# Fundamental architecture of optical scanning systems

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Configurations for active and passive optical scanning are categorized and unified with the use of a conjugate image model. Topics include architecture of scanners whose optical apertures may be overilluminated or underilluminated by flux, which is (or is not) radially symmetric, providing scan magnification and possible image rotation. A scan locus theorem is introduced. Scanner-lens configurations include flat fielding, telecentricity, double pass, and beam expansion/compression. The resolution invariant reveals beam propagation and anamorphic beam-handling consequences.

*Key words:* Scanning architecture, optical scanning, laser scanning, remote sensing. 

## 1. Introduction

Optical (spatial) scanning may be represented by the convolution of a point-spread function over an information-bearing surface. The scanning point may be one of a multiple array of points, or it may be an arbitrary distribution of light flux or radiometric sensitivity. The device that develops this relative motion is the scanner. Disposed in the optical path of an objective lens that forms a conjugate-imaging system, the scanner manipulates light before or after the objective lens. Additionally, the objective lens or the information surface may be moved. Such geometrical architectural options for scanning systems is the subject of this research.

When identified with laser scanning, this process is represented generically as active scanning, as distinguished from passive scanning, which is typified by the field of remote sensing. The conjugate-imaging architecture of passive scanning is similar to that for active scanning, with the ray directions reversed. Analogous to the moving point of the light of active (flying-spot) scanning is the moving remote-sensing aperture of passive scanning, which samples (captures) radiation from different portions of an object field, to be directed by the scanner and objective lens on typically fixed detector(s). Notable similarities and distinctions exist in their use, as developed here.

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Although serial scanning is addressed primarily, many of the expressed properties are applicable to general area scanning and to focal-plane imaging. The purpose of this research is to characterize fundamental architectural options for optical scanning and, within this structure, to unify the complementary nature of the two broad fields of active and passive optical scanning.

## 2. Representation of Optical Scanning Systems

### A. Conjugate Imaging

Extending the classical representation of conjugate optical imaging,[1] one can portray the scanning system in a similar form with added provision for beam manipulation.[2,3] Figure 1 illustrates a group of rays from a reference (object) point $P_o$, which is transferred lenticularly to the conjugate-image point at $P_i$ by means of scanning the ray group.

The ray directions make this illustration one of active scanning, because the light source (laser) forms the fixed object point (at $-\infty$ when they are collimated paraxially), and the rays are directed through the scan regions and an objective lens to converge at the moving focal point. Point $P_o$ is fixed, whereas $P_i$ is displaced on the information surface. In passive scanning the ray directions are reversed, sustaining the reciprocal nature of conjugate imaging. That is, the flux from each sampled point ($P_i$ on the right) is addressed by the scanner for its optical transfer to a fixed detector (at $P_o$ on the left). In addition to preobjective and postobjective scanning, Fig. 1 shows the option of objective scanning, which is discussed below. Preobjective and postobjective (active) scan-

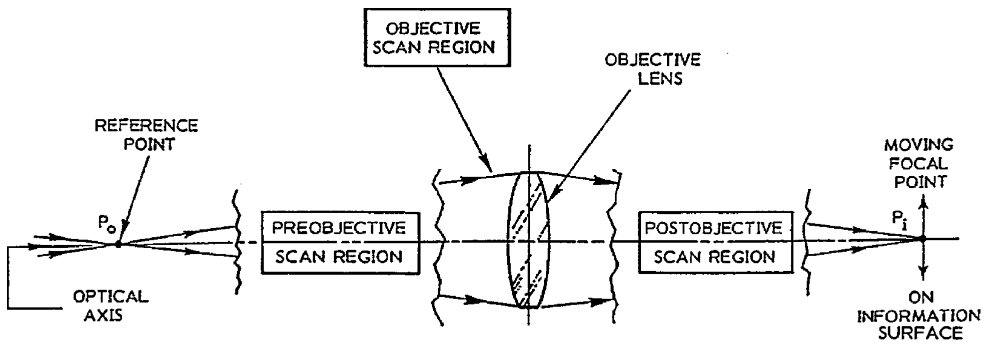


Fig. 1. Conjugate-imaging system with the scan regions identified. The ray directions represent active scanning when $P_o$ is a (point-object) source. The passive (remote-sensing) ray directions are reversed, representing the flux from $P_i$ propagating back through the optics to a detector at $P_o$.

ning have their (passive) remote-sensing counterparts, defined, respectively, as image-space and object-space scanning.[2]

### B. Functional Classifications

In the classification of scanner-system architecture, several distinctions are highlighted. Of translational and angular scanners, translational scanners are often themselves the objective lenses (reflective, refractive, diffractive, or hybrid). A familiar example is the objective lens of a compact disk reader, which scans the data with the transverse motion of either the lens or the disk. The motion in the depth direction for focus optimization could be considered spatial scanning if the focus change provides access to different image planes or disparate image locations in the depth. Angular scanners, imparting an angular change to the light flux, form the bulk of high-speed scanning, for they benefit from optical leverage[3] by displacement of the rays tangentially upon an image surface in proportion to the distance from its fulcrum.

Another distinction is between overillumination (overfilling) and underillumination (underfilling) of the scanning aperture. The operational consequences of this option (e.g., radiometric efficiency, radiometric uniformity, duty cycle, data rate, deflector size, resolution, and point-spread variations) are discussed in detail elsewhere.[2] The scanned underilluminated beam represents a portion of the overilluminated beam, terminating ultimately at the same focal point.[4] Thus a description of the overilluminated case, exemplified subsequently, often reveals more simply the focal trajectory resulting from underillumination. Passive scanning operates typically in the overilluminated mode, whereby the scanner aperture delimits the system aperture. Active scanning, however, may be conducted with either option. In notation $x$ represents the along-scan direction of the scanned line and $y$ represents the cross-scan direction, both normal to $x$. (In a nonlinear trajectory, $x$ and $y$ represent the corresponding instantaneous directions.) The $z$ direction is normal to the information surface. The axis of a rotational scanning system is designed as the $a$ axis.

