Athermal bond thickness for axisymmetric optical elements

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1. INTRODUCTION

Thermally induced stress is a primary concern in the bonded mounting of optical elements. Systems that are designed to endure particularly harsh thermal conditions must employ athermal bonding to protect against the risks of bond or optical element failures. There are several closed-form solutions for athermal bond thickness about an axisymmetric optical element. In this tutorial I will present the Bayar equation, van Bezooijen equation, modified van Bezooijen equation, and aspect ratio approximation solutions for athermal bond thickness. The assumptions and results associated with each formulation will be discussed and applied to an example system.

2. EXAMPLE SYSTEM

Consider a 25.4 mm diameter, 4 mm thick window made from NBK-7. As an example of athermal bonding consider the constraint of this optical element within a cylindrical mounting cell made of alloy 6061-T6. The simple window and cell are shown schematically in Figure 1. The window has flat edges, so the bond will have a cylindrical shape at the nominal bond curing temperature. The system is assumed to be free of stress at the nominal temperature. The bond thickness will be evaluated for two types of adhesive: 2216 B/A epoxy and RTV 566 silicone. Approximate values for material constants of the optical element, mounting cell, and two adhesives are given in Table 1.

Figure 1: Drawing of example system. Window in blue, bond in red, cell in gray.

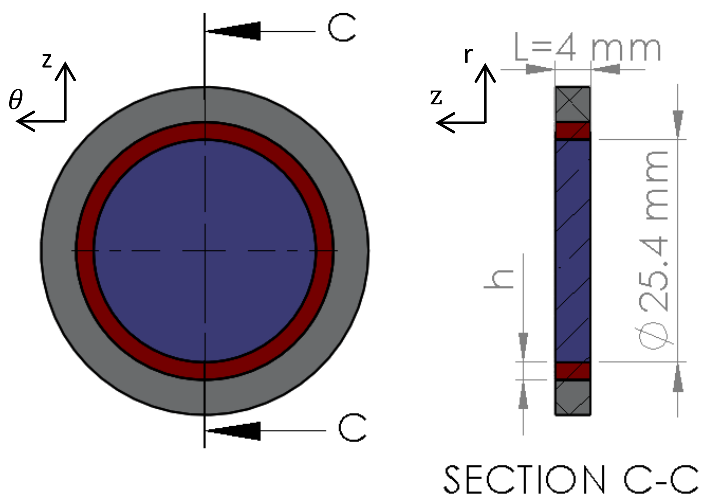


Table 1: Material constants used in example system

MATERIAL	α(ppm/°C)	Poisson ratio, ν	E (Gpa)
N-BK7	7.1	.21	82
6061-T6	24	.33	69
2216 B/A (gray)[1]	102	~.43	69
RTV 566	200	~.499	~.003

3. ATHERMAL BOND THICKNESS SOLUTIONS

3.1 Principles of athermal bonded axisymmetric elements

Athermal bonding requires that the differential expansions of the optical element, bond, and mounting cell are properly balanced to achieve zero radial stress in the optical system. In the case of the cylindrical geometry presented in this tutorial, we require that the bond thickness is chosen such that bond expansion compensates the temperature-dependent change in the gap between the optical element and cell. Note that we require $\alpha_o < \alpha_c < \alpha_b$ to create an athermal bond, where the subscripts o, c, b refer to the optical element, cell and bond, respectively. If $\alpha_o > \alpha_c$, then α_b would have to be negative, which is not practical. Matching all three CTE's is another possible solution to athermal bonding, but this is difficult to achieve with readily available materials. Luckily, common structural materials, like aluminum and steel, have a greater CTE than most glasses, while adhesives tend to have quite large CTE's. Note that the example system presented in this tutorial meets the criteria $\alpha_o < \alpha_c < \alpha_b$ with readily available materials.

