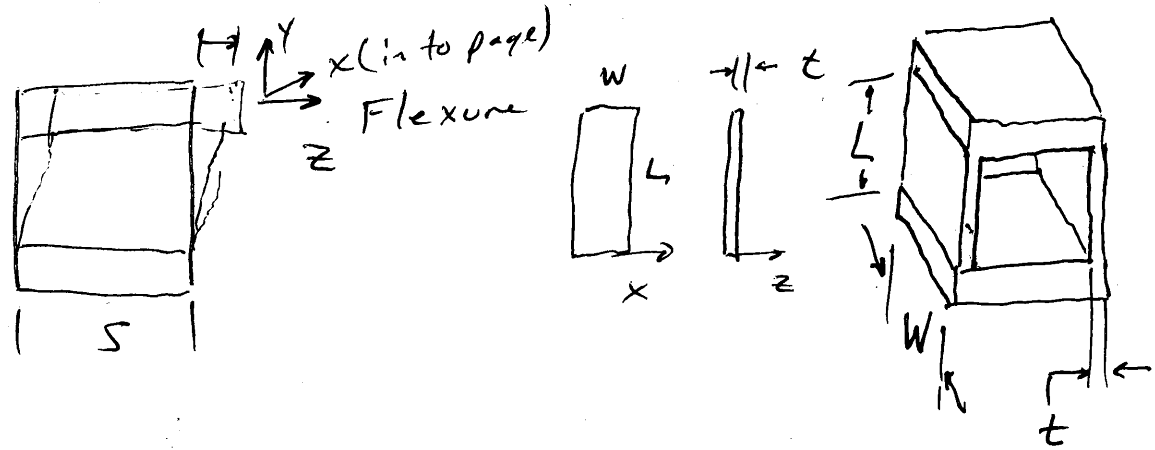
Optical Engineering 421/521 – Fall 2016
Midterm 2 75 minutes, closed book, closed notes, calculators will be provided.
November 18, 2016

1) Flexure stage design and analysis (25)

Sketch a design that uses two blade flexures to allow a small linear translation in the z direction, yet constrains all other degrees of freedom. On your sketch, define the x, y, and z axes and provide all important dimensions of the flexures. Use auxiliary views as necessary.



Provide the equations that would be used to determine k_x and k_y, the stiffness of the mechanism against forces in **each of the two directions** perpendicular to the motion allowed by the flexures.

k_x : Description:

Puts flexure in shear for $w \cong L$, bending in its stiff direction for $L \gg w$.

Equation:

$w \cong L$, stiffness per blade is from $\delta_x = \frac{F_x L}{GA}$

$k_{x_i} = \frac{GA}{L}$ per blade

two blades, stiffness adds

$k_x = 2k_{x_i} = \frac{2GA}{L}$

$= \frac{2Gwt}{L}$

k_y : Description:

Axial load on flexures

Equation:

per blade

$\delta_y = \frac{F_y L}{EA}$ $k_{y_i} = \frac{EA}{L}$

2 blades

$k_y = \frac{2Ewt}{L}$

2) <u>Flexure materials (15)</u>

a) List a material that is commonly used for flexures.

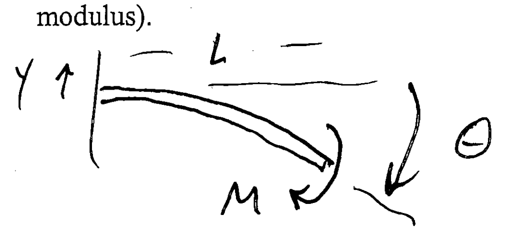
Stainless steel 17-4 PH, 15-5 PH

Be Co

Titanium

7075 Aluminum

b) From first principles, derive the relationship that relates the elastic range of bending a blade flexure to the geometry (Length, width, thickness) and material properties (yield strength, Young's modulus).



$$\theta = \frac{ML}{EI}$$

t = blade thickness

stress $\sigma(y) = \frac{My}{I}$

max stress at $y = t/2$

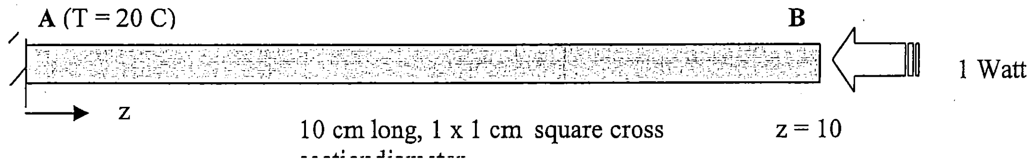
$$\sigma_{max} = \frac{t}{2I} \cdot \frac{EI\theta}{L} = \frac{1}{2}\frac{t}{L}E\theta$$

θ_{yield} for $\sigma_{max} = \sigma_{yield}$

$$\theta_{yield} = \frac{1}{2}\frac{t}{L}\frac{\sigma_{yield}}{E}$$

3) Heat flux and thermal expansion (25)

