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Predicting the vibration characteristics of elements incorporating Incompressible and Compressible Viscoelastic Materials

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ABSTRACT

In order to predict accurately the vibration characteristics of viscoelastic elements and viscoelastically-damped structures, the use of frequency-dependent parameters such as complex modulus and Poisson's ratio is important. Several techniques have been developed for measuring the frequency-dependent complex modulus of viscoelastic materials. However, the accurate determination of Poisson's ratio of viscoelastic materials is much less developed. This quantity is important as its commonly quoted value of 0.5 can be very different when a viscoelastic material is in its transition or glassy region or if the material is compressible.

In this paper, prismatic viscoelastic samples are employed to predict the value of Poisson's ratio using the Finite Element Method (FEM). The transmissibility characteristics of these prismatic samples are established experimentally and FEM is used in conjunction with measured complex Young's modulus and iterated values of Poisson's ratio such that the predicted FEM results agree as well as possible with the experimental data. It is shown that the method suggested is able to predict accurately the Poisson's ratio of incompressible and compressible viscoelastic materials.

1. INTRODUCTION

The use of frequency-dependent parameters such as the complex moduli and Poisson's ratio is important in terms of predicting the vibration characteristics of viscoelastic elements. Techniques used to determine the frequency-dependent complex moduli of viscoelastic materials have been well developed [1]. However, the same statement cannot be said about the accurate determination of Poisson's ratio of viscoelastic materials. The Poisson's ratio of a viscoelastic material can be determined by measuring the Young's and shear moduli of the material. Both moduli can be determined by conducting two separate tests. However, not only is the procedure tedious, but results for Poisson's ratio are very sensitive to errors in the measurement process. An alternative method is to use a Finite Element approach.

The aim of this paper is to show how prismatic viscoelastic samples are employed to predict the value of Poisson's ratio using the Finite Element Method, ABAQUS [2]. The transmissibility characteristics of these prismatic samples are established experimentally and FEM is used in conjunction with measured complex Young's modulus and iterated values of Poisson's ratio such that the predicted FEM results agree with the experimental data.

The Poisson's ratio of both incompressible and compressible materials are determined. The determined Poisson's ratio is used to predict the vibration characteristics of an anti-vibration mount which incorporates both incompressible and compressible viscoelastic materials.

2. POISSON'S RATIO EFFECT

From the theory of elasticity, the Young's modulus E, can be expressed as

$$E = 2G(1 + \nu) \tag{1}$$

or

$$E = 3K(1 - 2\nu) \tag{2}$$

hence

$$\nu = \frac{3 - 2G/K}{6 + 2G/K} \tag{3}$$

where
G is the shear modulus,
K is the bulk modulus,
and ν is the Poisson's ratio.

The dynamic moduli of viscoelastic materials may be divided into three regions, the rubbery region, the transition region and the glassy region. If a homogeneous viscoelastic material (e.g. rubberlike materials) is operating in the rubbery region where the value of K is several orders higher than G in magnitude, then from equation 3, the value of Poisson's ratio will be close to 0.5 since the ratio of G/K tends to zero. When the same homogeneous viscoelastic material operates in the transition or glassy region where the value of K is of the same order as G, then from equation 3, the value of Poisson's ratio will be less than 0.5.

If a viscoelastic material has a certain volume fill of air voids, the Poisson's ratio will be lower than a homogeneous rubberlike material operating in the same region.

Thus the Poisson's ratio of viscoelastic materials may change according to the operating conditions and the constituents and hence it is necessary to establish the value of Poisson's ratio of viscoelastic materials if accurate dynamic predictions are required. A viscoelastic material with a Poisson's ratio of close to 0.5 is considered as *incompressible*. If the Poisson's ratio is less than 0.5 it is considered as *compressible.*

3. DETERMINATION OF POISSON'S RATIO USING FEM

The procedure employed is as follows. The vibration transmissibility characteristics of two cylindrical prismatic viscoelastic samples, a "long" and a "short" sample, are determined experimentally. The vibration transmissibility characteristics of both samples must be determined under the same temperature and strain level to ensure consistency; "short" refers to a cylindrical sample with a diameter to length ratio of greater than unity and "long" refers to a cylindrical sample with a diameter to length ratio of less than unity.

When a prismatic viscoelastic sample is deformed in compression or tension and no slip condition is imposed at the ends, the resulting lateral strain to the longitudinal strain is described by Poisson's ratio, ν. However, at each end of the sample the Poisson's ratio effect will be prevented because the sample is bonded to the end plates. This constraint will tend to stiffen the sample in compression or tension. For a sufficiently " long " sample this effect is small but for a " short " sample the effect will be significant. Thus results for a " long " sample will be relatively insensitive to the value of Poisson's ratio used in the prediction whereas for a " short " sample, the value of Poisson's ratio used in the prediction is critical. The vibration transmissibility characteristics of these two samples can then be predicted using the finite element method incorporating the experimentally measured complex Young's modulus. The predicted vibration transmissibility characteristics obtained from the "long" sample act as a benchmark to indicate the accuracy of the measured Young's modulus; during this stage, values of Poisson's ratio used in the predictions of the "long" sample vibration transmissibility characteristics will have little effect on the predicted results. Using the same value of complex Young's modulus, the predicted results obtained from the "short" sample are "adjusted" to agree as well as possible with the experimental data by iterating the value of the Poisson's ratio.