### C. Radial Symmetry and Scan Magnification

Angular scanning devices function either mechanically or nonmechanically. The mechanical ones are often an array of mirrors formed into a multifaceted rotating polygon or utilized individually on a rotatable shaft. Holographic and diffractive deflectors can exhibit dynamic characteristics that are very similar to their mirrored counterparts.[4] The nonmechanical deflectors are primarily acousto-optic or electro-optic devices and may include such configurations as phased arrays and the minute decentering of microlens arrays. In addressing here the systematic functions of the components, I am deferring the exposition of detailed characteristics of deflecting devices.[2–7]

When appropriately illuminated, mechanical scanners can exhibit a basic property termed radial symmetry. When an illuminating beam converges to or diverges from the rotating axis of an angular scanner, the combination exhibits radial symmetry.[3,4] This is illustrated in Fig. 2 for an active scanner, in which the converging illuminating beam is directed to focus at point o on axis. A special case of radial symmetry occurs when the illuminating beam is collimated and paraxial, because it derives effectively from an on-axis point at $-\infty$. Radially symmetric systems provide

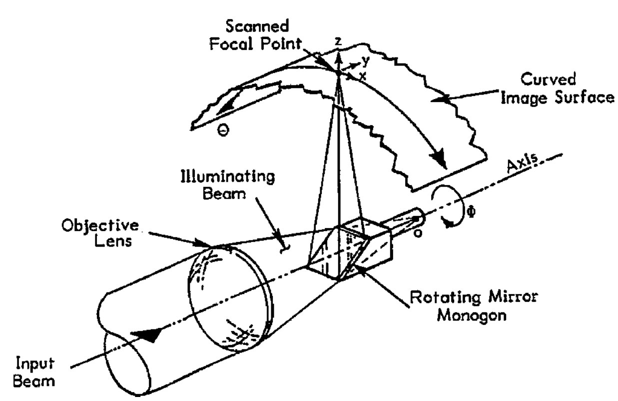


Fig. 2. Postobjective scan of a monogon. When the illuminating beam is focused on axis (at point o), the system is radially symmetric, when the image locus forms a section of a circle concentric with the axis. Scan magnification $m = d\Theta/d\Phi = 1$.

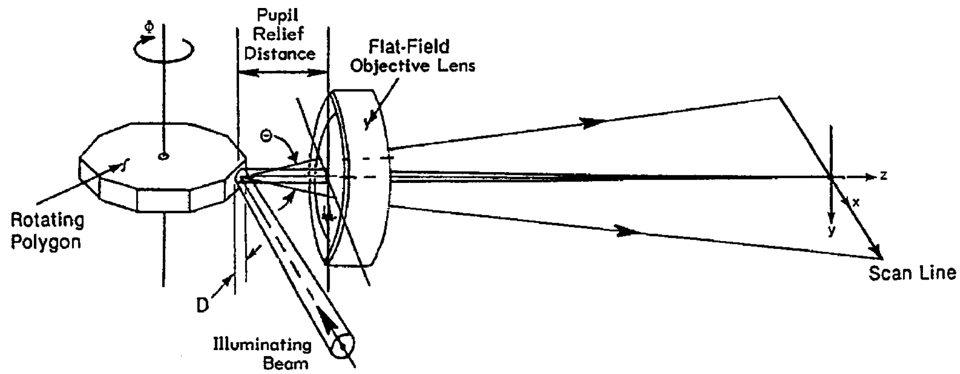


Fig. 3. Preobjective scan of a prismatic polygon underilluminated by beamwidth $D$. When the mirror surfaces are parallel to the rotating axis while the illuminating and reflected beams reside in a plane normal to the axis, scan magnification $m = 2$. The flat-field objective lens transforms the angular change $\Theta$ to the (ideally) proportionate $x$ displacement at constants $y$ and $z$.

unity optical angular change for unity mechanical angular change (Appendix B). Thus in radial symmetry,

$$d\Theta/d\Phi \equiv m = 1, \qquad (1)$$

where $\Phi$ is the mechanical scan angle and $\Theta$ is the resulting optical scan angle. Parameter $m$ is defined below.

### 1. *Scan Magnification*

Parameter $m$ is the scan magnification,[4] ranging typically from 1 to 2. The value of $m = 1$ is characteristic of radially symmetric systems, and $1 < m \leq 2$ appears in radially asymmetric systems. The latter case is represented in Fig. 3 by the prismatic polygon (facet planes parallel to the axis) in its typical configuration. When the illuminating beam is normal to a plane that includes the axis, $m = 2$; the optical angular change is twice the mechanical one. When illuminated radially symmetrically, however, $m = 1$, which is particularly evident with the pyramidal polygon of Fig. 4. The prismatic polygon can also be operated in radial symmetry, as in Fig. 5, with the same and an obvious result of $m = 1$ (Appendix B). In Fig. 4 a collimated input beam overilluminates two facets so that each facet selects its portion of the input beam to reflect it radially, forming $m = 1$. The flat-field objective lens converts the angular change in the scanned beam to a uniform displacement of the focal point along $x$ at a fixed $z$. In Fig. 5 the input beam that is focused toward point o overilluminates two facets, each of which selects and reflects from the converging beam a portion that forms point P, executing a circular arc on cylindrical surface S.[2,3] Again $m = 1$ for underillumination as well as for the illustrated overillumination. Although the above discussion exemplifies active scanning, it adapts to passive scanning when the ray paths are reversed and the light sources are replaced with detectors. Figures 4 and 5 illustrate overilluminated active facets. The optical apertures of passive systems are delimited typically by the same projected facet boundaries. In the context of remote sensing Figs. 3 and 4 are image-space scanners, and Figs. 2 and 5 are object-space scanners,[2] in which the remote object-space

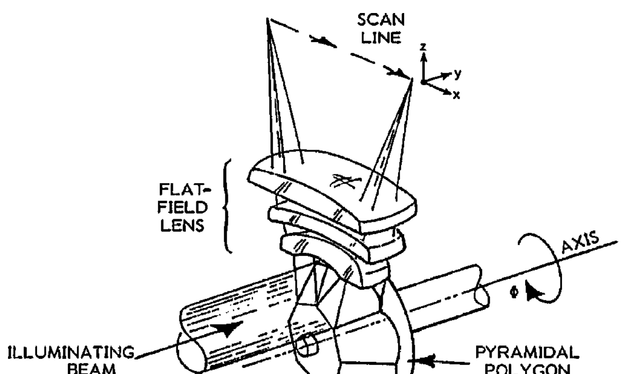


Fig. 4. Preobjective scan of a pyramidal polygon overilluminated (overfilled) with a paraxial collimated beam. The flat-field objective lens (elements cut for illustration) transforms the angular change to a nominally proportionate $x$ displacement of the focal point.