Consider a 10 cm long, 1 x 1 cm square cross section rod made of invar 36, heated from the end with 1 Watt of thermal power. (λ = 14 W/m-K for invar36) $= 0.14\ W/cm \cdot K$

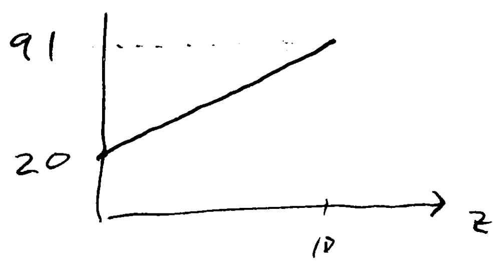


a) Define z=0 at the cantilever interface, which is always maintained at 20 C. Provide a plot that shows the temperature profile from z = 0 to z = 10 cm as the rod conducts 1 Watt of power steady state.

$$Q = \frac{H}{A} = \lambda \frac{\Delta T}{\Delta z}$$

for $\Delta z = 10\,cm$, $H = 1\,W$

$$\Delta T = \frac{1}{1}\ \frac{10}{.14}\ \frac{W}{cm^2}\ cm\ \frac{cm \cdot K}{W} = 71^{\circ}$$



$$\Delta T = 7.1 \cdot z$$

Compared to a 20 ° C isothermal state --

b) Determine the stress in the rod due to the thermal gradient.

Unconstrained, no stress

$\alpha \hat{=} 1\ ppm/^{o}C$

c) Determine the shift of point B at the end of the rod due to this thermal gradient.

$$\Delta L = \int \alpha\, \Delta T \cdot dz = \int_0^{10} 1 \cdot 7.1 \cdot z \cdot dz \quad \times 10^{-6}\ cm$$

$$= 7.1 \cdot \frac{1}{2} z^2 \Big|_0^{10} = 355 \times 10^{-6}\ cm$$

$$= 355 \times 10^{-5}\ mm$$

$$= 3.6\ \mu m$$

d) If a rigid constraint is added so that point B cannot move as the thermal gradient is applied, determine the reaction force at A.

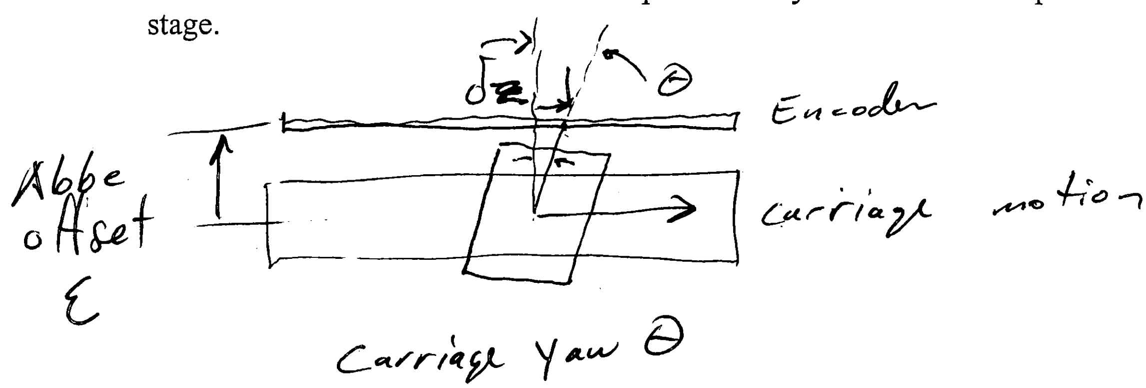
Super position $\qquad E \approx 150\,GPa = 21\,Msi$

$$F = \frac{EA}{L}\cdot\delta_z = \frac{150{,}000\ N/mm^2 \cdot 100\ mm^2}{100\ mm}\cdot 0.0036\ mm$$
$$= 540\,N$$

e) Determine the maximum stress in the rod for the case above with the overconstrained rod and the applied thermal gradient.

$$\sigma = F/A = \frac{540\,N}{100\,mm^2} = 5.4\,MPa$$

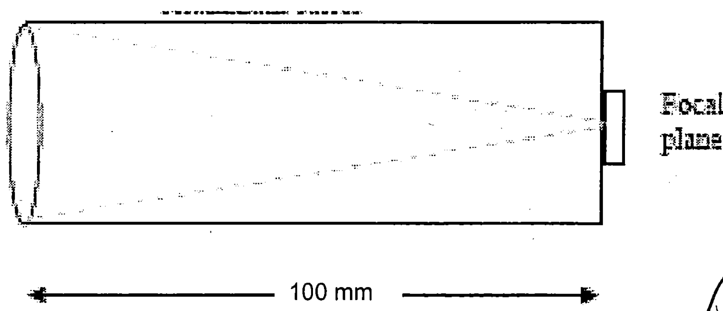
4) <u>Stages (5)</u>
In the design of linear stages, we strive to minimize the Abbe offset error. Make a sketch that defines the Abbe offset and show quantitatively how it can affect performance of a linear stage.