4. THE PRISMATIC SAMPLES

Two different types of viscoelastic materials, one homogeneous and the other composite (embedded with air voids) were used to verify the above method of determining Poisson's ratio. For each viscoelastic material, a "long" and a "short" cylindrical sample with a diameter of 30 mm, bonded at both ends to steel plates, was used. The lengths of the "long" and "short" sample were 40 mm and 15 mm respectively. The vibration transmissibility characteristics were obtained experimentally under the same test conditions with a mass of 0.135 kg at the free end.

The finite element predictions of the vibration transmissibility characteristics of these samples were carried out using the measured complex Young's modulus. Initially, Poisson's ratio was assumed to be a *real* value and *constant* within frequency range of interest.

5. THE EFFECT OF POISSON'S RATIO IN FE PREDICTIONS

The effect of Poisson's ratio can be different for different geometry. This can be demonstrated by predicting the static stiffnesses of the "long" and "short" samples. Both samples are assumed to have the same material properties. Figures 1a and 1b show the element model used in the predictions for the "long" and "short" samples. Axisymmetric elements were used to model these samples. Table 1 shows the percentage difference in the predicted static stiffnesses of both samples with different values of Poisson's ratio. In the case of the "long" sample, there is only a 10 % difference in the predicted static stiffness using Poisson's ratio of 0.3 and 0.499999. However a difference of 43 % is obtained in the case of the "short" sample.

Poisson's ratio also influences the dynamic characteristics. This can be demonstrated by predicting the vibration transmissibility characteristics of the two prismatic sample using different values of Poisson's ratio. Table 2 shows the percentage difference in the predicted first resonant frequency of the two samples with various values of Poisson's ratio. A percentage difference of 4.5 % is found in the predicted resonant frequency using Poisson's ratio of 0.3 and 0.499999 in the case of the "long" sample. This difference increases to a value of 18.3 % in the case of the "short" sample. Once again, the effect of Poisson's ratio is more significant in the "short" sample than the "long" sample results.

6. EXPERIMENTAL WORK

To carry out vibration transmissibility tests, the sample was mounted vertically on a shaker with the accelerometers attached to the input end and the free end (see figure 2). The input and output accelerations were measured and the ratio of the output to the input gave the vibration transmissibility of the sample. The rocking of the shaker table was checked and found to be neglible.

The acquisition and processing of test data were controlled by an HP 400 Series Unix Workstation running the commercial dynamic test software package from Leuven Measurement System (LMS) interfacing a DIFA

Scadas 2 16 channels data acquisition system. The response of the samples was measured using PCB336M13/M14 acceleration transducers. A Ling 500 series electrodynamic shaker was used to excite the samples.

7. POISSON'S RATIO OF INCOMPRESSIBLE MATERIALS

Two cylindrical prismatic samples of natural rubber were excited over a frequency range of 0 - 500 Hz, at room temperature (20 deg C). Figure 3 shows the measured complex Young's modulus and loss factor of the natural rubber obtained from the direct stiffness method.

Figures 4 and 5 show the predicted and measured results of the natural rubber "long" sample using a Poisson's ratio of 0.3 and 0.499999 respectively. Poisson's ratio of 0.499999 provides a better comparison between the predicted and the experimental results. The measured resonant frequency of 94 Hz is the mass-spring frequency of the "long" sample. The predicted resonant frequency shows a difference of 5 % using Poisson's ratio of 0.3 or 0.499999. Accepting this difference as the maximum, it can be argued that the "long" sample is fairly insensitive to Poisson's ratio effect and the measured Young's modulus used in the prediction is reliable. The same measured Young's modulus is then used to predict the vibration transmissibility characteristics of the "short" sample.

Figures 6 and 7 show the predicted and measured results of the natural rubber "short" sample using a Poisson's ratio of 0.3 and 0.499999 respectively. The measured resonant frequency of 174 Hz is the mass-spring frequency of the "short" sample. From figure 6, there is a difference of 8 % in terms of the resonant frequency between the predicted and measured results using Poisson's ratio of 0.3. However from figure 7, the predicted and measured results agree well when Poisson's ratio of 0.499999 is used. It can be concluded that the Poisson's ratio of this viscoelastic material has a value close to 0.5. This value of Poisson's ratio agrees with the often quoted value of 0.499999 for natural rubber operating in the rubbery region and is considered to represent *incompressible* materials.

8. POISSON'S RATIO OF COMPRESSIBLE MATERIALS

The same procedure was repeated with a viscoelastic epoxy resin material which had 20 % by volume of air voids. It displays a transition frequency at room temperature and high damping properties (with a loss factor of 1.5) as shown in figure 8. The epoxy resin "long" sample was excited over a frequency range of 0 - 1600 Hz and the "short" sample was excited over a frequency range of 0 - 2500 Hz. Both samples were tested at 20 deg C. Within the test frequency range, there is a rapid increase in the modulus and the loss factor increases to a maximum at the transition frequency and then decreases with frequency.

Figures 9, 10 and 11 show the predicted and measured results of the epoxy resin "long" sample using a Poisson's ratio of 0.3, 0.4 and 0.499999 respectively. The results from these three figures show that the sample is fairly insensitive to the value of Poisson's ratio used in the predictions. Good correlation between the experimental and the measured results indicate that the measured complex Young's modulus of this material is accurate and can be employed in the "short" sample analysis. However; ideally a somewhat longer sample would be preferred to reduce the sensitivity of Poisson's ratio further.

Figures 12, 13 and 14 show the predicted and measured results of the epoxy resin "short" sample using a Poisson's ratio of 0.3, 0.4 and 0.499999 respectively. From these three figures, the predictions using Poisson's ratio of 0.4 correlates well with the measured vibration transmissibility characteristics of the "short" sample whereas the results for 0.3 and 0.499999 are significantly in error. This viscoelastic material with a Poisson's ratio of 0.4 is considered as *compressible*. It can also be seen that Poisson's ratio of this composite viscoelastic material is constant over the range of frequency predicted.

