

Introduction to Designing Elastomeric Vibration Isolators

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Introduction:

An elastomer is any elastic polymer. They are used in many applications from adhesives to sealing. They are well suited to vibration isolation because properties such as young's modulus and internal dampening can be engineered to meet particular specifications, they are homogeneous and therefore compact, and they can be molded or formed in place and allowed to cure reducing manufacturing complexity. This paper will discuss how to design basic vibration isolators using elastomers. Anything more than using simple geometry requires complex analysis. Using simple geometries with rectangular or circular cross section will allow the use of the formulas presented here. By using the methods presented, isolation in single axis for a narrow range of excitation frequencies can be achieved or the calculations can be used as a starting point for sizing a more complex system. Many of the formulas used are approximations that are valid for specific conditions. These potential pitfalls will be identified so the designer ends up with a design that performs as expected.

This paper doesn't cover all of the various types of elastomers. Material selection depends on the application and properties such as: loading, sensitivity to thermal changes, sensitivity to strain, resistance to contaminants, material compatibility, internal damping, etc.

Design Parameters:

There are several key design parameters that go into designing an isolator. The first is what the physical configuration is. There are many forms depending on the application. Isolators can take on very complex shapes. However, for a simple introduction that relies on hand analysis, we'll consider three basic configurations: planer, laminate planer, and cylindrical. Another key parameter is the direction of the applied load on the isolator. Isolators can be loaded in shear, compression, and tension. Cylindrical isolators can also be loaded in torsion, but that is really just a type of shear. The direction of applied load ties into our other key design parameter, and perhaps our most important, spring rate. Designing for spring rates determines the system resonant frequency. That and dampening determine the amount of attenuation of the input amplitude. A couple of other factors that affect spring rate are material properties such as bulk and Young's modulus. Other key factors are maximum isolator displacement, both static and dynamic, and material properties like internal dampening and the ultimate strength.

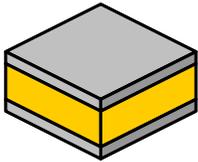
Isolator Configurations:

Isolators can come in many designs. Figure 1 shows an engine mount for an automobile. It is elastomer block bonded to two metal plates with mounting hardware to facilitate installation. This is a relatively simple design whose purpose is to keep high frequency vibrations generated by the engine from the vehicle frame. Isolators can be of more complex form and be designed to isolate particular frequencies or a range of frequencies.



Figure 1: Engine Mount

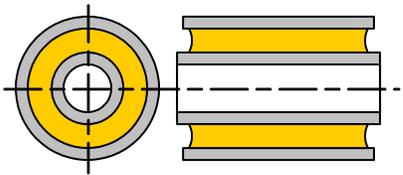
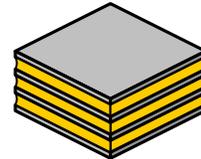
We'll consider three simple geometries: The planer, bearing, and cylindrical bearing. Bearing is just a term that describes any object that is designed to take a load.

**Planer Bearing:**

The planer bearing is perhaps the simplest. It consists of two plates with the elastomer sandwiched in between. It can be of any cross section; however, for design simplicity often it is round or rectangular.

Laminate Bearing:

The laminate bearing is a bunch of planer bearings stacked on top of each other. It has the advantage of being very stiff in compression which compliant in shear. Like the planer bearing, it can be of any cross section; however, for design simplicity often it is round or rectangular.

**Cylindrical Bearing:**

The cylindrical bearing takes axial loads and torsion in shear and radial loads in compression. It is unique in that it has a torsion spring constant.

Spring Rates:

As mentioned earlier, spring rate is probably the most important parameter in isolator design. Spring rate determines the system's resonant, also known as natural, frequency. This is assuming the isolator's compliance is much, much greater than any of the system's other structural components. Knowing the spring rate and dampening, the transmissibility can be calculated according to equation 1. Transmissibility is the ratio of input displacement to output displacement. Figure 2 shows the transmissibility curve as a function of the ratio of resonant to input frequency (f_r/f) for various loss factors (η).