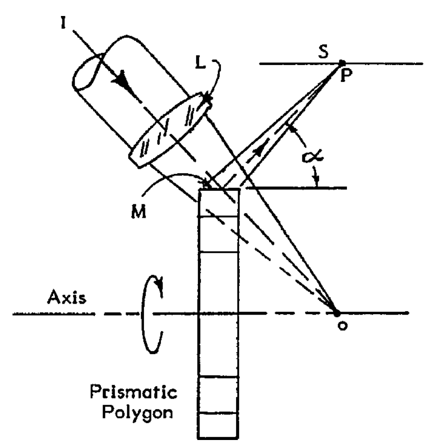


Fig. 5. Postobjective scan of a prismatic polygon operating in the radial symmetry. When the illuminating beam reflected from mirror M is focused originally by lens L on axis (at point o), the scanned beam generates an acute angle $\alpha$ about the axis and forms a circular locus of point P on surface S concentric with the axis. Scan magnification $m = 1$.

focal distance can be many times the image space distance.

Holographic scanners (rotating substrates with reflective facets replaced by diffractive elements) that are radially symmetric also exhibit $m = 1$ (Appendix B). For holographic scanners that are not radially symmetric, the nominal value of $m$ depends on the angles of incidence and diffraction of the input and output beams with respect to plane linear grating facets[2,4]:

$$m = \sin \Theta_i + \sin \Theta_o, \tag{2}$$

where $\Theta_i$ and $\Theta_o$ are the input and diffracted angles with respect to the grating surface normal. Thus, when $\Theta_i = \Theta_o = 45°$ and $30°$, $m = \sqrt{2}$ and 1, respectively.

### 2. *Sustenance of Radial Symmetry*

When radially symmetric performance is to be sustained through subsequent reflection, refraction, diffraction, or detection (termed operation), the operating function $f(r)$ must maintain symmetry with respect to the rotating axis. This is exemplified by the use of an auxiliary reflector following a radially symmetric holographic scanner,[4,8] represented in Fig. 6. Each scanner alone propagates the beam so that it maintains a constant acute angle to the axis, whereas the auxiliary reflector (AR) redirects the beam to propagate normal to the rotating axis. The AR's are spherical segments, (A) convex and (B) concave, whose radii of curvature ($R_A$ and $R_B$) originate on the rotating axis. They need not be spherical. They must, however, be figures of revolution or axiconal.[9] This general property may be expressed as

$$f(r, a)|_{a=a_i} = k_i, \tag{3}$$

in which $f(r, a)$ is the rotationally symmetric operating function, symmetric with respect to the rotating $a$ axis. At each $i$, $k$ is constant. This is exemplified further by the final image surface in Fig. 2, which shows a segment of the cylindrical detecting surface. An analogous passive system interrogates coaxial radiating sources.

Considering radial symmetry to include the operational components, we may now reenforce and augment the definition in Subsection 2.C above as follows: A scanning system exhibits radial symmetry when the illuminating beam converges to or diverges from the rotating axis of an angular scanner. Radial symmetry is sustained by operations on the propagating beam by axiconal components coaxial with the rotating axis.

## D. Image Rotation and Derotation

The rotation of an isotropic point-spread function (PSF) about its axis is nominally undetectable. If, however, the PSF is nonisotropic or polarized, or if an array of points is to be scanned, some arrangements can cause rotation of the image-spread function about its projected axis (i.e., the principal ray of the scanned beam).

Consider a postobjective monogon scanner, as shown in Fig. 2, illuminated radially symmetric with a grossly overilluminating beam, so that the mirror delimits the reflected beam to exhibit an (essentially) uniform intensity distribution. During scanning this rectangular cross-section beam is sustained radially in the near field. The scanned beam remains normal to and focused on the congruent cylindrical image surface. With the mirror uniformly illuminated, the scanned focal point (the diffraction-limited $\mathrm{sinc}^2\, x, y$ form of the PSF) remains stationary relative to its projected axis and scanned line (Appendix B). If, however, the same scanner is underilluminated with, for example, an elliptical cross-section Gaussian beam, the axes of the imaged elliptical Gaussian spot rotate directly with the mirror.[10] In a similar way, if the scanner is illuminated with preferential polarization or with multiple beams to generate an array of spots, the imaged polarization or array angle rotates directly with the mirror. This effect is identical in the pyramidal polygon of Fig. 4. Each mirror represents a marginal segment of the centered mirror of Fig. 2,

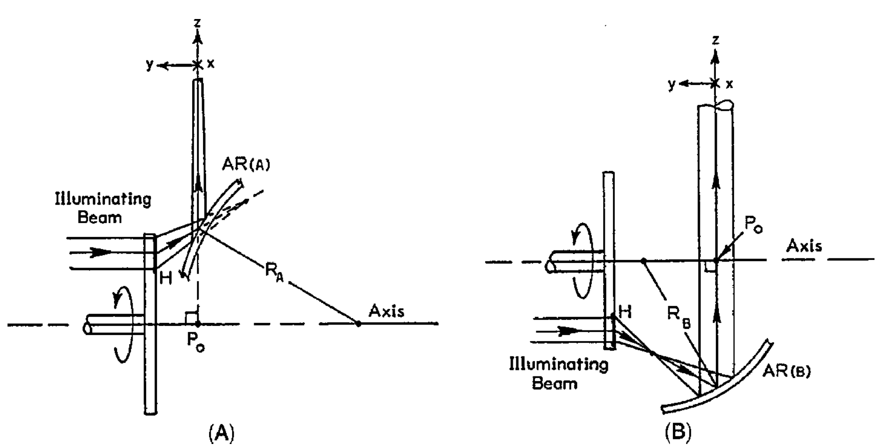


Fig. 6. Transmissive holographic scanner (showing holographic optical element H synthesizing lenticular facets): (A) convex AR of radius $R_A$, (B) concave AR of radius $R_B$. The AR transforms the beam scanned at an acute angle to the axis (similar to that in Fig. 5) to one typically normal to the axis, centered at point $P_o$. The radial symmetry of the beam is sustained by the AR centered on axis.

imparting the same rotation of a nonuniform illuminating beam. Although this is exemplary of active scanning, the corollary effect occurs in passive scanning.