9. ANTI-VIBRATION MOUNT

Finite element predictions of the vibration transmissibility characteristics of an anti-vibration mount commonly used in marine vessels were carried out using the measured complex modulus of the damping compounds.

The anti-vibration mount is made from stainless steel. The outer and inner U shaped leaf springs are rivetted together at the open ends with face plates to form an oval shaped assembly. The space between the inner and outer leaf springs is filled with an epoxy resin damping compound. In addition, the anti-vibration mount has a viscoelastic rubber block known as the accelerator unit which acts as a high frequency attenuator. This viscoelastic rubber block is made of natural rubber and has a square cross section with a hole through the middle of the block. The opposite ends of the rubber are bonded to thin mild steel plates.

10. FE MODEL OF THE ANTI-VIBRATION MOUNT

Figure 15 shows the FE model of the complete anti-vibration mount. The leaf springs of the mounts were modeled by three noded beam elements. Eight noded plane strain elements were used to model the damping compound (epoxy resin). Eight noded plane stress elements were used to model the viscoelastic rubber block. Due to the symmetry of the mounts, only half of the mounts were modelled.

The vibration characteristics of the anti-vibration mount were established experimentally to validate the predictions. The value of Poisson's ratio used in the predictions in the case of the epoxy resin damping compound and the viscoelastic rubber block were 0.4 and 0.499999 respectively. Figures 16 and 17 show the predicted and the measured vibration transmissibility characteristics of the complete anti-vibration mount without and with a preload of 0.5 kg respectively The predicted results correlate well with the measured results.

11. CONCLUSION

In order to predict accurately the vibration characteristics of any viscoelastic elements, a realistic value of Poisson's ratio must be known. The value of Poisson's ratio depends strongly on the operating conditions and the constituents of the viscoelastic materials. A method to establish the value of Poisson's ratio has been developed. The results obtained have clearly shown that the method is able to accurately determine the Poisson's ratio of both *incompressible* and *compressible* viscoelastic materials.

12. ACKNOWLEDGEMENT

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13. REFERENCES

1. S.O. Oyadiji and G.R. Tomlinson, 1991
   Establishing the Validity of the Master Curve Technique
   for Complex Modulus Data Reduction
   Proceedings of Damping 1991 Conference,San Diego, USA, February 1991,
   pp DBC1-20.