There are several ways to treat the bond constraints which provide different limiting cases for bond thickness. In all cases considered in this tutorial the radial strain of the bond is given by[4]

$$\varepsilon_r = \frac{\delta h}{h} = \Delta T\left(\alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o)\right),$$

where δh is the change in thickness of the bond, h is the nominal bond thickness, and r_o is the radius of the optical element.

The axial and tangential strains in the bond are directly influenced by the assumed constraints. Hooke's Law for stress yields the following formulation for radial stress σ_r in the bond in terms of axial strain ε_z and tangential strain ε_θ:[4]

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_r + \nu(\varepsilon_z + \varepsilon_\theta)\right].$$

Several treatments of bond constraints between the optical element and cell will be discussed in the following sections. In each case the constraints define ε_z and ε_θ and we solve for bond thickness h after setting radial stress equal to zero.

3.2 Bayar Equation

The Bayar equation[2] is the simplest formulation of athermal bond thickness. Axial and tangential constraints on the bond are entirely neglected so that $\varepsilon_z = \varepsilon_\theta = 0$. This assumption neglects all bulk effects in the bond and ignores any constraint of the bond in shear. These assumptions are clearly inaccurate, but the resultant bond thickness may be useful if small cubical bond areas are used rather than a continuous bond-line. According to the Bayar equation bond thickness is given by

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c}.$$

Results for example system: 2216 B/A, h= 2.75 mm. RTV 566, h=1.22 mm.

3.3 Van Bezooijen Equation

Van Bezooijen[3] offers a much more accurate formulation of bond thickness. In this formulation it is assumed that the bond is perfectly constrained to the surfaces of the optical element and cell. To determine the axial and tangential strains, the bond's length in the heated condition is taken as the average of the bond lengths at the expanded optical element and cell interfaces. This is a good approximation for the tangential and axial directions provided the bond is thin. The strain in both directions is, in fact, equal and given by

$$\varepsilon_z = \varepsilon_\theta = \Delta T\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right).$$

Using the strain equations and solving for zero radial stress, we get the van Bezooijen equation for bond thickness:[3]

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{2\nu}{1-\nu}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)}.$$

Note that the assumption of perfect bond constraint at each interface is not entirely accurate due to the possibility of bulging or contraction of the bond in the axial direction, especially near the center of the bond's thickness. Since this effect is neglected here, the bond thickness predicted by the van Bezooijen equation serves as a lower limit for athermal bond thickness.

Results for example system: 2216 B/A, h= 1.03 mm. RTV 566, h= 0.40 mm.

3.4 Modified van Bezooijen Equation

To address the bulk effect of bulging and contraction of the bond in the axial direction, Monti[4] gives another limiting case where axial strain is neglected. The formulation of tangential strain is exactly the same as it is in the derivation of the van Bezooijen equation. After solving for bond thickness as before, we have the modified van Bezooijen equation:[4]

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)}.$$

The only difference between this equation and the van Bezooijen equation is a factor of 2 discrepancy in the third term of the denominators. The predicted bond thickness is correspondingly larger, so this axially unconstrained formulation gives us an upper limit on athermal bond thickness.

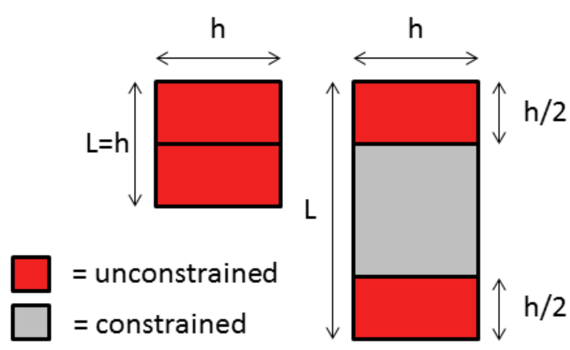
Results for example system: 2216 B/A, h= 1.50 mm. RTV 566, h=0.60 mm.

