

$$P_m=\frac{\int_0^L p(x)X_m(x)\,dx}{\int_0^L X_m^2(x)\,dx}\ , \quad (15)$$

where

$$\int_0^L p(x)X_m(x)\,dx=M_{eq}*[-X_m'(\zeta_1)+X_m'(\zeta_2)] \quad (16)$$

and

$$\int_0^L P(x)X_m(x)\,dx=P*X_m(X_L) \quad (17)$$

for the PZT and external excitations, respectively.[9]

Now, everything that is needed to find the total moment due to inertia for the two excitations has been obtained. By substituting the results from Eq. (15) into Eq. (11), then from Eqs. (11) and (9) into Eq. (3), and finally from Eq. (3) into Eq. (4), the total dynamic moment for each excitation can be determined. Each moment includes a complex part to determine the phase at each frequency, which in turn enables the dynamic moment (and the stress) to be minimized. To aid in the calculations, a computer program was written using Matlab™.

5. PRESENTATION OF PROGRAM

The Matlab™ program deals with matrix manipulations, so it lends itself readily to this type of work. This program also possesses a good graphics capability along with the ability to deal with complex numbers. Basically, the equations were input directly as derived above. First, the dimensions and material properties of the beam and actuators were input. Then, expressions for P_m and W_m were determined for both excitations. Each series will reach a point of convergence; so, for ease of calculations, they were truncated to eight terms. If done properly, this results in only a negligible loss of accuracy. The W_m terms were found over a wide range of frequencies to observe the frequency response of the moment at the beam root. The final step was to plug all the terms into Eq. (4). This produces an array consisting of the value of the moment at each frequency. This was done in a closed-form numerical manner by pre-integrating. The equation became

$$M=-a\rho\omega^2\sum_{m=1}^{8} W_m\int_0^L X_m(x)\,x dx. \qquad (18)$$

Next, it is important to designate what is meant by the phase of actuator excitation. If the externally applied load is varying as

$$P=P\sin(\omega t), \qquad (19)$$

then the applied voltage to the actuators varies as

$$V=V\sin(\omega t+\phi). \qquad (20)$$

What is really being attempted here is a form of open-loop control. At different frequencies and excitation locations, the beam will be more responsive to the actuators than the external load or vice versa. For this reason, the actuators need to apply more or less force to minimize the inertial moment, which in turn will minimize the stress. If the beam is at a frequency where it is more responsive to the actuators than the load, the actuators can actually do more harm than good if the applied voltage is not varied. This can happen even if they are excited at the proper phase. So, for a fixed magnitude, harmonic external load, all that can really be varied is the applied voltage, phase, and position of the actuators. Therefore, the program includes an algorithm to determine the proper phase and voltage at which to excite the actuators for a range of frequencies. One final note is that there will always be a maximum voltage that actuators are capable of handling; therefore, this limitation was also included in the program. If the actuators could handle unlimited voltage, and thus exert a potentially unlimited force, the stress could always be reduced to zero for all frequencies.

6. RESULTS

Once the program was completed, the results were verified with those obtained from BEAM VI. Then, a parametric study was conducted to determine which actuator positions provided the best reduction in the dynamic moment at various frequencies. Since the resonant frequencies usually cause the largest vibrations, they were of special concern. The following data were found using the same graphite/epoxy beam that was used in the experiment described in section 3. The PZT actuator patches were 5 cm long. The actuator positions investigated were as follows: Position #1 from 0.1 to 5.1 cm, Position #2 from 0.6 to 5.6 cm, and Position #3 from 2.1 to 7.1 cm. Special attention should be paid to the position of the nodes at different frequencies. If the actuators span a node, they will not be able to excite that mode effectively because the moment produced by the actuators will be self-cancelling. To check that these actuator locations were acceptable, the node points were found using BEAM VI. Additionally, a loss factor of 0.005 was used. Finally, the range of frequencies being investigated was from 20 to 1400 Hz. This range includes the first three natural frequencies of 65.9 Hz, 413.0 Hz, and 1156.3 Hz. Rarely in structural applications is there concern for any more modes.