2. Abaqus Theory Manual
   Version 5.2

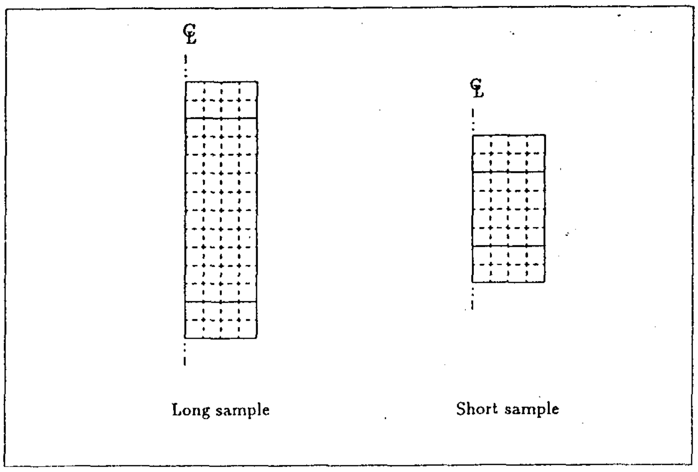

Figure 1: FE mesh of the long and short sample

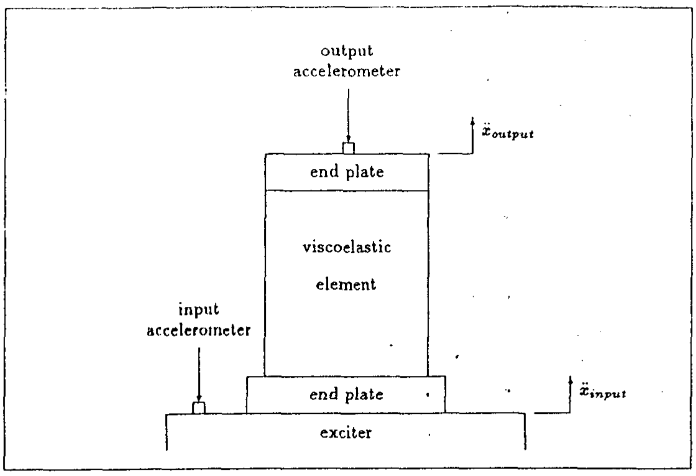

Figure 2: Experimental setup for benchmark work

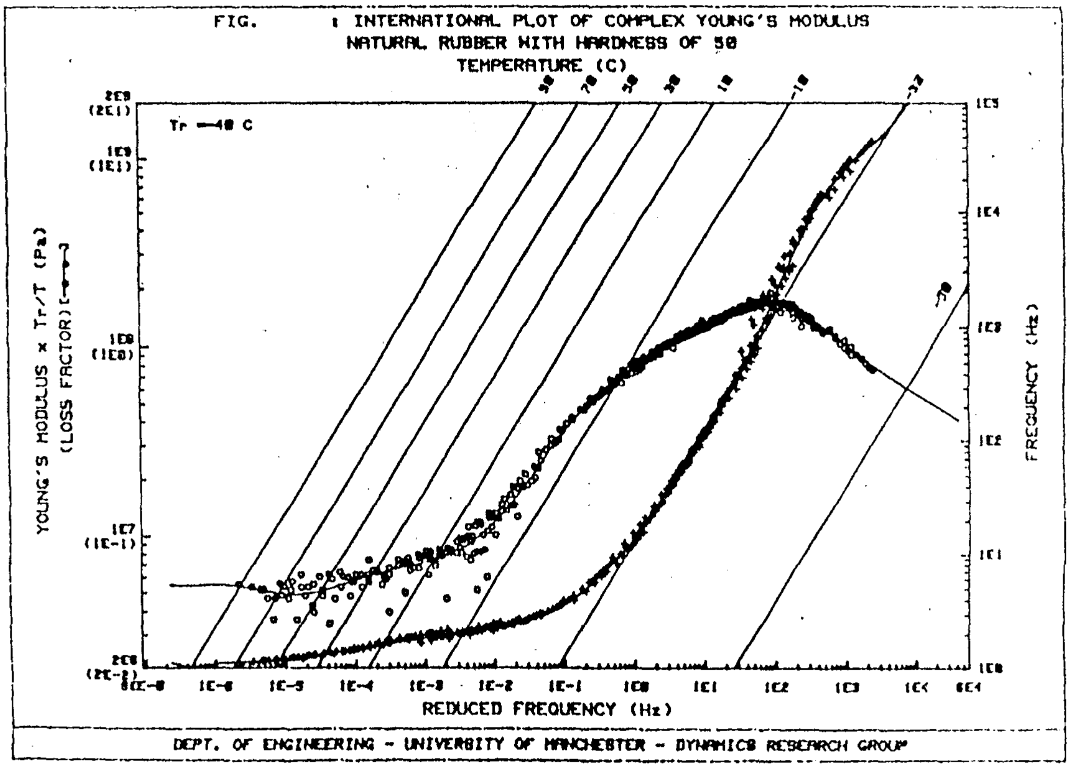

Figure 3: Master curve of the natural rubber

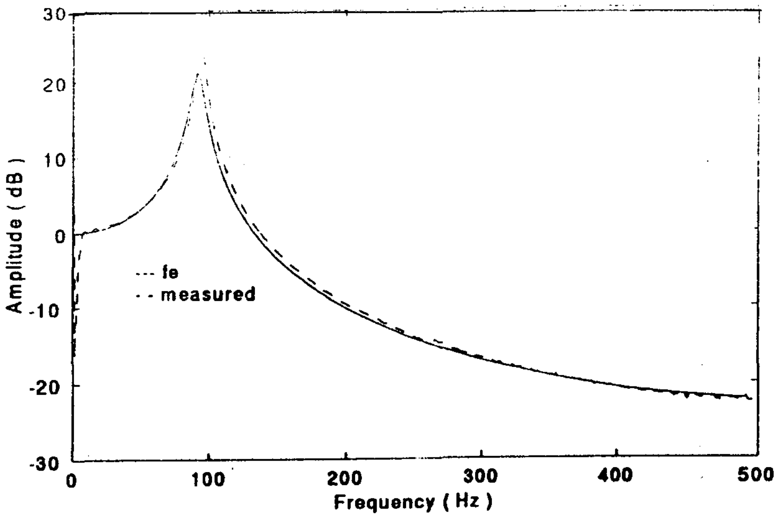

Figure 4: Predicted and measured vibration transmissibility characteristics of the long natural rubber sample using Poisson's ratio of 0.3

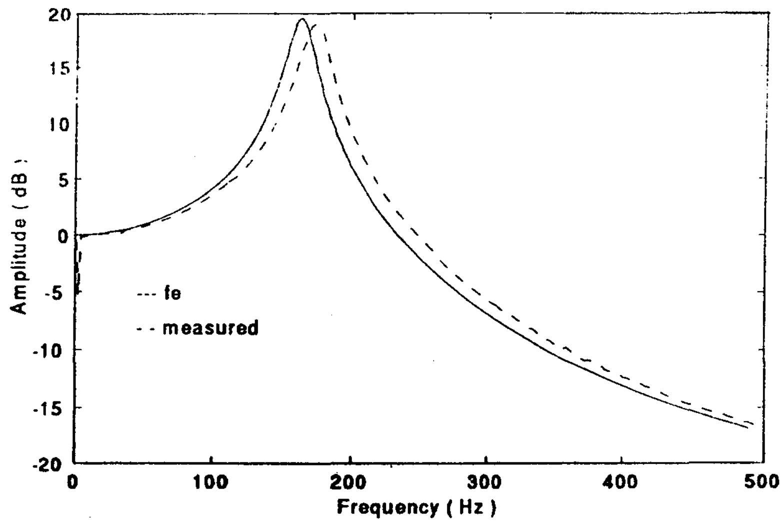

Figure 6: Predicted and measured vibration transmissibility characteristics of the short natural rubber sample using Poisson's ratio of 0.3