3.5 Aspect Ratio Approximation

To develop a closed-form solution that offers the best approximation of athermal bond thickness, Monti[4] considers the aspect ratio of the bond cross-section. For a bond length L in the axial direction, the aspect ratio of the bond is defined as $R_{aspect} = \frac{L}{h}$. Monti assumes that the modified van Bezooijen equation corresponds to an aspect ratio of 1, meaning that a bond with equal thickness and height acts as if it is axially unconstrained. As shown in Figure 2, the bond is decomposed into constrained and unconstrained portions. The ratio of axially constrained bond is then defined as follows:

$$R_{constrained} = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - R_{aspect}^{-1}.$$

Figure 2: Dimensions of unconstrained and constrained portions of bond in aspect ratio approximation



This coefficient is multiplied by the formula for axial strain ε_z in the van Bezooijen equation to produce a new set of strain relations

$$\varepsilon_z = \Delta T\left(1 - \frac{h}{L}\right)\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right),$$

$$\varepsilon_\theta = \Delta T\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right).$$

The bond thickness may be written, in direct analogy to the van Bezooijen equation, as

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu}\left(2 - \frac{h}{L}\right)\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right)}.$$

The coefficient $\left(2 - \frac{h}{L}\right)$ in the denominator ranges from 1 to 2 for aspect ratios in the range of 1 to infinity. This formula reduces to the van Bezooijen equation in the limit where R_{aspect} equals infinity and becomes the modified van Bezooijen equation for R_{aspect} =1. However, h appears in the right-hand-side of the equation, so the quadratic formula must be used to arrive at a closed-form solution. The aspect ratio approximation solution is most elegantly shown in its expanded form:[4]

$$h = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \begin{cases} a = \frac{-\nu}{L}\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right) \\ b = (1-\nu)(\alpha_b - \alpha_c) + 2\nu\left(\alpha_b - \frac{\alpha_o + \alpha_c}{2}\right) \\ c = -r_o(1-\nu)(\alpha_c - \alpha_o) \end{cases}$$

The aspect ratio approximation offers a closed-form approach to calculating bond thickness as an alternative to the application of empirically determined correction factors.

The empirical correction factors presented by Michels, Gregory and Doyle[5] depend on aspect ratio as well as poisson ratio of the bond material, and are applied in place of the coefficient $\left(2 - \frac{h}{L}\right)$ in the van Bezooijen equation. Monti[4] compares the results of the aspect ratio approximation and van Bezooijen equation with correction factors. The aspect ratio is a very good approximation for $\nu \in (.45, .5)$ and $R_{aspect} > 4$. Since these are relatively typical values for elastomeric bonding, the aspect ratio is a very useful formula for athermal bond design. If the aspect ratio is small it is best to use the empirical correction factors mentioned previously, or to model the design and optimize performance using FEA as feedback.

Results for example system: 2216 B/A, h= 1.13 mm. RTV 566, h=0.41 mm.

4. CONCLUSION

The results for athermal bond thickness in the example system are consolidated in Table 2. Note the stark difference between results from the Bayar equation and the other formulas. This should be a clear indication that axial and tangential strains are very important factors in athermal bond design. However, according to Monti[4] inaccuracies in cured adhesive mechanical constants may be the primary source of error in calculations of athermal bond thickness. The aspect ratio approximation is a great approach and offers an improvement in accuracy over the van Bezooijen equation. If thorough testing is performed to accurately determine the cured properties of an adhesive under controlled conditions, then FEA correction factors can offer a useful improvement over the aspect ratio approximation.

Table 2: Calculated athermal bond thicknesses results for example system

THICKNESS EQUATION	2216 B/A	RTV 566
Bayar	2.75 mm	1.22 mm
van Bezooijen	1.03 mm	0.40 mm
Modified van Bezooijen	1.50 mm	0.60 mm
Aspect ratio approximation	1.13 mm	0.41 mm

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