This is not so, however, for the (postobjective) mirror mounted as shown in Fig. 7 or for the (preobjective) prismatic polygon in Fig. 3. When the principal rays of the input and scanned beams are in a plane that is normal to the axis of rotation, execution of the scan does not alter the PSF (except for possible vignetting at the aperture and alteration of reflection characteristics with variation in the incident angle). In earlier cases of radial symmetry the incident angles remained constant while the image rotated. Here in the case of extreme radial asymmetry the incident angles change, while the image develops no rotation. Although mirrored scanners seldom operate in the regions between these extremes, holographic scanners can create possible complications with, for example, the polarization states. This is manifest in the variation in the diffraction efficiency of the gratings for $p$ and $s$ polarizations when they are rotated.[4]

Interposing complementary image rotation in the optical path can cancel the rotation caused by the scanner. The characteristic of a coaxial image rotator is that it inverts the image,[11] resulting in two complete rotations of the image per rotation of the component. Thus, when rotated at one-half of the angular velocity of the scanner, a rotator such as the Dove prism and several others[11] can provide coaxial image rotation or derotation.[2,12]

## 3. Characteristics of Scanned Systems

In reference to Fig. 1, in this section some distinctive properties of objective, preobjective, and postobjective (active) scanning are expressed. Passive scanning, having a complementary architecture, exhibits corresponding characteristics.

### A. Objective Scan (Translational)

Translation of an objective lens transverse to its axis (within its acceptable field) translates the imaged focal point. In a collimated beam the imaged focal point translates directly with the lens. Translation of the information surface with respect to a fixed objective lens renders the same effect, both termed the objective scan. Both forms can occur simultaneously, as exemplified in Subsection 2.B and in the external drum scanner shown in Fig. 8.

### B. Preobjective Scan (Angular)

The preobjective scan permits the objective lens to transform the angular displacement, $\Theta$, of an optical beam (beyond the small angle when $\Theta \approx \tan \Theta$ and having no dynamic focus) into a focused flat field. Figure 3 illustrates preobjective scanning into a flat-field objective lens.

### C. Postobjective Scan (Angular)

The postobjective scan, which is radially symmetric, as shown in Figs. 2 and 5, generates a perfectly circular locus of the scanned focal point (Appendix B). This permits conformance of the focal surface with a circular information surface. The departure from radial symmetry (i.e., the incident focal point not on axis at point o) generates a noncircular (e.g., limaçon) function,[2] except for the following special case.

A postobjective mirror with its surface on the rotating axis generates a perfectly circular scan locus when the input beam is normal to the axis and is focused beyond the axis. This is illustrated in Fig. 7, in which scan magnification $m = 2$.

### D. Objective Scan (Angular)

A perfectly circular locus results from a beam illuminating a scanner such that it is radially symmetric, while the objective lens is located in the output beam region of the scanner and coupled rigidly thereto. The scanned function is identical to that of the radially symmetric postobjective scan. It is instrumented functionally by movement of the objective lens of Fig. 2 (of proper focal length) into the axis of the projected beam and the mounting of the lens rigidly to the shaft.

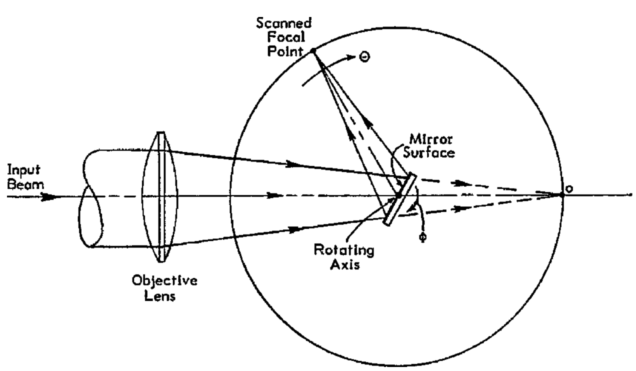


Fig. 7. Postobjective scan with the mirror surface on the rotating axis. The illuminating beam is intercepted by the mirror and reflected to scan a circular locus. Scan magnification $m = 2$.

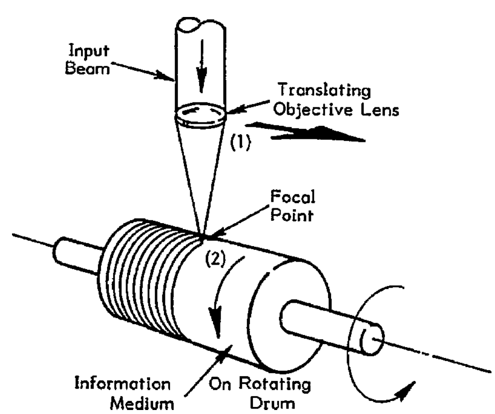


Fig. 8. Drum configuration executing two forms of the objective scan: (1) the lens and its focal point translated with respect to the information medium, (2) the information medium translated with respect to the lens during drum rotation.

## 4. Objective Optics

The objective lens transfers flux to or from the moving image point, forming an integral part of the architecture. In this section some characteristics of the objective optics in active systems are expressed. In passive systems they serve in a complementary manner.

### A. On-Axis Objective Optics

The simplest objective lens is one that operates on axis. Furthermore in laser scanning it may operate monochromatically. As in Figs. 2 and 5 the postobjective deflector intercepts a converging beam to scan its focal point, ideally stigmatically. When radially symmetric (Fig. 2), the perfectly circular scanned arc serves typically for the internal-drum scanner in which the information surface is cylindrical on a fixed drumlike support. In the corresponding passive (object-space) scanning the lens remains on axis but may be complicated by a required spectral compliance.

