A Comparison of numerous lap joint theories for adhesively bonded joints

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ABSTRACT

Numerous authors have investigated the state of stress in the adhesive of adhesively bonded joints. They have made various assumptions concerning the behavior of the adhesive and adherends to yield tractable differential equations which remove the stress singularities which occur at the edges of the bi-material interfaces. By examining several test problems, this paper investigates the effect of these assumptions on predicted adhesive stress.

INTRODUCTION

Lap joint theories predict the state of stress in the thin adhesive bonding adherend plates. In their classic paper, Goland and Reissner (1944)[1] presented the first modern lap joint theory. Subsequently, numerous authors have proposed theories which have improved upon Goland and Reissner's basic formulation (see Carpenter [1991][2]). The common feature of all these theories is that simplifying assumptions are made concerning the behavior of the adherends and of the adhesive. These assumptions remove the stress singularities which occur at the edges of the interfaces of the adhesive and the adherends and yield tractable differential equations which can be solved to yield stress in the adhesive. Maximum adhesive stresses from these solutions can then be used in joint design. The numerous authors who have used this approach in analyzing adhesively bonded joints have arrived at their basic differential equations by making varying simplifying assumptions. The manner in which these assumptions affect predicted adhesive stress is the topic of this paper.

The effect of a given assumption on predicted adhesive stress is difficult to determine with a differential equation approach. However, Carpenter and Barsoum (1989)[3] recently presented special adhesive finite elements which can be used to model the adhesive while plate or beam elements can be used to model the adherends. With these adhesive elements, control parameters are used to specify which assumptions are to be considered. It has been shown that results using this finite element approach converge to those of the lap joint theory having the same set of underlying assumptions. By examining the effect of control parameters on adhesive stress, the importance of any given assumption associated with a lap joint theory can be ascertained. In this paper, the common assumptions found in most lap joint theories are first discussed. Test problems are then described and the effect on predicted maximum adhesive stress of various assumptions is then investigated.

COMMONLY USED ASSUMPTIONS

The following is a description of common assumptions used in developing lap joint theories. In this paper, a set of control parameters (α, α_1, α_2, α_3, α_4, α_5, α_6, IFIN, and IPLANE) are used to prescribe what assumptions are currently being considered. These control parameters are nothing more than switches that can be turned on, off, or set to certain values, to effect a given assumption. The significance of these control parameters is next discussed.

DISPLACEMENT ASSUMPTION AND THE STRAIN-DISPLACEMENT EQUATIONS

Examine the lap joint of Figure 1. Let u be the displacement in the adhesive in the x direction and let w be the displacement in the z direction. Most lap joint theories assume that the displacements in the adhesive vary thus
where c_1, c_2, c_3, and c_4 are constants and where $f_1(x)$ and $f_2(x)$ are some function of x.

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$$u(x,z) = (c_1 + c_2 z)\, f_1(x)$$
$$w(x,z) = (c_3 + c_4 z)\, f_2(x) \qquad (1)$$

The strain-displacement equations for the adhesive are

$$\epsilon_x = \frac{\partial u}{\partial x}$$
$$\epsilon_z = \frac{\partial w}{\partial z} \qquad (2)$$
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \alpha_1 \frac{\partial w}{\partial x}$$

where α_1 is a control parameter which must be 1 if the complete shear strain-displacement equation is used but which is taken to be 0 by some authors.

Entering equation (1) into equation (2) gives

$$\epsilon_x = (c_1 + \alpha_2 c_2 z) f_1'(x), \qquad \epsilon_z = c_4 f_2(x)$$
$$\gamma_{xz} = c_2 f_1(x) + \alpha_1 (c_3 + \alpha_2 c_4 z) f_2'(x) \qquad (3)$$

where a prime denotes differentiation with respect to x and where $\alpha_2 = 1$ if no terms in the strain expressions are being neglected. The parameter α_2 can be set to zero to force the state of stress and strain in the adhesive to be constant through the thickness of the adhesive.

Authors such as Goland and Reissner and Delale and Erdogan (1981)[4] use an incomplete shear strain-displacement assumption and thus take $\alpha_1 = 0$ which gives a constant shear strain through the thickness of the adhesive as can be seen from equation (3). Authors such as Ojalvo and Eidinoff (1978)[5], on the other hand, take $\alpha_1 = 1$ which permits the adhesive shear strain to vary through the thickness of the adhesive. Authors such as Delale and Erdogan assume that strain does not vary through the thickness of the adhesive and thus take $\alpha_2 = 0$.