First, the dynamic moment was found. As stated previously, the program written for this work contains an algorithm to find the minimum moment possible for each actuator position. The external load used in this analysis had a magnitude of 0.8 N and was applied ¼ of the beam's length from the free end. Plots of the dynamic moment due to the external load and actuators at the three different locations are shown in Figures 4, 5, 6, and 7.

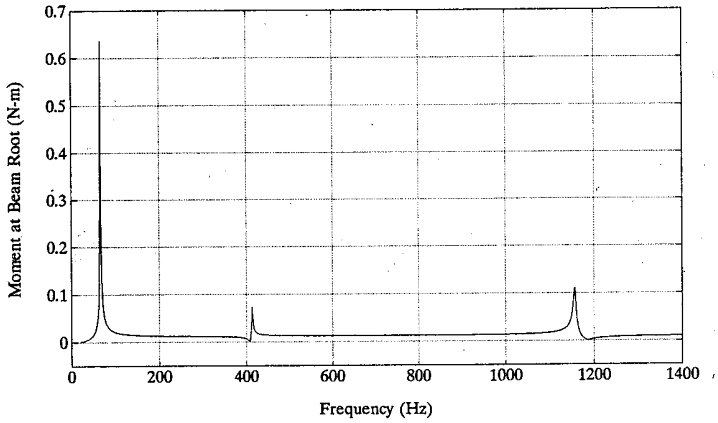

Figure 4: Dynamic moment due to external load only.

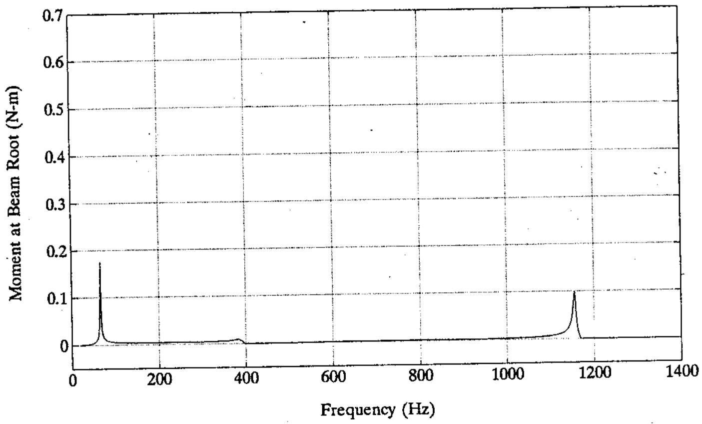

Figure 5: Total dynamic moment with actuators in Position #1.

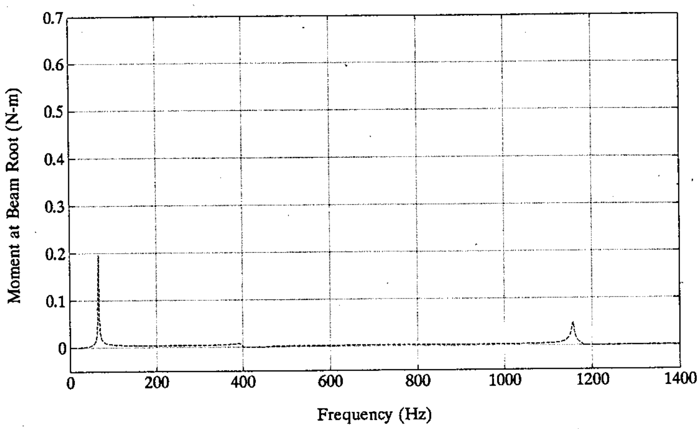

Figure 6: Total dynamic moment with actuators in Position #2.