$$T = \sqrt{\frac{1+(\eta r)^2}{(1-r^2)^2+(\eta r)^2}} \quad (1)$$

$$r = \frac{\text{resonant frequency}}{\text{input frequency}} = \frac{f_r}{f} \quad (2)$$

$$\eta = \frac{\text{dampening modulus}}{\text{dynamic elastic modulus}} = \frac{G''}{G'} \quad (3)$$

complex modulus:

$$G^* = G' + iG'' = G'(1+i\eta) \quad (4)$$

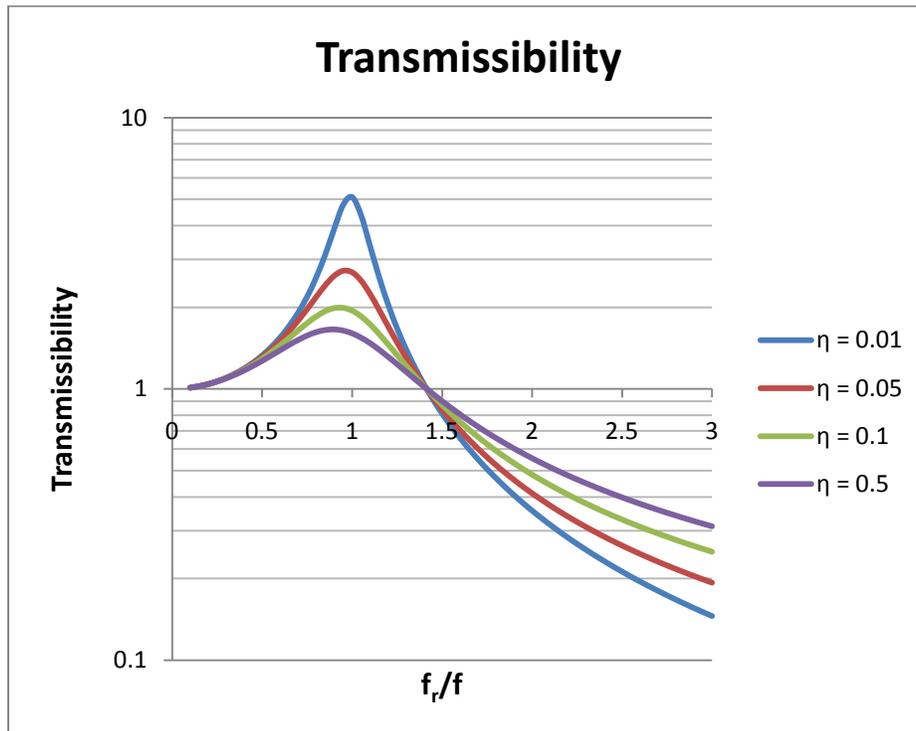


Figure 2: Transmissibility as a Function of the Frequency Ratio

It is important to note that when the ratio of the resonance frequency to the input frequency is less than $\sqrt{2}$ the input displacement is amplified. When it is greater than $\sqrt{2}$

the input displacement is attenuated. Knowing the resonant frequency, it is now possible to calculate the required spring rate according to equation 4 where m is the system mass.

$$f_r = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (5)$$

There is a spring rate for each of the three loading conditions: shear, compression, and tension. Each spring rate is also effected to some degree by the ratio of isolator load area to the area that the elastomer is free to move (bulge area). This ratio is known as the isolator's shape factor. For simple shapes this is easy to calculate. Equations 6 and 7 show how to calculate shape factor for rectangular and circular planer bearing respectively.

$$SF_{\text{rect}} = \frac{L \times W}{2t(L+W)} \quad (6)$$

$$SF_{\text{circ}} = \frac{\pi D^2}{4\pi D t} \quad (7)$$

For more complex geometries, such a cylindrical bearing the shape factor is calculated by approximating the rectangular form. The width approximation is given by equation 8 which is then used in equation 9. d_o and d_i are outside and inside diameters of the elastomer receptivity.

$$B = 1.12 \sqrt{d_o \cdot d_i} \quad (8)$$

$$SF_{\text{cyl}} = \frac{B \cdot L}{2(B+L) \left(\frac{d_o - d_i}{2} \right)} \quad (9)$$

Shear Spring Rate:

