1) **Flexure stage design and analysis (25)**

Sketch a design that uses two blade flexures to allow a small linear translation in the $z$ direction, yet constrains all other degrees of freedom. On your sketch, define the $x$, $y$, and $z$ axes and provide all important dimensions of the flexures. Use auxiliary views as necessary.

Provide the equations that would be used to determine $k_x$ and $k_y$, the stiffness of the mechanism against forces in each of the two directions perpendicular to the motion allowed by the flexures.

**$k_x$: Description:**

- Puts flexure in shear
- For $W = L$, bending in its stiff direction for $L \gg W$

**Equation:**

\[ w = \frac{W}{2L} \text{, stiffness per blade is from } \delta_x = \frac{F_x L}{G A} \]

\[ k_x = \frac{G A}{2L} \text{ per blade} \]

Two blades, stiffness adds

\[ k_x = 2k_x = \frac{2G A}{2L} = \frac{2G w t}{L} \]

**$k_y$: Description:**

- Axial load on flexures

**Equation:**

\[ \delta_y = \frac{F_x L}{E A} \]

\[ k_y = \frac{E A}{L} \]

2 blades

\[ k_y = 2 \frac{E w t}{L} \]
2) **Flexure materials (15)**

a) List a material that is commonly used for flexures.

Stainless steel 17-4 PH, 15-5 PH

Be Co

Titanium

7075 Aluminium

b) From first principles, derive the relationship that relates the elastic range of bending a blade flexure to the geometry (Length, width, thickness) and material properties (yield strength, Young’s modulus).

\[
\theta = \frac{ML}{EI}
\]

\[
\sigma = \frac{M y}{I}
\]

Max stress at \( y = t/2 \)

\[
\sigma_{max} = \frac{t}{2} \cdot \frac{E \theta}{L} = \frac{1}{2} \frac{t}{L} E \theta
\]

\[
\theta_{yield} \text{ for } \sigma_{max} = \sigma_{yield}
\]

\[
\theta_{yield} = \frac{1}{2} \frac{t}{L} \sigma_{yield} \frac{E}{L}
\]
3) **Heat flux and thermal expansion (25)**

Consider a 10 cm long, 1 x 1 cm square cross section rod made of invar 36, heated from the end with 1 Watt of thermal power. \( \lambda = 14 \text{ W/m-K for invar36} \)

\[ Q = 0.14 \text{ W/cm-K} \]

\( A \) (T = 20 C)  
\[ B \]

A 10 cm long, 1 x 1 cm square cross section rod  
\( z = 10 \) cm

a) Define \( z=0 \) at the cantilever interface, which is always maintained at 20 C. Provide a plot that shows the temperature profile from \( z = 0 \) to \( z = 10 \) cm as the rod conducts 1 Watt of power steady state.

\[ Q = \frac{H}{A} = \lambda \frac{\Delta T}{\Delta z} \]

For \( \Delta T = 10 \text{ cm}, H = 1 \text{ W} \)

\[ \Delta T = \frac{1}{10} \text{ cm cm^2 K} = 71^\circ \]

\[ \Delta T = 7.1 \cdot z \]

Compared to a 20 °C isothermal state --

b) Determine the stress in the rod due to the thermal gradient.

*Unconstrained, no stress*

c) Determine the shift of point B at the end of the rod due to this thermal gradient.

\[ \Delta L = 7.1 \cdot \frac{1}{2} z^2 \]

\[ = 7.1 \cdot \frac{1}{2} \times 10^{-6} \text{ cm} \]

\[ = 355 \times 10^{-6} \text{ cm} \]

\[ = 3.55 \mu\text{m} \]
d) If a rigid constraint is added so that point B cannot move as the thermal gradient is applied, determine the reaction force at A.

\[ F = \frac{E A}{L} \cdot \Delta T = \frac{150,000 \text{ N/mm}^2 \cdot 100 \text{ mm}^2}{100 \text{ mm} \cdot 0.0036 \text{ mm}} = 540 \text{ N} \]

\( E = 150,600 \text{ MPa} = 21 \text{ Ms} \)


e) Determine the maximum stress in the rod for the case above with the overconstrained rod and the applied thermal gradient.

\[ \sigma = \frac{F}{A} = \frac{540 \text{ N}}{100 \text{ mm}^2} = 5.4 \text{ MPa} \]

4) **Stages (5)**

In the design of linear stages, we strive to minimize the Abbe offset error. Make a sketch that defines the Abbe offset and show quantitatively how it can affect performance of a linear stage.

