Specification of the scattering characteristics of surfaces and systems, for use in the analysis of stray light

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Abstract

The measurement of the scattering characteristics of coatings and surfaces has not kept pace with the development of the software tools used to analyze stray light in an optical system. The use and importance of such measurements is emphasized by the use of a parametric analysis performed on the Spacelab Two telescope. The basic measurement of the BRDF need not be costly nor time consuming, but it is made clear that the data should be extensive enough to properly define the scattering function, not just a single profile of the function. The current forms of presentation used to define the stray light performance of optical systems are discussed. In most cases it is one of these definitions that is used to set the specification on the performance of a sensor. The definitions are all based on the point source transmittance of the system. When a broad source transmittance is used to define a specification many factors external to the system are involved and these mask the propagation characteristics of the sensor, and therefore make improvements more difficult.

Introduction

The investigation and analysis of an optical system to determine its propagation of unwanted energy by scattering or diffraction requires three basic types of information. These are the optical prescription, the mechanical design, and the scattering characteristics of the surface coatings. The first two can usually be supplied in almost limitless detail, for the optical or mechanical designer will set design specifications for the manufacturer to meet, and often the parts will be tested before being accepted. By contrast, there will be almost no information available about the surface coatings in a given optical system. For several reasons manufacturers have been reluctant to bid on parts for which the surface scattering characteristics are specified. Reasons for this reluctance include a general lack of experience in the measurement of scattering characteristics and a lack of data in the open literature that would indicate the difficulty of meeting the specification; in addition, until recently there has been only a nominal need and this need was met by the scratch-and-dig specification.

Utilization of Scattered Light Data in Stray Light Programs

In recent years, two computer programs have been developed for the analysis of stray light: the General Unwanted Energy Rejection Analysis Program (GUERAP) and Arizona's Paraxial Analysis of Radiation Transfer (APART). With the regular use of these programs as tools for evaluating the effectiveness of optical designs, the "back of the envelope" calculations have given way to a computer technology that can efficiently use accurately measured scattering data. However, judging by the small amount of scattering data available, the measurement of the scattering characteristics of surface coatings and substrates has not kept pace with the development of the computer analysis, even though such measurements are in some cases the limiting factor in the analysis.

As the author of the APART program, I will use it to discuss how scattered light data are used in a computer analysis. In APART the basic radiometric equation is

$$dP_c = \frac{L_s(\theta, \phi) \cos \theta \, dA_s \, \cos \phi \, dA_c}{R_{sc}^2},$$  

(1)

where $L_s$ is the source radiance, $A_s$ and $A_c$ are the areas of the source and collector sections respectively, and $R_{sc}$ is the distance between the source and the collector. This basic equation is transformed to the product of three terms:

$$dP_c = \text{BRDF} \times dP_s \times \text{GCF}.$$  

(2)

The first term represents the scattering characteristics of the surface as a function of the input and output angles; generally this is referred to as the Bidirectional Reflectance Distribution Function (BRDF). The second term, $dP_s$, is the power incident on a specified area of some element in the system; in an analysis, a point source or surface is initially assigned some arbitrary amount of power. The third term, GCF, is the Geometrical

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Configuration Factor. In APART(3,4) the accuracy of this last term is enhanced greatly through the use of sectioning and subsectioning of the surface elements.

In APART, Equation (2) is used iteratively, the way the optical designer uses transfer and refraction equations in a lens design program. Therefore the power collected on one section of an element becomes the source for the next transfer. The accuracy of the subsequent power calculations is limited by the accuracy of the BRDF that the program determines as a function of both input and output angles. This BRDF is the specification that seems to give manufacturers second thoughts about accepting a job. I hope to show two basic points that would help to relieve their fears: (1) BRDF measurements are needed, they are used accurately, and they make a difference; and (2) the basic measurements are neither difficult nor costly. I also hope to answer the questions "What is a valid surface scattering specification?" and "What is a valid system scattering specification?"