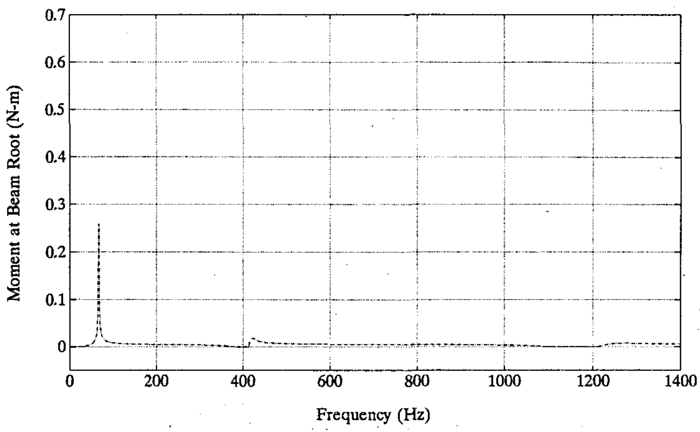

Figure 7: Total dynamic moment with actuators in Position #3.

From analyzing these results, it can be seen that the moment due to the external load is greatest at the first resonant frequency, followed by the third, then the second. Naturally, since the entire beam vibrates up and down together in the first mode, the best actuator position would be as close to the wall as possible. This is confirmed by the plots above. However, if the control of the vibration at the third natural frequency is desired, having the actuator very close to the wall would be one of the worst positions. In fact, the effect that the actuators at Position #1 have on the third mode is almost negligible. By positioning the actuators a little farther away like in Position #3, the magnitude of the moment at the third mode can be greatly reduced; however, the moment at the fundamental frequency is slightly higher. Also, if control of all three modes was needed, a compromise position like Position #2 would be the best. At this location, there is marginal reduction in the moment at both the first and third modes. Additionally, it should be noted that all three positions provide adequate reduction of the moment at the second mode. The beam seems to be much more responsive to the actuators than to the external load at this frequency. Finally, the moment reduction with the two symmetrically bonded PZT actuators is significant when a very low magnitude load is applied; however, with greater applied loads, other orientations such as PZT stacks may be required in order to obtain sufficient stress reduction.

The next item of interest is the ideal voltage that is required to minimize the moment due to inertia. The maximum voltage was limited to 300 V in the program. The full voltage is not required at all frequencies in order to reduce the net moment to zero. The plot of the voltage required to minimize the moment at the beam root is shown in Figure 8.

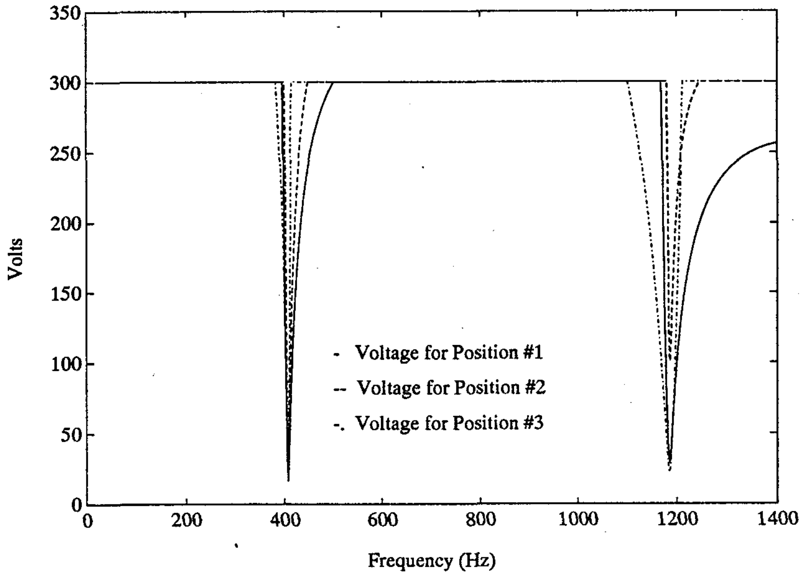

Figure 8: Actuation voltage required to minimize the dynamic moment.