### B. Flat-Field Objective Optics

This important lens forms a flat field by transforming an angularly scanned beam into a straight-line focal locus. It is often provided with an $f$-$\Theta$ transfer function to denote the proportionality of image displacement $x$ with scan angle $\Theta$ (versus the uncorrected function $x = f \tan \Theta$). As represented in Figs. 3 and 4 the deflector (in active systems) is preobjective, and, as is most prevalent, the scanned input beam is assumed collimated. The pupil must be outside the lens, and, as in Fig. 3, the lens must be spaced at a pupil relief distance to provide clear passage of the input beam.[6] Adaptable to a variety of scanner types, the lens must accommodate the field angle, satisfy the numerical aperture of the scanned flux cone, and provide integrity of line focus and linearity. Although the arrangement of three elements in Fig. 4 is typical, this lens can become considerably more complex for high-resolution service. Also, in the corresponding passive (image-space) scanning, although the object-space numerical aperture may be relaxed, the lens can be complicated by spectral conformity.

### C. Telecentricity

Telecentric optics[3,12,13] provides for the chief ray of a scanned beam to translate parallel to the optical axis, as represented schematically in Fig. 9. Interposed one focal length between the pupil, $D$, and the flat information surface, the ideal lens transforms the angular change at $D$ to a linear translation of the output cone. It restricts the angular deviation of the cone and/or retroreflect a probing beam for calibration or measurement. The correction for $f$-$\Theta$ operation and spectral match can add significant complexity to the telecentric lens.

### D. Double-Pass Optics and Beam Expansion

A nominal beamwidth is fundamental to the acquisition of scanned resolution, defined as the number $N$ of detectable angles $\Delta\Theta$ disposed within a total scanned field angle $\Theta$. That is, $N = \Theta/\Delta\Theta$, shown proportional to the product of angle $\Theta$ and optical aperture width $D$.[3,4] The beam expander,[3,13] introduced in 1964 (Ref. 4, Appendix 1) is an afocal lens group with its shorter focal length followed by a longer one, serving to enlarge a small optical beam to width $D$, as required to achieve a resolution $N$ for a given scan angle $\Theta$.

When the objective lens is adapted to double pass, as depicted in Fig. 10, it serves first as the collimating portion of a lenticular beam expander and, on reflection of the beam by the scanner, as a conventional flat-field lens. This provides compaction not only by folding the beam expander within the objective optics but by avoiding extra pupil relief distance for passage of the illuminating beam.[3] Also, because the illuminating beam approaches normal landing upon the undeflected (neutral position) facet, the beam and facet undergo minimum enlargement, conserving the size of the deflector, which is valuable for high-speed operation.

A slight skew of the input and output planes avoids obstruction of the scanned beam by the folding mirror but introduces a slight bow to the scanned line. If excessive, the bow can be canceled with a shallow field prism or by providing the deflector facets with a small pyramidal tilt.[13] An alternative double-

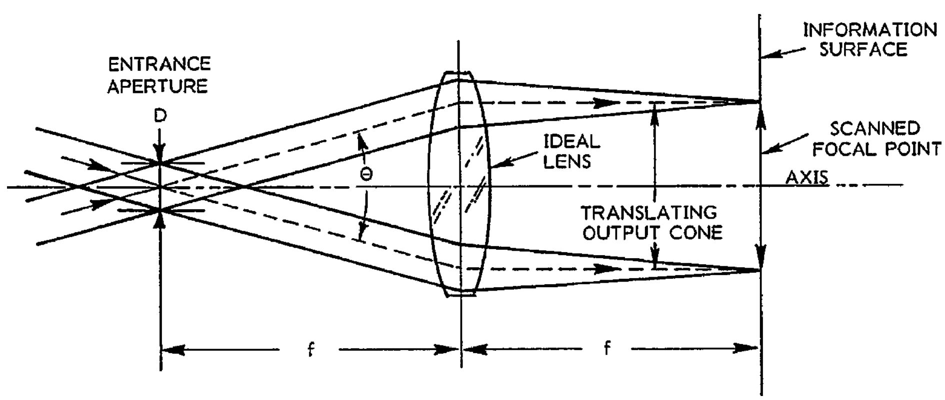


Fig. 9. Telecentric optical schematic. The ideal lens of focal length $f$ transforms the angular scan $\Theta$ from aperture $D$ to the translational scan of the focused beam, whose principal ray is nominally maintained normal to the information surface.

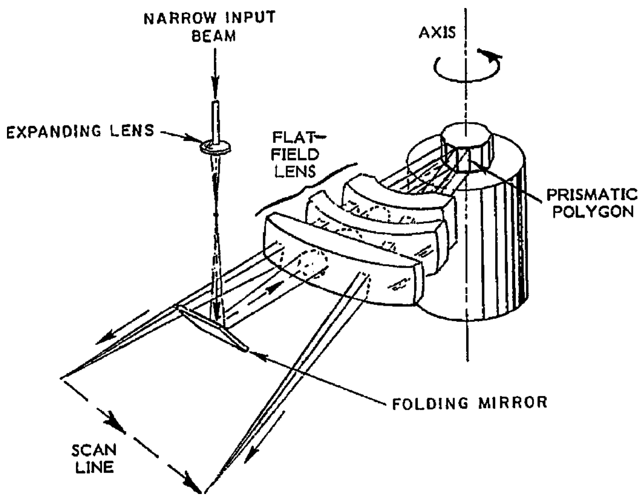


Fig. 10. Prismatic polygon of the double-pass configuration. The input beam is expanded and picked off beyond the focus by the folding mirror to be directed through the flat-field lens and collimated thereby to illuminate polygon facets. The beam reflected and scanned by the polygon reconverges at the flat-field lens to focus on the scan line. Illuminating and output beams are slightly skewed above and below the lens axis for separation by the folding mirror. (The lens elements are shown cut for illustration.)

pass method is to widen the lens field slightly to permit side injection of the input beam off axis, so that all beams remain in the same plane normal to the axis. Although this imposes some potential off-axis beam aberration, it avoids a scanned bow.