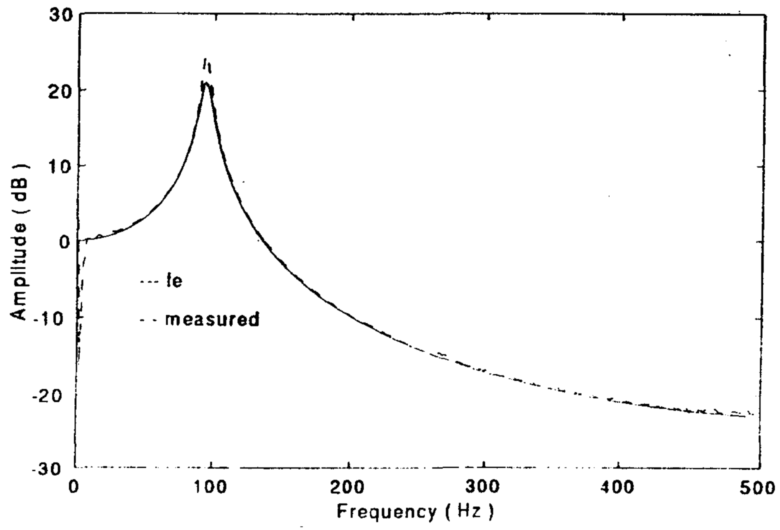

Figure 5: Predicted and measured vibration transmissibility characteristics of the long natural rubber sample using Poisson's ratio of 0.499999

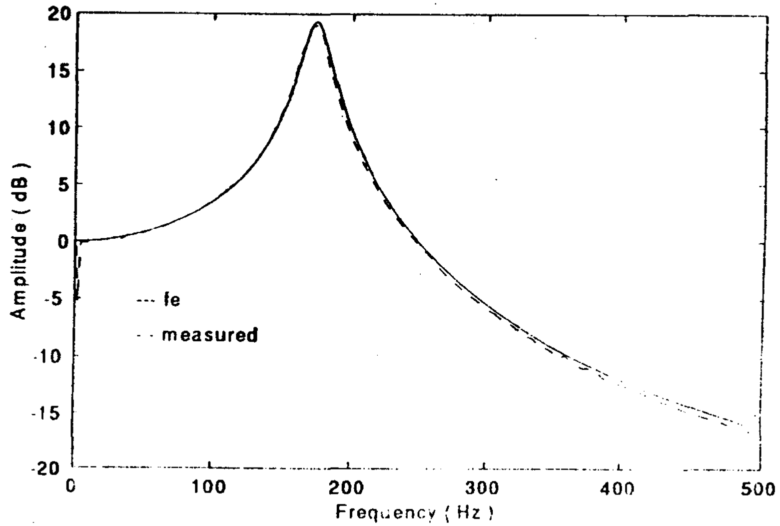

Figure 7: Predicted and measured vibration transmissibility characteristics of the short natural rubber sample using Poisson's ratio of 0.499999

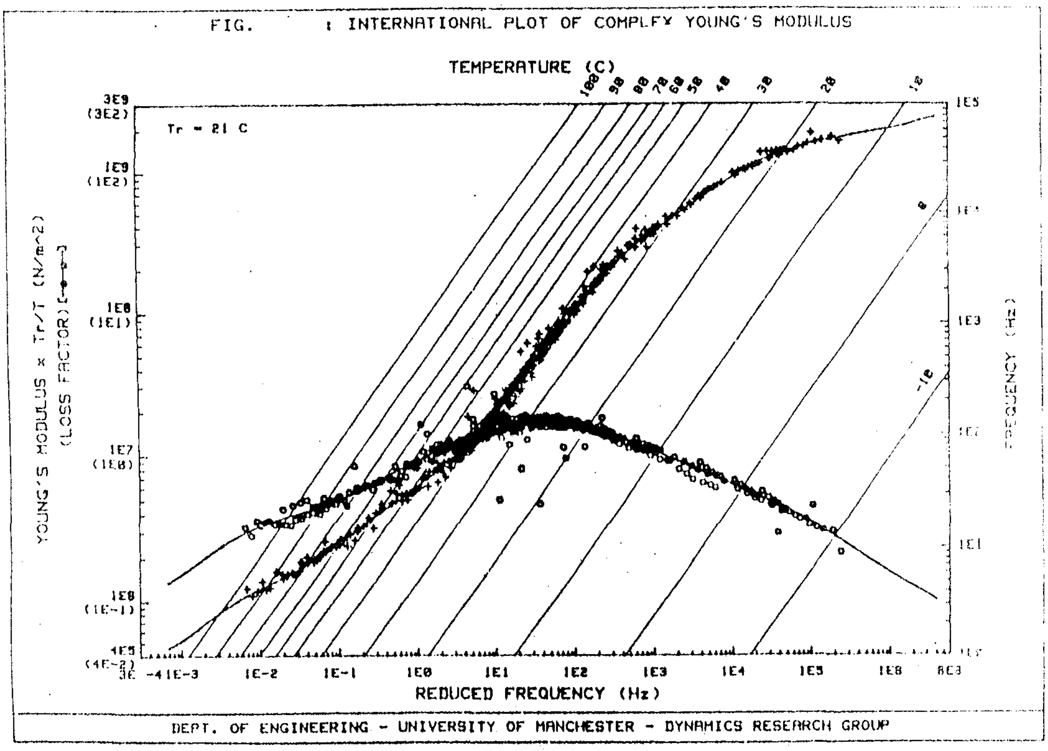

Figure 8: Master curve of the epoxy resin

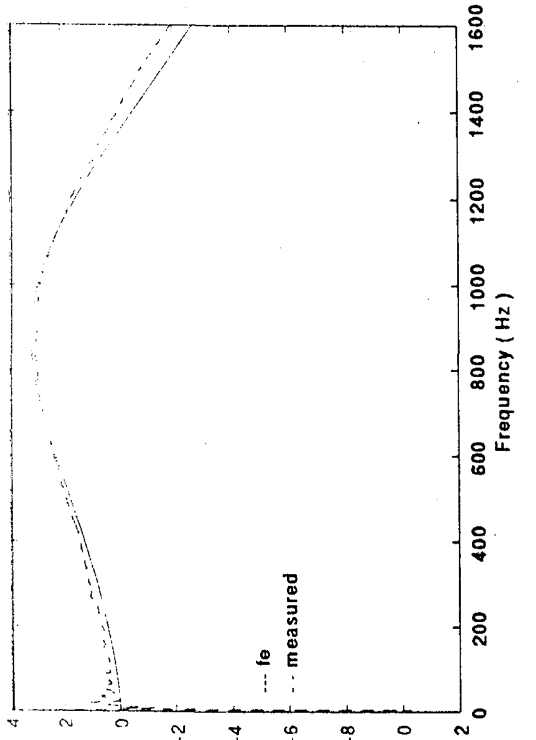

Figure 9: Predicted and measured vibration transmissibility characteristics of the long epoxy resin sample using Poisson's ratio of 0.3