SHEAR DEFORMATION OF THE ADHEREND

With lap joint theories, the adherends are treated as beams or plates. All modern lap joint theories consider bending and axial deformation of the adherends. Some consider shear deformation of the adherends as well while others neglect shear deformation. In this paper, the parameter α_3 controls whether shear deformation is considered or not. If $\alpha_3 = 1$, shear deformation is considered and if $\alpha_3 = 0$, shear deformation is neglected.

INCONSISTENT PLANE STRESS/PLANE STRAIN ASSUMPTION FOR THE ADHERENDS

In earlier lap joint theories such as that of Goland and Reissner, the adherends were taken to be in plane strain when considering bending but were taken to be in plane stress when considering axial forces. To be consistent, for plane stress

$$I^* = I, \qquad A^* = A \qquad (4)$$

and for plane strain

$$I^* = \frac{I}{1 - \nu^2} \quad and \quad A^* = \frac{\alpha_6 A}{1 - \nu^2}, \qquad \alpha_6 = 1 \tag{5}$$

Goland and Reissner used

$$I^* = \frac{I}{I - \nu^2} \quad and \quad A^* = A \tag{6}$$

which corresponds to using in equation (10)

$$\alpha_6 = 1 - \nu^2 \tag{7}$$

STRESS-STRAIN EQUATIONS FOR THE ADHESIVE

The bonded lap joint is assumed to be elastic and is assumed to be under either plane stress or plane strain conditions. A control parameter IPLANE is used in this paper to specify which condition is being considered. If IPLANE=0, plane stress is assumed and if IPLANE=1, plane strain is assumed.

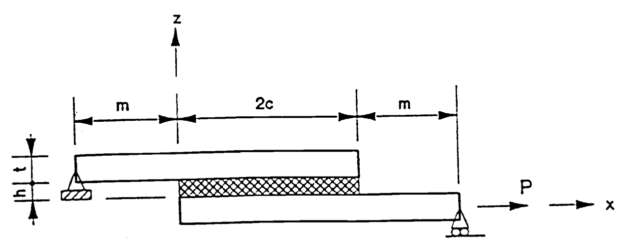

Figure 1. Lap joint

Adhesive stress and strain are related thus

$$\begin{Bmatrix} \sigma_z \\ \sigma_x \\ \tau_{xz} \end{Bmatrix} = [\sigma] = [D] \begin{Bmatrix} \epsilon_z \\ \epsilon_x \\ \gamma_{xz} \end{Bmatrix} \tag{8}$$

where for plane stress (IPLANE=0)

$$[D] = \frac{E_a}{1 - \nu_a^2} \begin{bmatrix} \alpha_5 & \alpha_4 \nu_a & 0 \\ \alpha_4 \nu_a & \alpha_4 & 0 \\ 0 & 0 & \frac{1-\nu_a}{2} \end{bmatrix} \tag{9}$$

and where for plane strain (IPLANE=1)

$$[D] = \frac{E_a(1 - \nu_a)}{(1+\nu_a)(1-2\nu_a)} \begin{bmatrix} \alpha_5 & \frac{\alpha_4 \nu_a}{1-\nu_a} & 0 \\ \frac{\alpha_4 \nu_a}{1-\nu_a} & \alpha_4 & 0 \\ 0 & 0 & \frac{1-2\nu_a}{2(1-\nu_a)} \end{bmatrix} \tag{10}$$

Goland and Reissner assumed the following stress-strain relationship for the adhesive

$$\sigma_z = E_a \epsilon_z \tag{11}$$

To model this violation of the stress-strain equations one should take for plane stress

$$\alpha_5 = 1 - \nu_a^2 \quad and \quad \alpha_4 = 0 \tag{12}$$

or for plane strain

$$\alpha_5 = \frac{(1+\nu_a)(1-2\nu_a)}{(1 - \nu_a)} \quad and \quad \alpha_4 = 0 \tag{13}$$

ZERO ADHESIVE THICKNESS ASSUMPTION

In theories which consider that the stress in the adhesive is constant through its thickness, the deformation characteristics of the adhesive are defined by the quantities E_a/h and G_a/h and not by the parameters E_a, G_a, and h themselves. Thus, it is possible to treat the adhesive as having zero thickness with properties defined by E_a/h and G_a/h. Goland and Reissner and Delale and Erdogan treat the adhesive in this way. This situation is referred to in this paper as the zero adhesive thickness assumption and in this paper this assumption is effected by setting the control parameter IFIN to 0. In cases where the adhesive is treated as having a finite thickness, such as with the theory of Ojalvo and Eidinoff, the situation is referred to as the finite adhesive thickness assumption and IFIN is set to 1.