For an isolator loaded in shear the spring rate is given by equation 10. Where A is the area, G is the shear modulus, and t is the thickness. For an elastomer, where Poisson's ratio is near 0.5 (ratios can be on the order of 0.49), the shear modulus is approximately 1/3 of Young's modulus.

$$k_s = \frac{AG}{t} \quad (10)$$

Some important considerations are that shear modulus can be considered linear up to 75% to 100% strain and can change for high compressive load given large shape factors. If possible try to avoid either situation. Also, where the aspect ratio of thickness to length is greater than 25% a correction factor must be added to equation 10 that accounts for partial normal stress coupling into shear stress (equation 11). r_g is the radius of gyration.

$$k_s = \frac{AG}{t} \cdot \frac{1}{1+t^2/36r_g^2} \quad (11)$$

Compression Spring Rate:

For an isolator loaded in compression, equation 12 is used to calculate spring rate. A is area, E_c is the effective compression modulus, and t is the thickness.

$$k_c = \frac{AE_c}{t} \quad (12)$$

The effective compression modulus is linear up to 30% strain. It is also highly dependent on the isolator's shape factor. For large shape factors, where the elastomer is very thin, the effective compression modulus is equal to the bulk modulus. For small shape factors the effective compression modulus is approximately the same value as the Young's modulus. There is also a transition region between the two. It is important to know which region your isolator falls into to correctly estimate the effective compression modulus.

Calculation of the Effective Compression Modulus:

Before we can calculate the effective compression modulus, we'll have to determine the bulk and Young's modulus for our material. Some manufacturers include this information in their datasheets. If they don't, then it may be necessary to contact them directly. Another option is to test the material using samples. Many times, manufacturers will present the Durometer Shore A hardness value and an ultimate strength. Gent derived an empirical formula (equation 13) for finding Young's modulus from Shore A hardness (s). This can be used for initial calculations. Values correlate well for Shore A values greater than 40.

$$E(MPa) = \frac{0.0981(56+7.66s)}{0.137505(254-2.54s)} \quad (13)$$

Remember that shear hardness is approximately 1/3 Young's modulus. Using the graph from Gent (figure 3), effective compression can be found knowing the shape factor.

An alternate method was developed by Hathway. He found the slope of the transition zone given in equation 14. He used the ratio t/D of a circular planer isolator which is similar to the shape factor term.

$$\frac{E_c}{E} \cdot \left(\frac{t}{D}\right)^{1.583} = 0.336 \quad (14)$$

$$\frac{t}{D} \propto \frac{1}{SF} \quad (15)$$

Knowing that for small values of t/D , the effective compression modulus is the same as bulk modulus, the point where transition region begins can be determined by equation 16.

$$\left(\frac{t}{D}\right)_1 = \left[0.336 \cdot \frac{E}{E_{bulk}}\right]^{\frac{1}{0.583}} \quad (16)$$

The end of the transition zone occurs where E_c/E is one (equation 17). The results are shown in figure 4.