## 5. Optical and Resolution Invariant

In Subsection 4.D we express the proportionality of the $\Theta D$ product to the number $N$ of detectable angles in the scanned image field. In the far field this $N$ corresponds to the number of resolvable elements per scanned line—the scanned resolution.[4] Thus this resolution is determined at the scanner and not by subsequent (ideal) optics. This important factor is reenforced by the optical or Lagrange invariant,[11,14,15] expressed in this nomenclature as

$$n\Theta D = n'\Theta' D', \tag{4}$$

where the primed terms are the refractive index, angular deviation, and aperture width, respectively, in the final image space. For the common condition of $n = n' = 1$, this forms the resolution invariant,[2,4,6]

$$I = \Theta D = \Theta' D', \tag{5}$$

illustrated effectively with an afocal relay or beam expander/compressor. Figure 11 represents a telescopic (compressor) relay in which a small angular displacement $\Theta/2$ at pupil aperture $D$ is increased to $\Theta'/2$ at the proportionately reduced pupil image $D'$ so that Eq. (5) is sustained.

The proportionality of the $\Theta D$ product to scanned resolution, and its invariance, results in some significant architectural consequences:

(1) Beam expansion, compression, or aperture relaying, represented by $D'/D = \Theta/\Theta'$, expresses the complementary relationship between the aperture width and the angular propagation of a beam delimited by that aperture.

(2) The scanned resolution (in $N$ elements per scan) is developed at the deflector. The application of (1) following the deflector permits the transfer of the resolution into effectively any field angle or format width (within practical limitations, such as wide format microimaging).

(3) Because $\Theta$ can include any incremental angular deviation, such as from diffraction or systematic error, (1) represents the manner of propagation of these components as well.

## 6. Anamorphic Error Correction

The purpose of anamorphic beam handling is to provide control over one of the most insidious beam-pointing errors in cross-scan beam placement (Subsection 2.B), resulting from imperfect repeatability of

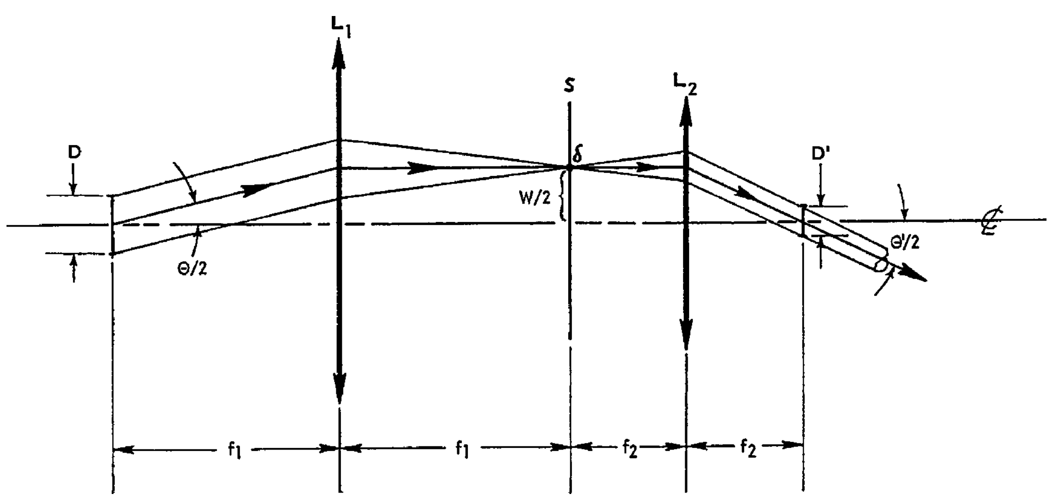


Fig. 11. Invariance $I = \Theta D = \Theta' D'$ with telescopic transfer of the scanned small angle $\Theta/2$ from aperture $D$ to output angle $\Theta'/2$ at output aperture $D'$ so that $\Theta'/\Theta = D/D' = f_1/f_2$.

the deflector's angular position in that direction. Although the function of the deflector is to render a useful angular deviation $\Theta$ in the along-scan direction, it can impart an incremental error in the quadrature cross-scan direction as a function of mechanical imperfections in that direction. These errors can be represented in the arcsecond and fractional arcsecond angular domains.

Since the introduction of cylindrical optics in 1973 for stabilizing such beam placement,[16] many variations have been incorporated into this important process. Most of its explanations, however, cling to the original one, which, although valid as a special case, fails to analyze the parameters that determine the magnitude of the error, the magnitude of its correction, and the available architectural options. In this section these factors are addressed.

### A. Basis of Anamorphic Error Correction

The original explanation of anamorphic error correction describes a collimated isotropic illuminating beam which is then focused cylindrically in $y$ to form a line image on the deflector aperture. The along-scan aperture illumination remains at width $D$. Following deflection, the expanding beam (in $y$) is intercepted by another cylinder (with positive power in $y$) to restore collimation and isotropy to the scanning beam, which is transferred by an objective lens to focus ideally stigmatically on the image plane. Because the line illumination forms essentially no $y$ subtense on the scanner aperture and remains ideally fixed, any erroneous cross-scan angular deviation is directed by the lens to reimage with no $y$-displacement error. That is, a stable conjugate relationship is formed in $y$ between the scanner and the image plane.

A more general representation that identifies the error source and the magnitude of error and its correction is obtained by separating the resolution equation into quadrature components, $N_x$ and $N_y$.[2,4] This effectively forms quadrature resolution invariants:

$$I_x = \Theta_x D_x, \tag{6a}$$

$$I_y = \Theta_y D_y. \tag{6b}$$

Equation (6a) is proportional to the along-scan (desired) resolution $N_x$, and Eq. (6b) is proportional to the (undesired) cross-scan error. Whereas $I_x$ seeks a systematic large number, $I_y$ ideally seeks to approach zero.[17] That is, in a one-dimensional scan there shall be no discernible displacement of recurrent scan lines. The factor $\Theta_y$ represents the source of the error, and the factor $D_y \rightarrow 0$ represents the classic method of negating the error.