The Need, Use, and Importance of BRDF Data

To show that BRDF measurements are needed, I will use data from an analysis made on the Space Lab 2 telescope(5). The basic design features of the main body of the telescope are shown in Figure 1. It is an off-axis Newtonian with straight 90 degree baffles on the main tube, and, in its initial design, it had a black diffuse sunshield. Initially, the system was evaluated using a BRDF model in APART that is based upon measured(6) Martin Black coating data. The plots, representing several profiles of the BRDF, are shown in Figure 2. The relationship between a "Lambertian" black hemispherical reflectance, \( \rho \), and the BRDF is:

\[
\text{BRDF} = \frac{\rho}{\pi} \text{sr}^{-1}
\]  

(3)

The word Lambertian is in quotes because there does not exist a Lambertian surface of low reflectance. However, an approximate value of \( \rho = 0.01 \) for the hemispherical diffuse reflectance of Martin Black was used in the analysis. For near normal angles of incidence (Figure 2) this should be quite reasonable. The parametric analysis of the system as a function of the off-axis angle of a point source is shown in Figure 3.

For all intents and purposes the Lambertian approximation was excellent for all off-axis angles as shown in Figure 5. Hence it would appear that the additional costs of making BRDF measurements, or of using complicated BRDF models in APART, are not worthwhile. However, just as some of the "back of the envelope" calculations are only sometimes correct, this closeness only sometimes exists. The same system was reevaluated with the vanes removed from the main baffle. Once again the Lambertian value of 0.01 was compared to the Martin Black model (Figure 4). Figure 4 shows a difference of two or three orders of magnitude between the two representations of the surface scattering characteristics. An error of this magnitude, if discovered in the final testing, is not readily correctable. Figures 3 and 4 show how the use of vanes on the main baffle affect the overall performance of the system (Figure 5). The reason the initial approximation (Figure 3) was close was that all the scattering either was backscatter or had near-normal angles of incidence. As shown in Figure 2, all the BRDF values converge to low values (\( \rho = 0.01 \)) at large backscatter angles, or remain low for near-normal angles of incidence. Such representative scattering paths are shown in Figure 6.
Figure 2: BRDF profiles of Martin Black. 
$\theta$ equals the sine of the angle of incidence. 
$\theta_0$ equals the sine of the angle of observation. $\lambda = 10.6 \mu m$.

Figure 3: Spacelab 2 PSPT for vanes and Martin Black versus .01 diffuse.

Figure 4: Spacelab 2 PSPT for Martin Black versus .01 diffuse.
Figure 5 Spacelab 2 PSPT for Martin Black coating with and without vanes.

Figure 6 A Lambertian approximation to the BRDF gave very good relative results because all the surfaces had near normal angles of incidence for the scattered radiation.

In more complicated systems there is almost always a mixture of the two kinds of paths -- those where some approximations are valid and those where they are not. Generally, in stray light analysis, one must have reliable measured BRDF data. Otherwise the usefulness of the computer analysis program is compromised.
The Basic Scattering Measurements

By definition, the BRDF is a function, and rarely will a single profile or a single value represent the BRDF for all input and output angles. However, such is the case in the theory worked out by Harvey and Shack for scattering from mirror surfaces. In Figure 7, BRDF profiles for several angles of incidence (Figure 7a) are replotted as a function of $\beta$-$\beta_0$ (Figure 7b). When the observation point is in the plane of incidence, $\beta_0$ equals the sine of the angles of incidence and $\beta$ equals the sine of the angle of observation. Here the single profile represents the BRDF. As a result of this theory, the single BRDF profiles generated in the past, which depict the BRDF as a function of one observation angle only, can be converted to a true BRDF for all angles of incidence. The theory goes further, so that the given curve is the true three-dimensional representation of the function (see Ref. 7). This also has an impact on the measurements that need to be made by a manufacturer. This will be discussed in greater detail shortly.

![Figure 7 Illustration of the importance of the coordinate system within which the scattering process is discussed. (a) Relative intensity plotted versus scattering angle. (b) Scattering function plotted versus $\beta$-$\beta_0$.](image)

Even more rare are the times when a single number can be used to represent the BRDF. Figure 8 depicts several BRDF profiles for NaCl at 10.6 $\mu$m. A constant BRDF of 0.03 would reasonably represent the function.

Some types of BRDF measurements that are sometimes supplied to me are of little value. When the relationship between the detector and sample remains fixed while only the laser is scanned, as shown in Figure 9, almost no useful information is gained. The polar plots shown in Figure 10 show where three such data points would appear. They would not warn of any great variation in the BRDF if one existed. Only when the detector is moved off the normal position for several angles would the variation become obvious. Likewise, the corresponding measurement of scanning the detector at a single angle of incidence is equally uninformative when measuring low diffuse scattering coatings.

