Review: Design strength of optical glass

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The paper "Design strength of optical glass" by Keith B. Doyle and Mark A. Kahan describes the mathematical principles used in characterizing the strength of optical glasses, as well as the types of experimental data needed to perform such analysis. Since glass is a brittle material, its strength is limited probabilistically by the formation of cracks and other surface defects. The relevant parameters are categorized as fracture toughness, static fatigue, inert strength, time to failure, and cyclic fatigue crack growth. The paper is intended for entry-level optical engineers as an overview of the process needed to analyze an optical glass for mechanical strength and evaluate its reliability under the stresses in a particular system design.

The analysis begins from the fundamentals of fracture mechanics. The stress intensity factor depends on crack size and geometry as given by Griffith's law,

$$K_I = Y\sigma\sqrt{a}$$

(eq. 2.2 in the paper), where Y is a crack geometry factor, σ is the nominal stress, and a is the flaw size. We then define K_{IC} to be the critical stress intensity factor, that is the stress intensity at which a crack is just able to propagate. (The subscript "I" denotes the first of the three modes of crack propagation, in which tensile stress normal to the crack plane produces crack opening.) As fracture toughness data are not readily available for all glasses, it is often computed from a published lapping hardness, which indicates the volume of material removed under given processing conditions. It relates to elastic modulus (E), hardness (H), and fracture toughness (K_{IC}) as given by

$$\frac{E^{7/6}}{K_{IC}^{1/2} H_k^{23/12}}$$

(eq. 2.3 in the paper).

Stress corrosion due to moisture is a significant contributor to the failure rate of optical glasses under static tension. Stress corrosion is measured by time-to-failure studies, in which the time to failure is recorded under various combinations of stress and moisture levels. Since this type of study is quite time-consuming, another method known as dynamic fatigue testing is used for accelerated studies. In dynamic fatigue testing, the applied stress is ramped up at a constant rate, so that the failure threshold shows a direct dependence on the ramp rate, which results because higher ramp rates allow less time for crack propagation. The plot of average stress at failure versus the ramp rate on a log-log scale is approximately linear, with the slope known as the fatigue-resistance parameter. The linear relationship holds only in the regime where the entire crack surface is available to react with moisture. As the stress intensity increases, the crack velocity plateaus at a point where moisture acts only on part of the crack. The velocity briefly rises again, now too fast for moisture to be a significant contributor, before failure ultimately occurs. For typical stress intensities, dynamic fatigue testing shows fused silica to have a lower crack velocity than BK7 glass, and SF1 glass a higher velocity.

To predict design strength for the purpose of designing an optical system, statistical analysis of test-specimen data is performed. One analysis method describes the inert strength of the component (that is, under conditions where stress corrosion is absent) by a Weibull distribution, as given by

$$P_f = 1.0 - e^{\left(-\left(\frac{\sigma}{\sigma_0}\right)^m\right)}$$

(eq. 4.1 of the paper), where P_f is the probability of failure, σ_0 is the characteristic strength (i.e. the stress per area at which 63.2% of specimens fail), and m is the Weibull modulus, which describes the spread of the data. A glass choice can then be validated by checking the probability of failure against a chosen threshold for a given stress level. A more accurate method takes into account the propagation characteristics of subcritical cracks for the material in question, in which the total time to failure is predicted from the crack velocity's exponential or power-law dependence on the initial stress intensity factor. The resulting relationship takes the form given as eq. 4.10,

$$K_{Ii} = K_{IC}\left(\frac{\sigma}{\sigma_0}\right)\left(\ln\frac{1}{1-P_f}\right)^{-1/m}$$

In addition to constant or steadily ramping stresses, optical parts in realistic systems often experience cyclic loading. Early methods approximated the cyclic stress as a summation of static stresses, giving a constant factor that relates the failure time under cyclic stress to that under static stress. This appears on a log-log plot of maximum stress intensity vs. crack propagation rate that the line for cyclic loading is shifted leftward relative to that for each equivalent static load. These methods have been superseded by those that take explicitly into account the fatigue characteristics of the material, which involve subcritical crack growth. One such equation is a modification of the Paris Law used for metals, given by

$$\frac{da}{dN} = B(K_{max})^g(\Delta K)^q$$

(eq 5.4 in the paper), where g and q are the static crack growth exponent and cyclic crack growth exponent respectively; da/dN is the crack propagation rate per fatigue cycle. Typically, g is much larger than q, indicating that crack growth depends more on the peak stress intensity factor than on the amplitude of the cycles. As for the frequency of a cyclic load, it is generally possible for an increase in frequency to have, a beneficial (positive) effect on time to failure, a detrimental (negative) effect, or no discernible effect.

As they approach the conclusion, the authors give an example of the design analysis for a BK7 Schmidt corrector plate used in a Schmidt telescope assembly. The dimensions of the plate are 10 in diameter, 0.75 in thick. The plate is mounted within an aluminum ring frame with six tangential flexures of titanium bonded to the edge. The strength data from a study of test specimens were modeled by a Weibull distribution with characteristic strength of 10.2 ksi and Weibull modulus of 30.4, after applying appropriate corrections for the effective surface area given the nonuniformity of the stress distribution. The resulting design curves are of the form previously given as eq. 4.10. The authors determine a characteristic strength of 9.7 ksi for the part. The authors choose to target a probability of failure 10^{-5} over a survival time of 10 years, assuming that the moisture level during crack-growth testing represents that encountered during orbital operation, which the authors judge to be a conservative assumption. Based on the calculated strength curves, a design strength of 1.85 ksi is judged. Detailed calculations are not given for this section; the full data may be in another reference, and the application of the aforementioned equations is left as an exercise to the reader.

In the conclusion section, the authors remind us that rigorous statistical analysis of glass strength is vital when stress levels over 1000 psi are encountered in an optical system. The reader is also advised to consider an appropriate margin of safety in the choice of optical mounting methods. The surface finish, surface area under tensile stress, glass composition, and type of loading are also mentioned as relevant parameters. Flaw size is to be estimated from the size of the grinding particles, and the flaw size is then to be compared againt the material's fracture toughness. When probabilistic methods are used, the Weibull distribution is often used to describe the distribution of flaws. Such analysis assumes no crack growth occurs while the part is in use; when the stress over time is large enough to challenge that assumption, test data covering subcritical crack growth must be analyzed to predict the time to failure. The analysis is more complicated for glass elements under dynamic or cyclic stresses, for which material-specific fatigue properties are an important consideration. In such cases, one is advised to estimate the strength of the glass conservatively.

All in all, "Design strength of optical glass" gives a brief but comprehensive overview of the physical principles of crack formation and propagation that limit the strength of optical glasses, as well as of the equations used to analyze such physical processes. The provided analysis is intended mainly to reinforce one's introductory knowledge, and the reader is expected to consult other sources for detailed discussion of complicated cases.

References

All material from:

Doyle, K. B., and Kahan, M. A., "Design strength of optical glass", Proc. of SPIE Vol. 5176 (2003).