$$\left(\frac{t}{D}\right)_2 = (0.336)^{\frac{1}{0.583}} \quad (17)$$

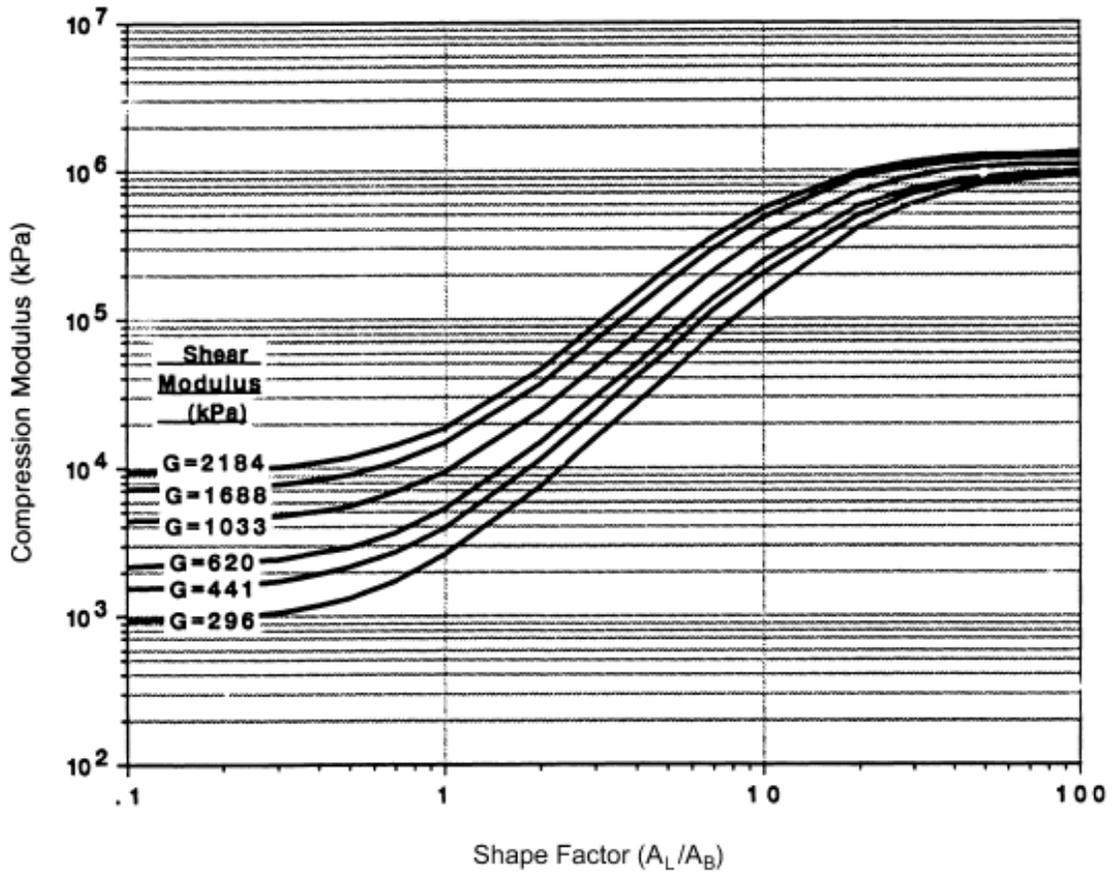


Figure 3

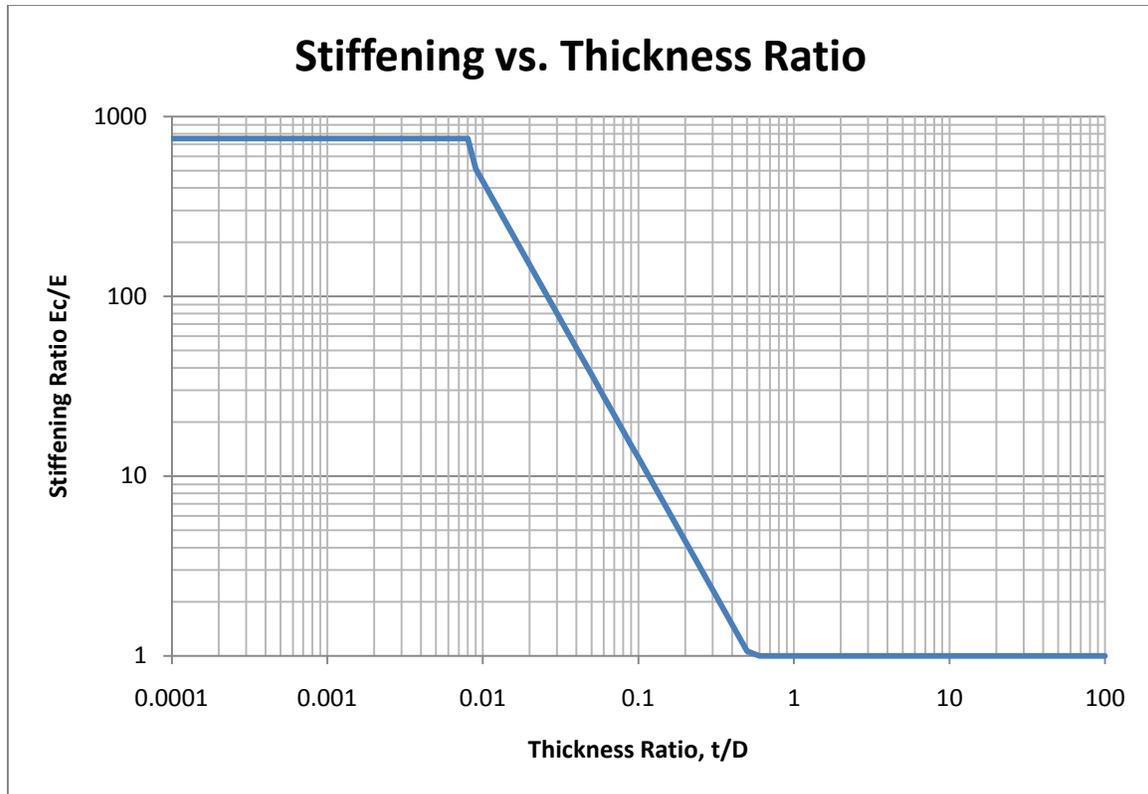


Figure 4

Tension Spring Rate:

The final spring rate for an isolator in tension is given by equation 17. Try to avoid loading in tension. Isolators in tension have a low modulus and can be damaged by relatively low loads.

$$k_t = \frac{AE_t}{t} \quad (17)$$

One method to avoid loading in tension is to apply a compressive preload. It is relatively easy to preload cylindrical bearings. After the cylindrical bearing is constructed, either the outer sleeve can be compressed or the inner tube can be expanded. For planer bearings, careful design of the installation must be done so that a preload is applied.

Laminate Bearings:

Laminate bearings are made up of planer bearings stacked on top of each other. There are alternating layers of elastomer and a relatively stiff material such as metal. The shear and compression spring rates are given as follows:

$$k_s = \frac{AG}{tN} \quad (18)$$

$$k_c = \frac{AE_c}{tN} \quad (19)$$

N is the number of layers and t is the thickness of each layer. Each layer typically has a very large shape factor which means the effective compressive modulus is equal to the bulk modulus. The shear modulus is 1/3 Young's modulus. This means the laminate bearing is very stiff in compression while being compliant in shear. This feature can be taken advantage of to isolate in a particular direction only.

Cylindrical Bearing:

The cylindrical bearing is two concentric tubes with elastomer in between. It can take radial loads in compression and axial load and torsion in shear. It is often used in automotive suspension linkages where the joint must rotate as well as provide some measure of isolation. It has a torsion spring rate given by equation 20 where d_o and d_i are the outside and inside diameters of the elastomer. When calculating the effective compression modulus, use equations 8 and 9 to find the shape factor.

$$k_T = \frac{T}{\theta} = \frac{\pi GL}{(d_o^{-2} - d_i^{-2})} \quad (20)$$