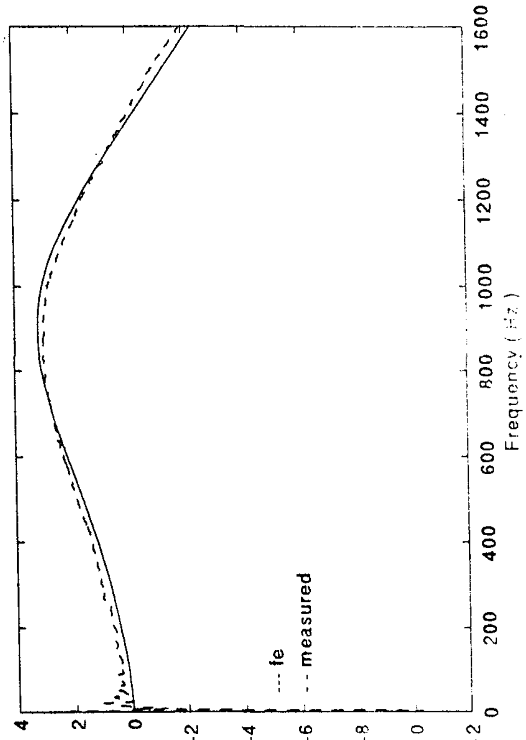

Figure 10: Predicted and measured vibration transmissibility characteristics of the long epoxy resin sample using Poisson's ratio of 0.4

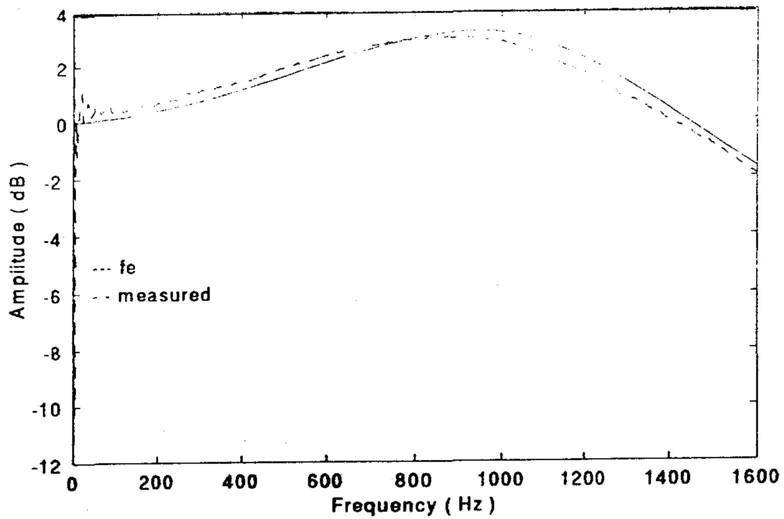

Figure 11: Predicted and measured vibration transmissibility characteristics of the long epoxy resin sample using Poisson's ratio of 0.499999

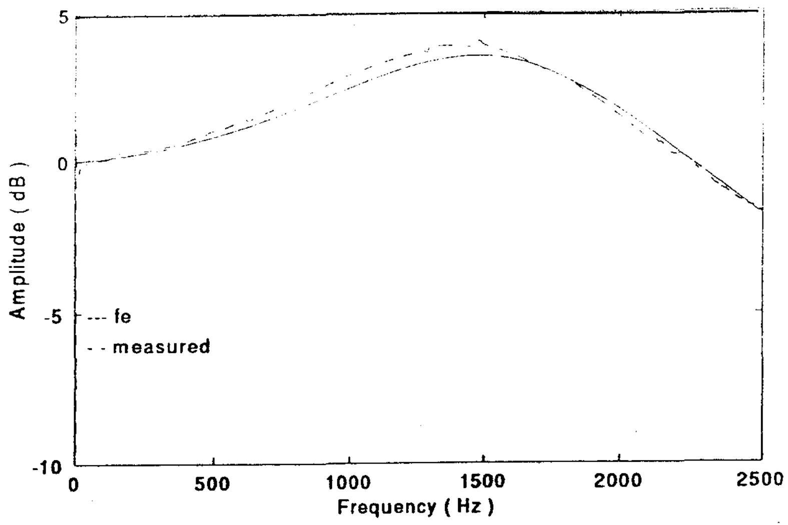

Figure 13: Predicted and measured vibration transmissibility characteristics of the short epoxy resin sample using Poisson's ratio of 0.4