### B. General Anamorphic Error Control

Denoting the reduced $D_y$ as a variable $D_\perp$ and keeping all other resolution factors constant,[17] Eq. (6b) is restated to form an error resolution invariant:

$$I_\perp = \Theta_y D_\perp, \tag{7}$$

where $\Theta_y$ is the angle of perturbation in the $y$ direction of the $x$-deflected beam, a function of allowable systematic tolerances. The ratio of $I_\perp$ to $I_y$ represents therefore the perturbation reduction factor $R$, yielding

$$R = D_\perp / D_y, \tag{8}$$

where $D_\perp$ is the reduced beam height and $D_y$ is the normal beam height at the deflector. Beam isotropy (or anisotropy, if desired) is restored with anamorphic optics following deflection.

The simple relationships of Eq. (8) and the cross-scan resolution equation[17] provide critical information on the magnitude of the problem and the (idealized) degree of beam compression. If the system requires only a limited magnitude of reduction in error, there is no need to impart an arbitrarily excessive compression of the illuminating beam. Excessive compression can impose not only aberration with the use of high-powered anamorphics over the scan field, but inconsistency in the compressed line height on a deflector resulting from its possible $z$ translation during scan can create perturbing focal variations on the image plane.

The invariance characteristics of the architecture permit an exercise of freedom in the orientation of the restoring anamorphic optics. Depending on factors of convenience and minimization of aberration (in both $x$ and $y$), the restoring anamorphic optics may be located before, after, or within the objective lens, or in combinations thereof.

## 7. Conclusion and Summary

A conjugate image model is adapted to represent the scanning of an optical flux that propagates to or from a focal point. The reciprocal properties of conjugate imaging unify the fields of active and passive optical scanning, identifying complementary processes and nomenclature. Distinctions are expressed in the relative orientation of the scanner with respect to its objective optics and with utilization of the limiting aperture. Relationships relating to the sequence of the scanner and its objective lens are identified and classified. The Lagrange-resolution invariant reveals important consequences regarding the transfer of resolution and its error components. The conditions for arcuate scan, in particular, along a circular focal locus, are identified and supported by a circular locus theorem (Appendix B). Notably the circular locus may be developed from an illuminating beam whose principal ray is not paraxial with the axis of the scanner.

In Appendix A some of the definitions, axioms, and relationships developed among the objective lens, the scanning elements, and the optical and rotational axes are summarized, and the resolution invariant and its consequences are highlighted.

## Appendix A.

### 1. Definitions

(1) Radial symmetry is the condition of an illuminating beam converging to or diverging from the axis of a rotational scanner. Paraxial collimated illumination is a special case of radial symmetry.

(2) Scan magnification is represented by

$$m = \mathrm{d}\Theta/\mathrm{d}\Phi,$$

in which $\Theta$ is the optical scan angle resulting from a mechanical scan angle $\Phi$.

(3) Axiconal components are functional figures of revolution about an axis generalized to axiconal, expressed by

$$f(r, a)|_{a=a_i} = k_i$$

in which $f(r, a)$ is a general function of radius $r$ and axial position $a$. At each $a_i$, $k_i$ is constant.

### 2. Axioms

(1) Radially symmetric systems exhibit $m = 1$.

(2) Radial symmetry is sustained by operations on the propagating beam by axiconal components that are coaxial with the rotating axis.

(3) A rotational scanner configured for radial symmetry forms an image that rotates about its projected axis at an angular velocity equal to that of the scanner.

(4) When the principal rays of the input and scanned beams of a rotational scanner reside in a plane that is normal to its axis of rotation, $m = 2$, and the scanned image remains stationary with respect to its projected axis.

(5) Overilluminated (overfilled) and underilluminated (underfilled) optical scanners generate the same focal-point trajectory.

### 3. Objective Lens Relationships

(1) Executing the objective scan by translation of an objective lens transverse to its axis translates the image focal point. Executing the objective scan by translation of the information surface with respect to the objective lens renders the same effect.

(2) (a) A scanner oriented prior to its objective lens is designated as preobjective. (b) One oriented following its objective lens is designated as postobjective.

(3) Preobjective scanning permits subsequent lenticular formation of a preferred field such as flat and/or telecentric and of arbitrary format.

(4) Postobjective scanning generates a generally curved locus of the scanned focal point. When radially symmetric, the locus is circular [$m = 1$, per Axiom (1)].

### 4. Scanned Resolution Invariant

With the scanned resolution proportional to the optical scan angle and the aperture/pupil distribution $D$ at the scanner (in the direction of $\Theta$), the $\Theta D$ product becomes the basis of the resolution invariant,

$$I = \Theta D = \Theta' D',$$

in which the primed terms appear in the final aperture/pupil space of a stigmatic, nonvignetting optical transfer system.

As a consequence, we have the following:

(1) The scanned resolution ($N$ elements per scan) is determined at the deflector, not by subsequent (stigmatic and nonvignetting) transfer optics.

(2) An incremental angular change $\Delta\Theta$ (such as a deflected error component) propagates according to invariant $I$ through subsequent optics.

## Appendix B: Circular Locus Theorem

Certain optical scanning systems require that the scanned focal point execute a locus that matches a circular arc segment.[2,3] This permits the formation of a congruent information-bearing surface for precise focal conformity. For high resolution to be achieved, diffraction limitations require the use of converging cones of light derived from large numerical apertures (low $f$ numbers) with shallow depths of focus. For a high resolution to be sustained, the light cone must be deflected so that its focal surface maintains precise conformity with that of the information-bearing medium. Furthermore the velocity of the moving focal point must often be uniform. This theorem, initiated in 1965 to support a scanner development and a subsequent patent filing,[18] describes the formation of a circular scan locus from an apparent off-axis illuminating beam. The illumination is defined as exhibiting radial symmetry. The original research, which was directed toward active single-point scanning by mirror reflection, is extended here to express the characteristics of radially symmetric systems with arbitrary reflective/refractive/diffractive properties.

Three recurring phrases are defined here:

Rotational system: A group of optical elements rigidly disposed with respect to an axis and rotating about said axis.

Radial symmetry: The condition of an illuminating beam converging to or diverging from the axis of a rotational system. Paraxial collimated illumination is a special case of radial symmetry.