Attention has been focused on the direct measurement of the BRDF. Harvey's work dealt with the inverse scattering problem, that is, making BRDF measurements and then predicting the RMS roughness and the autocovariance function. However, an equally acceptable indirect method of determining the BRDF is through measurement of the RMS roughness and the spectral density function. As shown by Church, the data can be converted to Total Integrated Scatter (TIS) and angular scatter patterns. In scattered light analysis, what is necessary is to be able to determine the BRDF from the measured data.

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The cost of making these measurements can be quite low. For surfaces and coatings for which the Harvey-Shack theory applies, and under ideal conditions, three or four data points would specify the BRDF—the entire three-dimensional form! In the visible this would require a laser source, a detector with its associated power supply and readout apparatus, and some focusing optics. All of these are off-the-shelf items. The input beam could be baffled with a single plane baffle. The three or four measurements could be made in a plane parallel to an optical bench table top (Figure II).

The sophisticated and expensive BRDF measuring devices that are described in the literature are glorified versions of the above setup. No disrespect is meant for these systems with the additional bells and whistles. They are necessary tools for extensive research analysis and for repetitive measurements of different sample substrates at several wavelengths and polarization directions. These devices represent the state of the art. Unfortunately the term BRDF seems to imply the use of this type of instrument exclusively, and by experienced users. It is true that in some spectral regions one will need a dewar, a chopper, a lock-in amplifier, perhaps baffled collector optics before the detector, and a few other pieces of equipment. But if it is agreed that the manufacturer's responsibility is limited to making some clearly specified measurements, the cost, time, and difficulty will be low. The measurements could then be compared to more extensive BRDF measurements made on similar types of coatings to determine obvious discrepancies, or to determine the need to make more extensive measurements beyond those specified as the manufacturer's responsibility. The end result should be a host of brief, but accurate, BRDF data points on many new materials. This data base will help the manufacturer develop the ability to accurately cost out a scattering specification depending upon the difficulty to meet it. It also could be used to direct attention to interesting topics of extensive evaluation and in the development of useful theories.

Figure 8 BRDF of Powdered NaCl, λ=10.6μ

Figure 9 Scattering measurements as depicted here are of little value.

Figure 10 Scattering measurements of 3M. Characteristic of most blacks, it shows little variation in the near normal direction for all angles of incidence. Data taken as shown in Fig 9 would only show the variation from points 1, 2, and 3. θ equals the angle of incidence.
Although the use of Lambertian approximations may pose problems, there are times when such approximations are all that are available. There is no measured BRDF data beyond 10.6 μm. However, emissivity and TIS measurements made at long wavelengths help warn of significant changes in the scattering characteristics. TIS measurements may indicate the need for some specialized piece of equipment to unravel the physics involved. They may also be a signal that the usual BRDF data extrapolated to these wavelengths would give an erroneous stray light analysis.

**What Is a Valid BRDF Specification For a Surface?**

We will restrict the question to the application of the BRDF in a stray light suppression analysis. The broadest specification—where the BRDF cannot exceed a certain value for all input and output angles—would be the easiest to make but in most cases hardest to meet. However the analysis programs mentioned earlier can make use of the directional characteristics of the scatter. To ease the requirements, a good BRDF specification will include information as a function of the input and output angles, for quite often the directions of scatter that have high BRDF values may not involve significant propagation paths.

Using the Harvey-Shack notation to plot BRDF for a mirror surface, the value of the BRDF at θ-θ₀ = 0.01 and the slope in the θ-θ₀ plot will totally specify the function. To be avoided is the specification that reads: The BRDF shall be less than 1.E-3 at 20° from specular. This specification lacks the angles of incidence and some sort of information on falloff with angle. It would most likely result in an overspecification that would be harder to meet.

For a black coating, the BRDF specification should include several observation angles in the plane of incidence, for several angles of incidence, and ideally some azimuthal angles. Typically the specification must permit the BRDF to increase with increasing angle of incidence—again to make it realistic, easier, and more likely to be met.

The specification of a BRDF for use in a scattered light analysis should be complete enough to depict the following:

1. Variations as a function of the observation angle (azimuth angles included).
2. Variations as a function of the angle of incidence.
3. The wavelength used to measure (or define) the BRDF.
4. The coating name, process, or manufacturer.