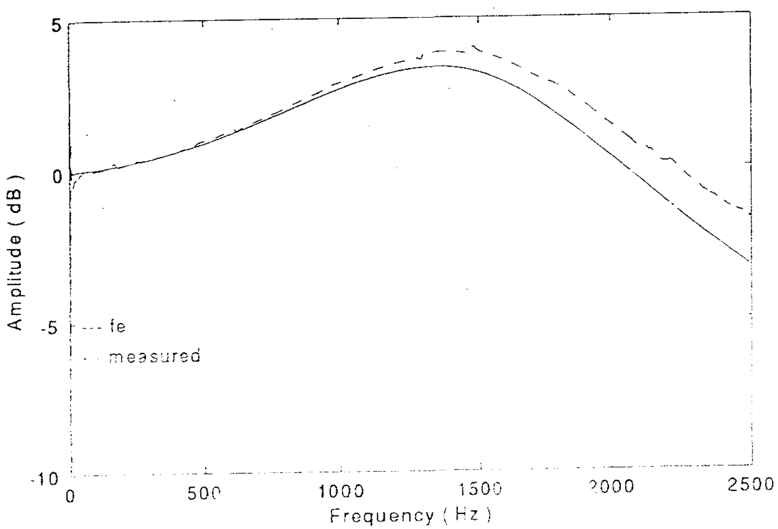

Figure 12: Predicted and measured vibration transmissibility characteristics of the short epoxy resin sample using Poisson's ratio of 0.3

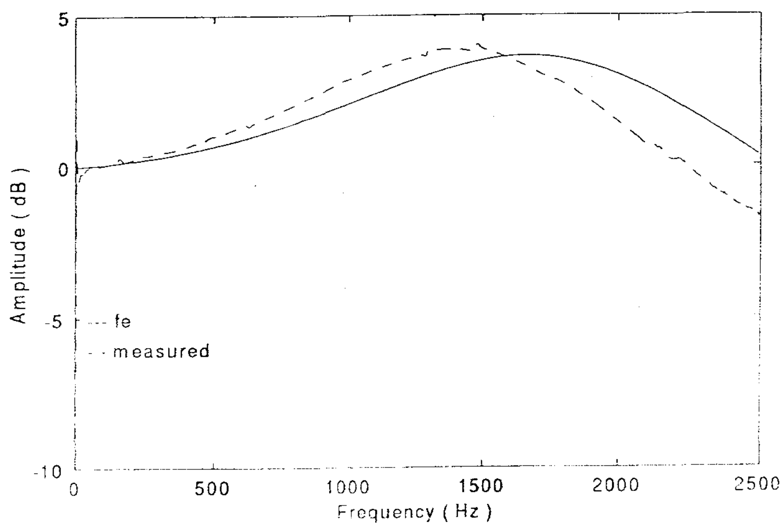

Figure 14: Predicted and measured vibration transmissibility characteristics of the short epoxy resin sample using Poisson's ratio of 0.499999

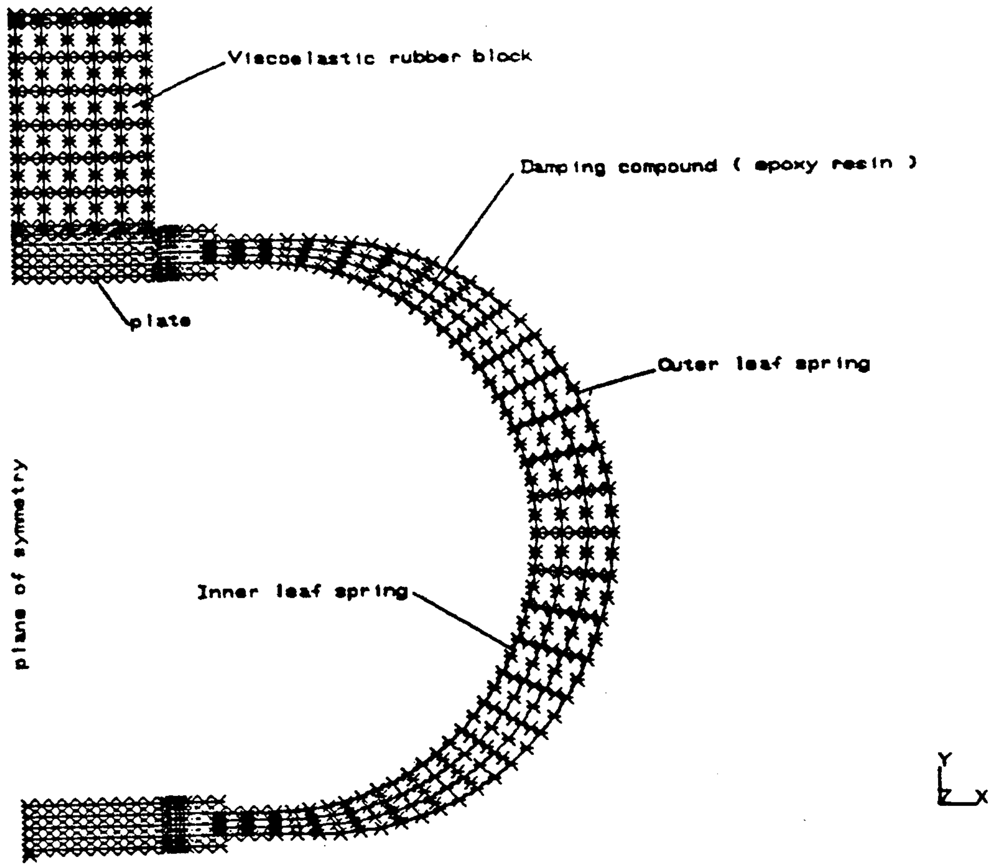

Figure 15: FE model of the anti-vibration mount

	% difference w.r.t. $\nu = 0.499999$	
Poisson's ratio, ν	long sample	short sample
0.3	10.6	43.2
0.4	6.4	25.6

Table 1: Predicted static stiffnesses

	% difference w.r.t. $\nu = 0.499999$	
Poisson's ratio, ν	long sample	short sample
0.3	4.5	18.3
0.4	2.8	10.9