EXAMPLE

In this investigation, the lap joint of Figure 1 with m=0 was considered. Particulars of the lap joint are given by Carpenter (1991)[2]. The lap joint was subjected to the following load cases:

1. Membrane-Shear loading,
2. Membrane-Bending loading,
3. Shear loading, and
4. Bending loading.

These loading cases are shown in Figure 2.

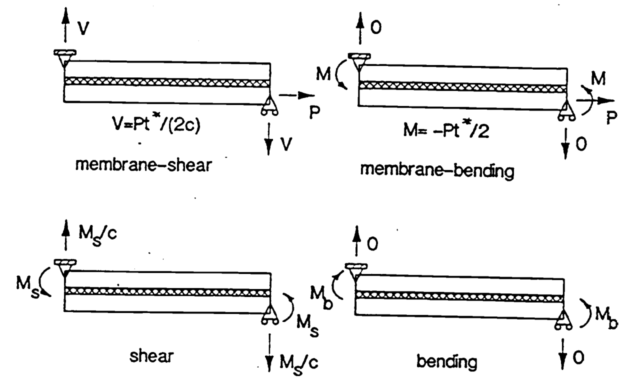

Figure 2. Various loadings

So as to examine the full range of possible boundary conditions for the membrane case, as shown in Figure 3a, moments of $-\beta Pt^*/2$ are applied to the ends of the adherends where $t^* = t+h$. Reactions and thus the amount of shear applied to the ends of the adherends depends on the parameter, β. When $\beta=0$, the loading corresponds to the Membrane-Shear loading and when $\beta=1$, the loading corresponds to the Membrane-Bending loading. To examine the effects of shear and bending loadings on the adherends in the absence of axial forces, load cases 3 and 4 were combined as shown in Figure 3b. When the parameter $\gamma=0$ in Figure 3b, the loading corresponds to the Bending loading case and when $\gamma=1$, the loading corresponds to the Shear loading case.

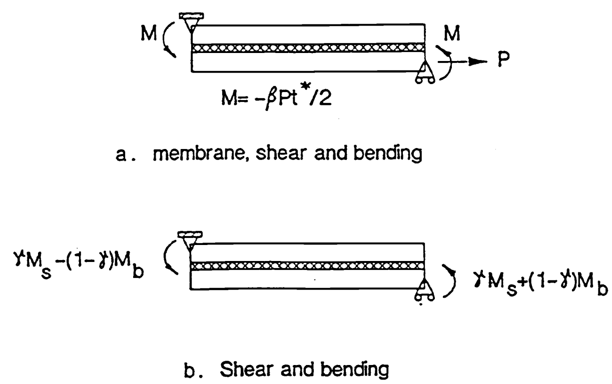

Figure 3. Combined loadings

For both the Shear-Bending study and the Membrane Shear-Bending study, 10 sets of assumptions were examined. These assumption sets are listed as Cases 1 through 10 in Table 1. In Case 1, all parameters are set to 1. In Cases 2-8, all parameters but one are set to 1 while the remaining parameters in turn are set to zero. Case 9 corresponds to the assumptions of Delale and Erdogan. This case was investigated as the theory of Delale and Erdogan is considered one of the best modern day lap joint theories. Case 10 corresponds to the assumptions made by Goland Reissner in their classic paper.

Table I

Control parameters considered

Case	IPLANE	IFIN	α_1	α_2	α_3	α_4	α_5	α_6
1	1	1	1	1	1	1	1	1
2	0	1	1	1	1	1	1	1
3	1	0	1	1	1	1	1	1
4	1	1	0	1	1	1	1	1
5	1	1	1	0	1	1	1	1
6	1	1	1	1	0	1	1	1
7	1	1	1	1	1	0	1	1
8	1	1	1	1	1	1	1	0
9	1	0	0	0	1	1	1	1
10	1	0	0	0	0	0	.743	.910

Figures 4-7 shown maximum adhesive peel stress (σ_{zz}) and maximum adhesive shear stress. These maximum stresses occur at the edges of the joint. Throughout the examples of this paper, maximum adhesive stresses reported are for the left end of the joint at the top adherend-adhesive interface. Notice that in the Membrane Shear-Bending study as well as in the Shear-Bending study, there was almost no difference in predicted maximum adhesive shear stress for the assumption cases examined.

In both the Membrane Shear-Bending study and the Shear-Bending study, the maximum adhesive peel stress was affected very little by most assumptions. The factors which did affect the maximum adhesive peel stress were

1. whether the plane stress or plane strain was being assumed,
2. whether shear deformation of the adherends was being considered or not, and
3. whether a consistent shear stress-shear strain equation was being employed.