If any of the following information is available from measurements it should accompany the data as an aid for future areas of research. Some of the information may also be used as part of a specification.
(1) Variations from sample to sample.
(2) The solid angle subtended by the detector as seen from the sample.
(3) Coating thickness.
(4) Polarization direction of the input beam, if any.
(5) Handling characteristics.
(6) Variations with wavelength.

Initially I indicated that manufacturers generally show some reluctance to accept a scattering specification on system elements. If a large enough database could be gathered, a manufacturer could associate an appropriate cost increase with the specification, just as he now knows how much more to charge for a \( \lambda/2 \) surface figure versus a \( \lambda/3 \) figure.

**Specifying the Scattering and Diffraction Propagation Characteristics of an Optical System**

There are two ways one might specify the stray light propagation characteristics of a system. First, a given design is presented, evaluated, modified, and then reevaluated. The performance of the system is then "defined" as a result of the analysis. The background noise calculated by the program becomes an acceptable noise level. The stray light analysis is usually relatively straightforward.

In contrast to this is the system whose stray light propagation characteristics must achieve a certain signal-to-noise level in a specified environment. Then such parameters as the size of the central obscuration, the stop location, dimensional restrictions, coatings, fields of view, subsequent reimaging devices, and off-axis positions of sources of unwanted energy come into play to reach the specified goal.

This analysis and the initial writing of the system specification are more difficult. In most systems, the Point Source Transmittance, \( \text{PST}(\theta) \), of unwanted energy drops off rapidly as the source is moved farther off axis. However, if the source is the earth or some other broad source, this dropoff may be compensated for by the increase in the solid angle that the source subivdes so that the total power into the system may be rapidly increasing with the off-axis angle. To set a specification, one must determine the incident power, \( \Phi_{\text{in}}(\theta) \), into the system as a function of the off-axis angle, \( \theta \), and the maximum power, \( \Phi_1 \), of unwanted energy allowed to reach the image plane. \( \Phi_1 \) may also be a function of angle but for simplicity here it is assumed to be a constant. These can be used to define the stray light specification, \( f(\theta) \):

\[
PST(\theta) = \frac{f(\theta)}{\Phi_{\text{in}}(\theta)}.
\]

**Definitions of System Performance**

Several definitions are used to specify the stray light performance of a system. None of the definitions is more meaningful than any of the others.

The first one (Figure 12), already mentioned is,

\[
f(\theta) = \Phi_{\text{det}}(\theta)/\Phi_{\text{in}}(\theta),
\]

where \( \Phi_{\text{det}}(\theta) \) is the power incident on the detector from an unwanted source \( \theta \) degrees from the optical axis. This is called the Point Source Power Transmittance, \( \text{PSPT}(\theta) \). The values of \( \text{PSPT}(\theta) \) are typically in the range of \( 10^{-6} \) to \( 10^{-15} \), and are unitless. This term is sometimes awkward to use, one reason being that \( \Phi_{\text{det}}(\theta) \) depends upon the area of the detectors.

The term "attenuation" is sometimes used to define the performance of a system. The power attenuation is usually defined as the ratio of the power into the system to the power reaching the detector, the reciprocal of \( \text{PSPT} \). These factors therefore are typically very large, \( 10^6 \) to \( 10^{14} \), and increase as less power reaches the detectors. This type of relation-ship is not conducive to a clear understanding and application of the attenuation factors. Occasionally the attenuation is incorrectly defined, usually as the \( \text{PSPT} \) is defined here, thereby causing further misunderstanding.

The Point Source Irradiance Transmittance, \( \text{PSIT}(\theta) \), is defined as

\[
\text{PSIT}(\theta) = \frac{E_{\text{det}}(\theta)}{E(\theta)},
\]
where $E_{\text{det}}(\theta)$ is the irradiance on the detector from a point source that puts a flux density of $E(\theta)$ on some defined entrance port, not necessarily the entrance aperture (Figure 13).

![Diagram of point source transmittance](image)

**Figure 12** Point Source Power Transmittance (PSPT)
= Power on the detector/ Power in. PSPT=1/Attenuation

Both PSPT and PSIT, though useful, require some well-defined entrance port—which some designs don't have. For example, for a Cassegrain design that lacks a main baffle around the primary, there is unwanted energy coming in from all angles.