Design Considerations:

When designing an isolator it is necessary to consider the following:

- What's being isolated?
 - mass, center of gravity, maybe moments of inertia
- What are the inputs?
 - input frequency and amplitude, shock loads
- Are there static loads?
 - steady state or slow time varying accelerations (such as on an aircraft)
- What are the environmental conditions?
 - temp, humidity, damaging fluids
- What is the allowable system response?
 - attenuation, maximum deflection
- What is the service life?
 - partially determined by input energy and duty cycle

Conclusion:

All of the formulas used here make very specific assumptions. The key is to keep the design simple. To avoid non-linear effects, keep loads low to moderate and stay away from high strain values. Keep the aspect ratio less than 25% for shear geometries. For complicated geometries, it will be necessary to perform a finite element analysis and testing, but you have tools to first get you in the ballpark. Also, it is important to know about what your effective modulus is. You might think that you can use bulk modulus, but in reality the effective modulus might be in the transition zone. One other variable that wasn't discussed much, but plays a large part in transmissibility is the loss factor (η). If you find that at certain frequencies the isolator is amplifying the input displacement to unacceptable levels, then increasing internal dampening (finding a material with a larger η) may fix the problem. Keep in mind that this comes at the cost of less attenuation at

higher input frequencies. Finally, the isolator may also have additional, more stringent requirements imposed. For mirror mounting, there may a requirement on how much temperature induced stress due to CTE mismatching is allowed. Or, if the design is supposed to be athermal, other material properties become very important. But, if the design is kept simple then this a good starting point for sizing an isolator.

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