![Abbe Offset Diagram](image)

\[ \text{Carriage yaw } \theta \]

\[ \text{Encoder error } \delta z = \varepsilon \cdot \theta \]
5) Change in focus with temperature (10)

Consider a 100 mm focal length lens made from BK7 and support tube made from aluminum. The aperture is 25 mm.

\[ \text{Al} \quad \alpha = 23 \text{ ppm/°C} \]
\[ \text{BK7} \quad \alpha = 7 \text{ ppm/°C} \]
\[ \frac{du}{dT} = 3 \text{ ppm/°C} \]

\[ \beta = \alpha - \frac{1}{n-1} \frac{du}{dT} = 1 \text{ ppm/°C} \]

For 20°C temperature change, determine the change in focus in mm.

\[ \text{Lens} \quad \Delta f = \beta f \Delta T = 1 \cdot 100 \cdot 20 = 2000 \text{ E-6 mm} \]
\[ = 2 \mu m \]

\[ \text{Ballard} \quad \Delta L = \alpha L \Delta T = 23 \cdot 100 \cdot 20 = 46 \mu m \]

Difference \[ \Delta f - \Delta L \]

System de focus is \[ 44 \mu m \]
6) Vibrations (20)
A 100 lb weight hangs at the end of a 100 inch long, 0.2 in diameter steel cable

\[ A = \frac{\pi D^2}{4} = 0.03 \text{ in}^2 \]

a) Determine the natural resonant frequency of the mass in the vertical, “bounce” mode

\[ k = \frac{E A}{L} = \frac{300 \cdot 10^6 \cdot 0.03}{100} = 9400 \text{ lb/in} \]

\[ m = \frac{W}{g} = \frac{100 \text{ lb}}{386 \text{ in/s}^2} = \frac{f_n}{2\pi \sqrt{k/m}} = \frac{1}{2\pi} \sqrt{\frac{9400}{100/386}} = 30 \text{ Hz} \]

b) Assume a Q of 50 for this mode. Plot the transmissibility, defined as ratio of mass motion to base motion. Label your plot and give units. Show the correct behavior at low frequencies, at resonance, and at high frequencies.

\[ f (\text{in log scale}) \]

\[ \text{Log Scale} \]

\[ f_n = 30 \text{ Hz} \]

\[ T = 10^{-8} \]

\[ \Delta x_{rms} = \frac{1}{(2\pi f)^2} \cdot x_{rms} = 75 \text{ in} \cdot \text{rms} \]

c) For the case where the base motion has flat PSD with \( \text{10}^8 \text{ G/Hz} \), calculate the rms displacement of the mass in inches.

\[ \text{PSD of mass} = T^2 \cdot \text{PSD base} \]

\[ \text{use Miles eqn.} \]

\[ \Delta x_{rms} = \sqrt{T^2 \cdot \text{PSD base}} = 0.00696 \]

\[ \Delta x_{rms} = \sqrt{\frac{1}{2} \cdot 30 \cdot 50 \cdot 10^{-8}} = 2.65 \text{ in/s}^2 \]

Most motion at resonance \( f_n = 30 \text{ Hz} \)

\[ \Delta x_{rms} = \frac{1}{(2\pi f)^2} \cdot x_{rms} = 75 \text{ in} \cdot \text{rms} \]

d) Determine the resonant frequency of the mass in the lateral, “swing” mode.

(Hint, you can find the effective stiffness by either solving for the force required to displace the mass laterally or by using energy methods).

\[ F_x = W \cdot \theta = W \frac{x}{L} \]

\[ k_x = \frac{F_x}{x} = \frac{W}{L} \]

\[ f_n = \frac{1}{2\pi} \sqrt{k/m} = \frac{1}{2\pi} \sqrt{\frac{W/L}{W/16}} \]

\[ = \frac{1}{2\pi} \sqrt{386} = 0.3 \text{ Hz} \]

\[ \Delta x = \frac{x^2}{2 \cdot L} \]

\[ k = \frac{W}{L} \]