Table 2: Predicted first resonant frequency

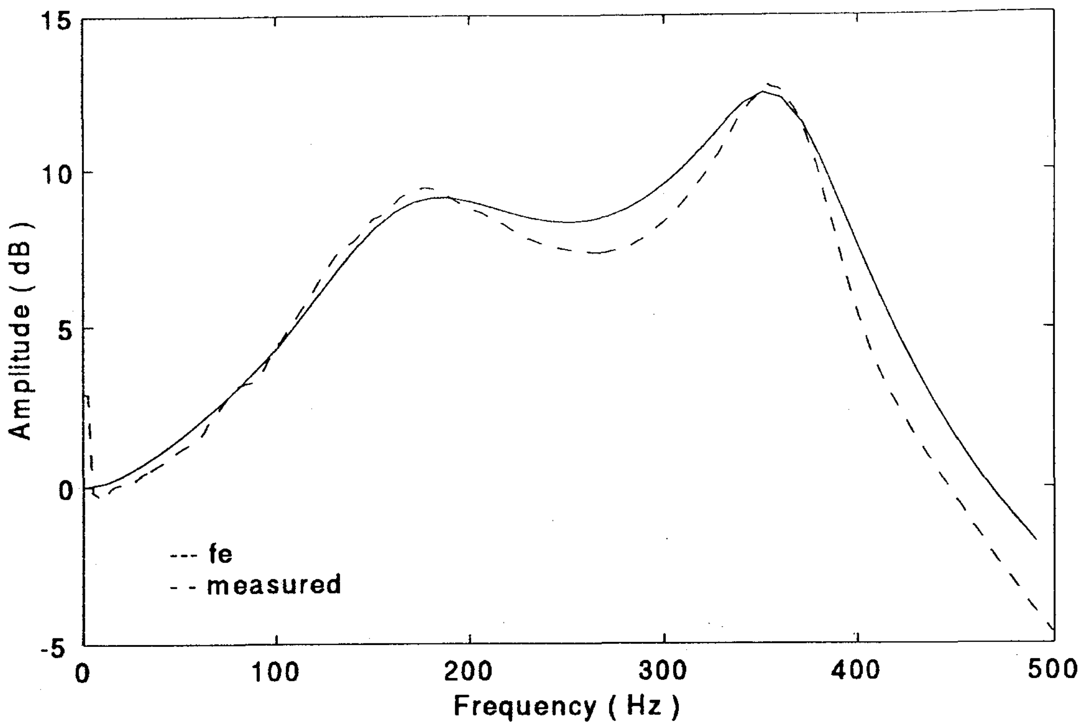

Figure 16: Predicted and measured vibration transmissibility characteristics of the anti-vibration mount without preload

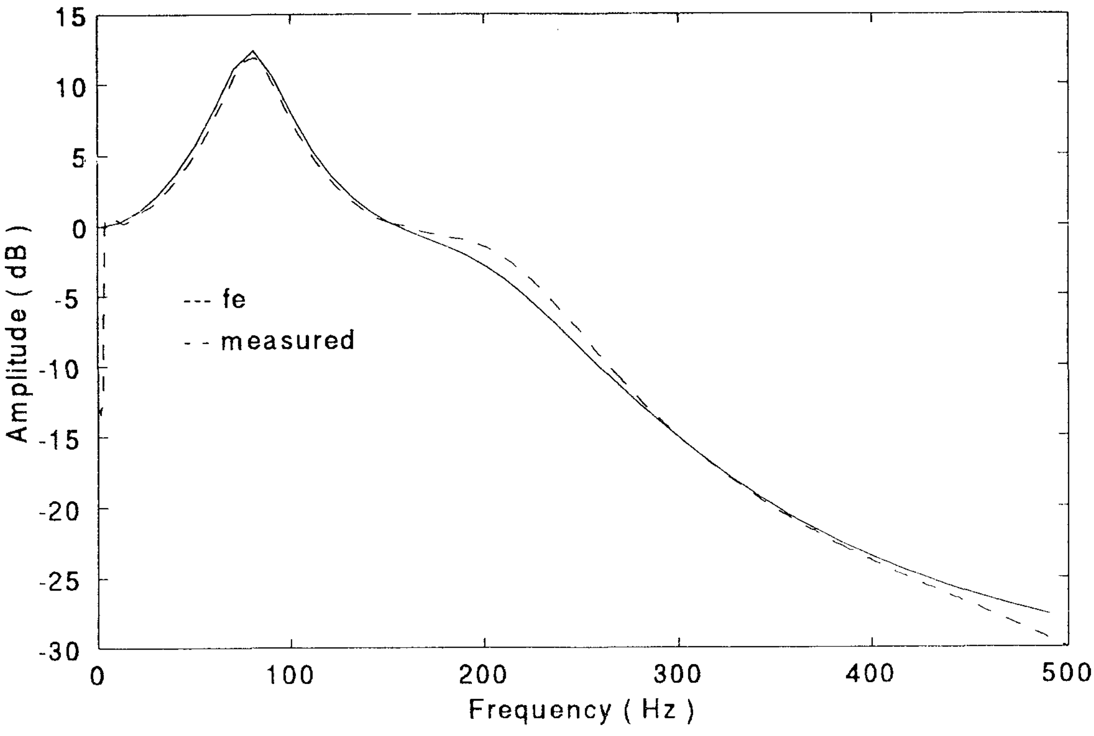

Figure 17: Predicted and measured vibration transmissibility characteristics of the anti-vibration mount with 0.5 kg of preload