By looking at these plots, it can clearly be seen that the maximum voltage is not required at all frequencies. The large drops in required voltage are located at the anti-resonant frequencies for the second and third modes. At these frequencies, the moment is very small to begin with (see Figure 4), so not much voltage is required to reduce it to zero. One item of interest regarding the required voltage is that the stress can always be reduced to zero if the actuators are capable of handling unlimited voltage. This is because, theoretically, they would be capable of putting out unlimited force. Furthermore, in this example, only a 0.8 N load was used. If sufficiently large loads are applied, this voltage curve would just be a straight line at 300 V. This is because the magnitude of the moment due to the external excitation would be so great that even at the anti-resonant frequencies that the net moment could not be reduced to zero even with the maximum allowable voltage.

The final concern in minimizing the inertial moment is the proper phase of activation. Again the pre-described program contains an algorithm to find this. Common sense tells us that at the first resonant frequency, a 180° phase difference is needed, as was used in the initial experiment. However, it is not quite as clear what is needed at the higher modes. The plot of the required phases of activation to minimize the moment is shown in Figure 9.

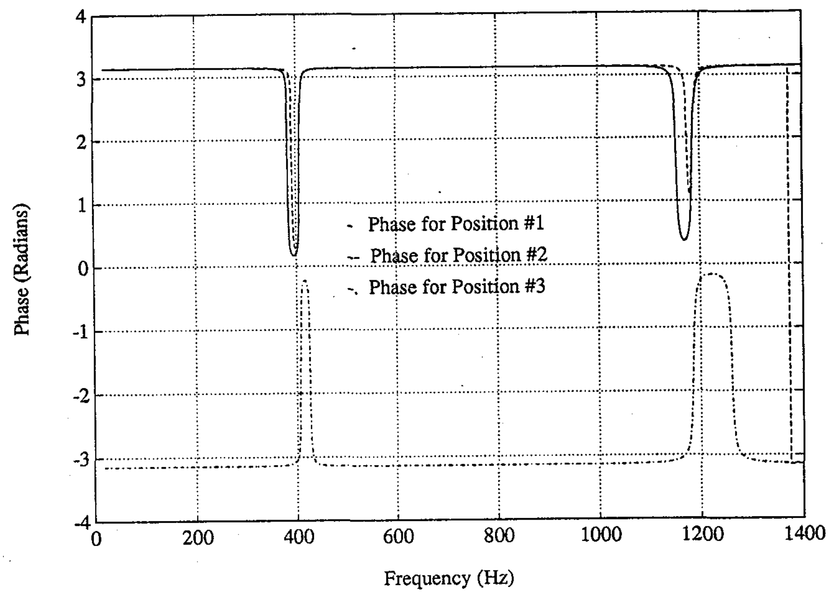

Figure 9: Phase of actuator excitation required to minimize the dynamic moment.

An examination of this plot indicates that the actuators should be excited 180° out-of-phase except near the second and third natural frequencies. Around these modes, the excitations need to be almost, but not completely, in-phase to properly minimize the stress.

7. CONCLUSION

In this paper, a way to increase the fatigue life of beam structures with adhesive bonding by using intelligent material systems has been provided. It was first shown experimentally that the fatigue life of an adhesive bond of a cantilever beam can be increased by nearly an order of magnitude using the approach of indirect stress cancellation. Then, the mathematics concerning the dynamic response of the beam were derived and a computer program was presented which illustrated the active stress reduction phenomenon. In this program, the effect of actuator position on the total net moment at the root of the beam was addressed. Additionally, algorithms were written to find the proper applied voltage and phase of activation of the actuators. In essence, a form of open-loop control is presented from which the moment due to inertia at the beam root, and thus the peeling stress, can be minimized. This will in turn dramatically increase the fatigue life of the adhesive bond.

8. ACKNOWLEDGMENTS

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