Original focal point: The waist of a converging flux tube that would exist if an obstructing barrier were removed.

### 1. Plane Mirror Case (Fig. 12)

Mirror M intercepts a uniform distribution of rays converging to an original focal point p on axis and reflects the rays to point P′ so that

$$|\mathbf{m}_i\mathbf{p}| = |\mathbf{m}_i\mathbf{p}'|, \tag{B1}$$

where the $m_i$ positions are the $(1, 2, 3, \ldots, i)$ ray intercepts on M.

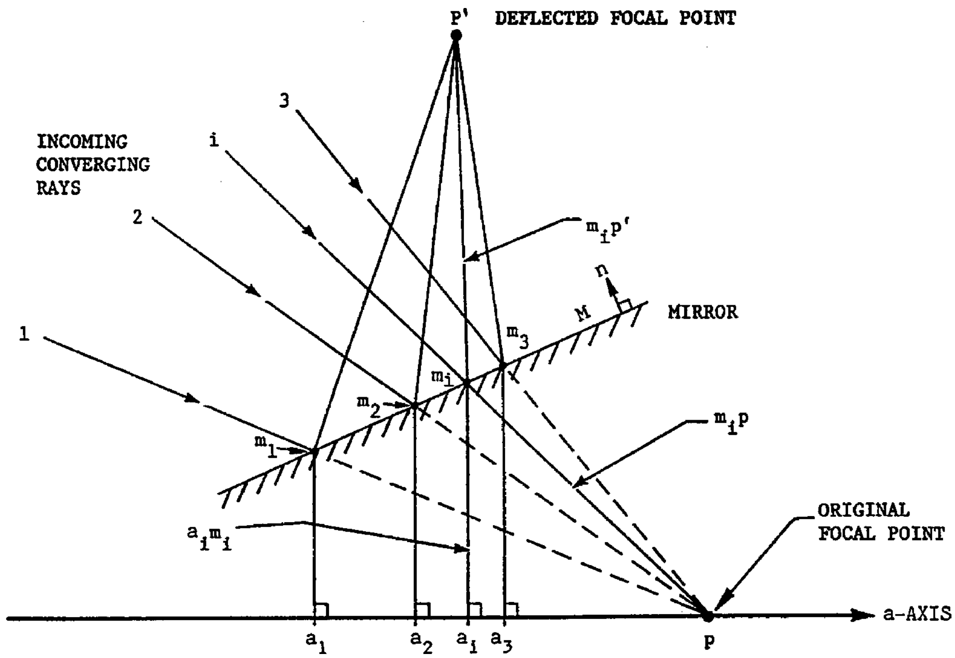


Fig. 12. Section of the rotational system containing the normal n to mirror M. Rays 1, 2, 3, . . . , *i* are selected from the overilluminating beam (converging at point p) to intersect at $m_1$, $m_2$, $m_3$, . . . , $m_i$ during rotation of M, which maintains a fixed distance $a_i m_i$ to the axis. Consider that the normal to M remains in the plane of the illustration while the illuminating flux rotates about the *a* axis (pivoting about point p).

During the rotation of *M* about the axis, each radial distance from the axis to each of the $m_i$ positions on M remains a constant $k_i$. That is,

$$\mathbf{a}_i\mathbf{m}_i = \mathbf{k}_i. \tag{B2}$$

Also during rotation the new rays selected from the incoming uniform distribution of converging flux intersect at $m_i$ on their paths to point p, maintaining constants $c_i$. That is,

$$\mathbf{m}_i\mathbf{p} = \mathbf{c}_i. \tag{B3}$$

Because

$$\mathbf{a}_i\mathbf{m}_i + \mathbf{m}_i\mathbf{p} = \mathbf{k}_i + \mathbf{c}_i, \tag{B4}$$

then, by Eqs. (B1), (B2), and (B3),

$$\mathbf{a}_i\mathbf{m}_i + \mathbf{m}_i\mathbf{p}' = \mathbf{a}_i\mathbf{p}' = \mathbf{K}_i, \tag{B5}$$

where each $K_i$ represents a constant distance from the axis to point P′, describing a circular locus of P′ about the axis. Because P′ remains in the plane that includes the normal to M, its angular displacement about the axis equals that of M.

If the original focal point p is not on axis, then contrary to Eq. (B3), distances $\mathbf{m}_i\mathbf{p}$ are not constant, whereupon $\mathbf{a}_i\mathbf{p}'$ of Eq. (5) is not constant and the locus is not circular. Thus the plane-mirror circular locus theorem is stated as follows:

A plane mirror in a radially symmetric system reflects converging rays to focus on a circular locus about the axis and to execute an angular displacement that is equal to that of the rotational system.

### 2. *Arbitrary Optical Element Case*

Replacing plane mirror M with a reflector or reflective diffractor element having arbitrary optical characteristics provides reflection/diffraction of a flux function, which is determined by the optical element. Each set of virtual/real foci developed by the arbitrary reflector/diffractor executes its circular locus superimposed in a fixed relationship on all others of the set. With the incoming uniform illumination maintained invariant to the rotating scanner, the resulting reflected/diffracted flux distribution remains stationary during execution of its circular locus about the axis.

Similarly, replacement of mirror M with a transmissive refractor/diffractor having arbitrary optical characteristics and illuminating it uniformly from point p develops a scanning flux function that maintains a fixed spatial relationship to the axis. With the illumination to or from point p establishing radial symmetry, this leads to the general form of the circular locus theorem, which can be stated as follows:

A rotational system illuminated in radial symmetry develops the distribution of a scanned flux that executes a circular locus about the axis, with an angular displacement that is equal to that of the rotational system.

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$$N_y = \frac{\Theta_y D_y}{a_y \lambda},$$

in which the numerator is Eq. (6b), $a_y$ is the aperture shape factor ($0.75 \leq a < 2$), and $\lambda$ is the wavelength. $N_y \rightarrow 0$ seeks no displacement between ideally superimposed recurrent scan lines. The numerical evaluation of this equation yields an actual displacement in number of lines (or the relative value of the cross-scan error, typically as a fractional displacement of the linewidth or pitch).
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