Another often-used system stray light specification is the Point Source Normalized Power Transmittance, PSNPT (Figure 14):

$$PSNPT(\theta) = \frac{\Phi_{\text{det}}(\theta)}{\Phi_{\text{in}}(\theta=0)}.$$  \hspace{1cm} (7)

![Diagram of point source normalized power transmittance](image)

**Figure 14** Point Source Normalized Power Transmittance = PSNPT = Off Axis Rejection (OAR)=Power on the detector/ Power on the detector when the source is on axis.
This is the power on the detector for a point source \( \theta \) degrees off the optical axis normalized by the input power on axis--i.e., the signal that would be produced by a source if it had been axis. Although the PSNPT is commonly referred to as the Off-Axis Rejection (OAR) of the system, this terminology is misleading because, first, the system generally does not reject the energy but rather absorbs most of it, and second, the value itself does not represent what is being "rejected" but rather what is not being "rejected."

A fourth way to specify the stray radiation characteristics of a system is by the ratio of the irradiance on the image plane to the irradiance at the sensor due to an off-axis point source. The plane on which the irradiance from the point source is determined is normal to the line of sight from the point source to the target. The Point Source Normal Irradiance Transmittance, PSNIT(\( \theta \)) (Figure 15) is:

\[
PSNIT(\theta) = \frac{E_{\text{det}}(\theta)}{E_{11}(\theta)}.
\] (8)

![Diagram of Point Source Normal Irradiance Transmittance (PSNIT)](image)

**Figure 15** Point Source Normal Irradiance Transmittance (PSNIT) 
- Detector Irradiance/ Irradiance at the sensor - normal to the line of sight.

This definition has several advantages. It does not depend upon a well defined entrance port, as does the PSITT, and it is easily applied to an array of detectors of various sizes. Also the irradiance at the sensor \( E_{11}(\theta) \) from a source of unwanted energy can usually be easily calculated. The product of \( E_{11}(\theta) \) and the PSNIT will give the image plane irradiance contribution due to this particular point source.

Another method that specifies the system's performance is the system BRDF(\( \theta \)) (Figure 16).

![Diagram of System Bidirectional Reflectance Distribution Function (BRDF)](image)

**Figure 16** The System Bidirectional Reflectance Distribution Function (BRDF)

\[
\text{BRDF}_{\text{sys}}(\theta) = \text{Power on the detector from the off axis source} \quad \frac{P(\theta)}{P_0} \quad \frac{1}{\theta_{\text{det}}} \quad \text{sr}
\] (9)

Although the detector's relationship to the elements that contribute power to it may be fixed, the amount of power reaching the detector \( P(\theta) \), will be a function of the direction \( \theta \) and magnitude of the total input power \( P_0 \). If \( \theta_{\text{det}} \) is the solid angle subtended by the detector then
The term is really a system bidirectional transmittance function.

Certainly other terms may be necessary for other users or special systems. Nevertheless, this set of definitions encompasses most of the terms that I have encountered.

In addition to specifying limiting system transmittance values for a range of angles, some additional information is useful or necessary. The unwanted point source is usually assumed to be at a large distance from the sensor. Such a source, at an angle 9 from the optical axis, will have a different transmittance factor than a point source at the same off-axis angle but relatively near the sensor—as might occur in a test environment. Under the latter circumstance the specification should include the distance. Conversely, using point source transmittance factors (d^--=) to determine the contributions from broad sources of very large area will usually not be valid because of the rapid falloff of the Point Source Transmittance Factors (PSTF) with angle.

All of the terms used to define a system's performance have been defined for point sources of unwanted energy. This is because the point source propagation paths through the system are more clearly defined than for broad sources. The PSNIT is used in APART to calculate broad source rejection ratios. Broad source rejection ratios have much less general applicability. They are for a particular source that has a specified radius and emissivity, and for the system at some orbital altitude and pointed some specified number of degrees from the horizon of the source. Although it may take a few hundred seconds to calculate a range of PSTF, they in turn can be used over and over again to calculate varying Broad Source Transmittance (BST) factors. These broad source calculations take only a few additional seconds on the computer.

In conclusion, any one of the above terms may be appropriate to use when specifying the scattering characteristics of a system. The range of off angles should always be included along with any coating specifications or limitations. Information about the operating environment should be included because it may restrict the meaningful modifications that can be made to a system. The wavelengths or wave band must be specified.

It is beyond the scope of this paper to describe how system specifications, once made, can be met. The optical design, location of stops, obscuration ratios, coatings, etc., will play an important role in determining whether the specification can be met and with how much redesign effort. Such topics are covered in detail elsewhere.

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References

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