Encoder error $\delta z = \varepsilon \cdot \theta$

5) Change in focus with temperature (10)

Consider a 100 mm focal length lens made from BK7 and support tube made from aluminum. The aperture is 25 mm.



Al $\alpha = 23$ ppm/°C

BK7 $\alpha = 7$ ppm/°C

$\frac{dn}{dT} = 3$ ppm/°C

$\beta = \alpha - \frac{1}{n-1}\frac{dn}{dT} = 1$ ppm/°C

For 20°C temperature change, determine the change in focus in mm.

Lens $\Delta f = \beta f \Delta T = 1 \cdot 100 \cdot 20 = 2000E\text{-}6$ mm

$= 2\ \mu m$

Barrel $\Delta L = \alpha L \Delta T = 23 \cdot 100 \cdot 20 = 46\ \mu m$

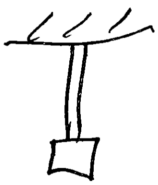
Difference $\Delta f - \Delta L$

System defocus is $44\ \mu m$

6) Vibrations (20)

A 100 lb weight hangs at the end of a 100 inch long, 0.2 in diameter steel cable $A = \frac{\pi D^2}{4} = 0.03\ in^2$

a) Determine the natural resonant frequency of the mass in the vertical, "bounce" mode

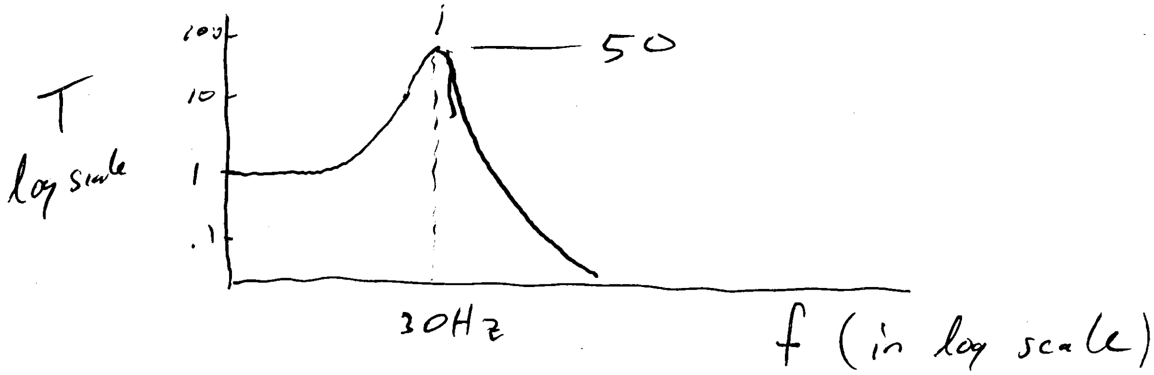


$$k = \frac{EA}{L} = \frac{30E6 \cdot 0.03}{100} = 9400\ lb/in$$

$$m = W/G = \frac{100\,lb}{386\ in/s^2} =$$

$$f_n = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{\frac{9400}{100/386}} = 30\ Hz$$

b) Assume a Q of 50 for this mode. Plot the transmissibility, defined as ratio of mass motion to base motion. Label your plot and give units. Show the correct behavior at low frequencies, at resonance, and at high frequencies



c) For the case where the base motion has flat PSD with 10^8 G^2/Hz, calculate the rms displacement of the mass in inches.
should be 10^{-8}

PSD of mass = $T^2 \cdot$ PSD base

use Miles eqn. $a_{rms} = \sqrt{\frac{\pi}{2} f_n \cdot Q \cdot PSD} = 0.0069\,G$

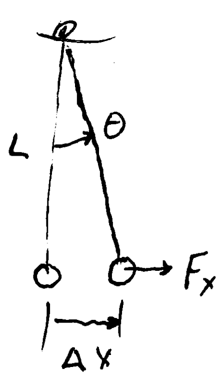
$$= \sqrt{\frac{\pi}{2}\, 30 \cdot 50 \cdot 10^{-8}} = 2.65\ in/s^2$$

Most motion at resonance $f_n = 30\ Hz$,

$$\Delta X_{rms} = \frac{1}{(2\pi f)^2} \cdot \ddot{X}_{rms} = 75E{-6}\ in\ rms$$

$T = W/\cos\theta$
$\cong W$

d) Determine the resonant frequency of the mass in the lateral, "swing" mode.
(Hint, you can find the effective stiffness by either solving for the force required to displace the mass laterally or by using energy methods).



$$F_x \cong W \cdot \theta = W \frac{x}{L}$$

$$k_x = \frac{F_x}{x} = \frac{W}{L}$$

$$f_n = \frac{1}{2\pi}\sqrt{k_x/m} = \frac{1}{2\pi}\sqrt{\frac{W/L}{W/G}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{G}{L}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{386}{100}} = 0.3\ Hz$$

or E: $\frac{1}{2} k x^2 = W \Delta z$

$$\Delta z = \frac{x^2}{2 \cdot L}$$

$$k = W/L$$