Comparing results to the results of Case 1, the assumption with regard to plane stress or plane strain affected results by approximately 6% while neglecting shear deformation of the adherends affected results up to 30%. The widely used theory of Goland and Reissner neglects shear deformation of the adherends, inconsistently uses plane stress and plane strain for the adherends, and uses an inconsistent shear stress-shear strain equation for the adhesive. It had a maximum deviation from standard Case 1 of approximately 30% but for most values of β or γ the effects of the inconsistencies cancel the effects of neglecting the shear deformation of the adherends and the deviation was less than 15%.

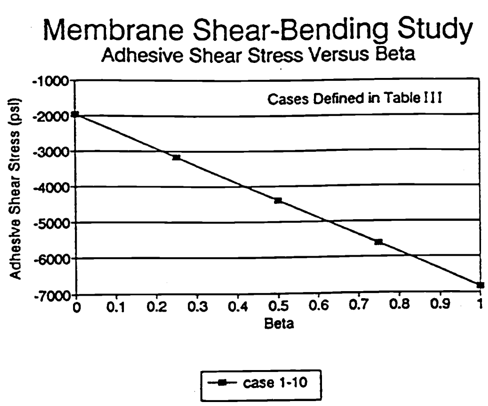

Figure 4. Adhesive shear stress

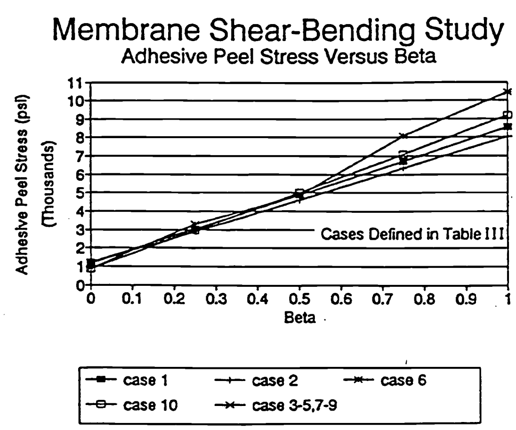

Figure 5. Adhesive peel stress

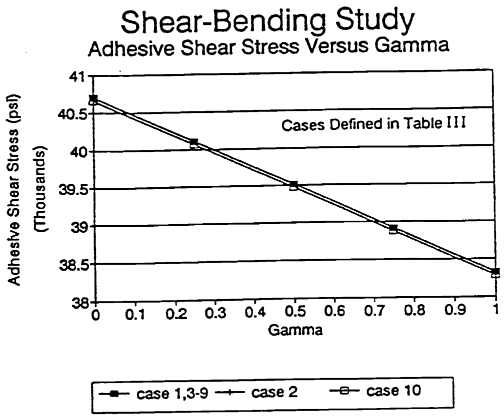

Figure 6. Adhesive shear stress

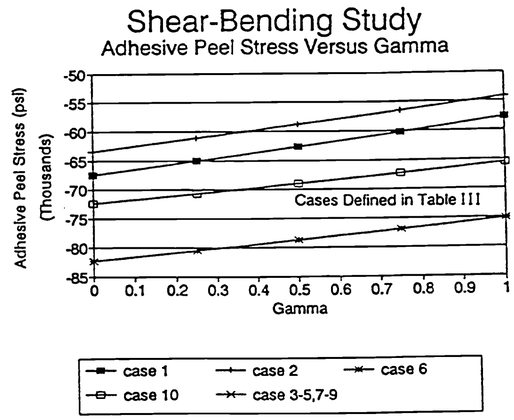

Figure 7. Adhesive peel stress

CONCLUSION

Over the last several decades, various authors have developed lap joint theories to predict stresses in the adhesive of bonded lap joints. The effect of various assumptions associated with lap joint theories has been studied in this paper. It was found that many of the sundry assumptions made by various authors have insignificant effect on maximum shear stress. Several well known theories neglect the effect of shear deformation of the adherends. It was found that neglecting shear deformation had little effect on adhesive shear stress but could affect the adhesive peel stress by as much as 30%. The classic theory of Goland and Reissner neglects shear deformation of the adherends, inconsistently uses plane stress and plane strain for the adherends, and uses an inconsistent shear stress-shear strain equation for the adhesive. The theory of Goland and Reissner gave adhesive shear stress results that were approximately the same as from theories without those assumptions but the maximum adhesive peel stresses were as much as 30% different. However, in most cases effects of the inconsistencies cancel the effects of neglecting shear deformation of the adherends and the difference was less than 15%.

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