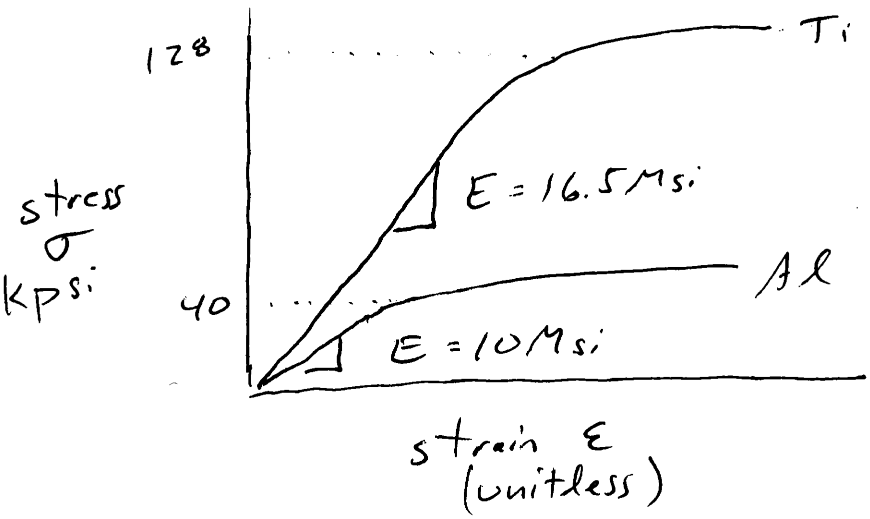
Name:

Optical Engineering 421/521 – Fall 2015
Midterm 2 **60 minutes, closed book, closed notes, calculators to be provided**
November 20, 2015

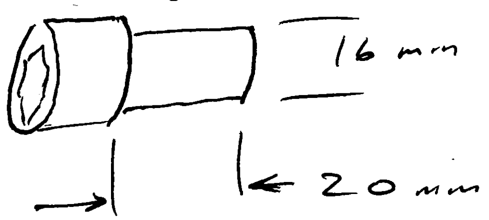
1.) (8) Draw a single plot that shows the relationship between stress and strain for both 6061-T6 aluminum and 6Al-4V titanium. Label your axes and provide units. Show how the yield strength and Young's modulus relate to this graph. Give the approximate values for E and yield strength for both materials.

Al 6061-T6 : $E = 10\,Msi$ $\sigma_y = 40\,ksi$
Ti 6Al - 4V : $E = 16.5\,Msi$ $\sigma_y = 128\,ksi$



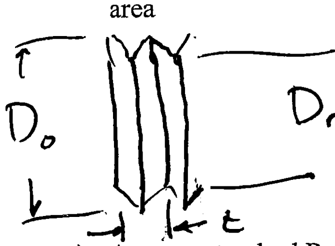
2.) (6) Consider an M16 x 2 x 20 mm socket head cap screw

a) Sketch the screw and give the critical dimensions



2 mm per thread

b) Determine the approximate minor (or root) diameter and corresponding minimum cross sectional area



$$D_r \cong D_o - t = 16 - 2 = 14\,mm$$

$$A = \pi \frac{14^2}{4} = 154\,mm^2$$

c) Assume standard Property Class 5.6 with 500 MPa ultimate material strength, calculate the approximate tensile strength of the screw in Newtons.

$$F = \sigma \cdot A = 500 \frac{N}{mm^2} \cdot 154\,mm^2$$

$$= 77{,}000\,N$$

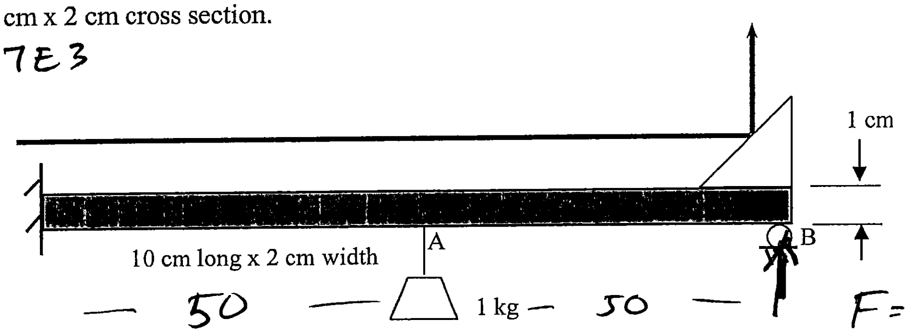
5) (16) Consider a flat mirror mounted at the end of a 10 cm long rail made from 7075 aluminum alloy. The rail is overconstrained as shown. The mirror is used to redirect a collimated laser beam by 90°. The rail has 1 cm x 2 cm cross section.

$I = \frac{1}{12}(10)^3(20) = 1.7E3$

$\cancel{10^4}$ mm^4

$E = 70{,}000\ N/mm^2$

$EI = \cancel{7E8}$ $1.17E8$

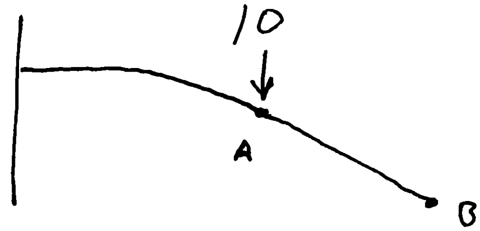


$F = 1\,kg \times 9.8 \cong 10\,N$

A 1 kg weight is attached at point A in the middle of the beam, causing it to distort.

A) Use superposition to determine the reaction force at B.
B) Determine the change in pointing angle for the reflected laser beam.

A)



$+$

$$\delta_A = \frac{F(L/2)^3}{3EI} \qquad \theta_A = \frac{F(L/2)^2}{2EI}$$

$$\delta_B = \delta_A + \theta_A \cdot \frac{L}{2} \qquad \theta_B = \theta_A$$

$$= \frac{F(L/2)^3}{EI}\left(\frac{1}{3}+\frac{1}{2}\right)$$

$$= \frac{10 \cdot 50^3}{1.17E8} \cdot \frac{5}{6} = 8.9\,\mu m$$

$$\delta = \frac{R_b L^3}{3EI} = 8.9E{-3} \quad \therefore R_b = \frac{8.9E{-3} \cdot 3 \cdot 1.17E8}{100^3} = 3.1\,N$$

B) $\Delta\theta_B = \Delta\theta$ due to F_A $+\ \Delta\theta$ due to R_b

$$= \frac{F(L/2)^2}{2EI} - \frac{R_b L^2}{2EI} = \frac{L^2}{2EI}\left[10 \cdot \frac{1}{2}^2 - 3.1\right]$$

$$= \frac{100^2}{2 \cdot 1.17E8}\left[-0.6\right]$$

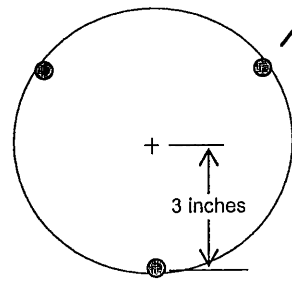
$$= 2.56E{-5}\ rad$$

deviation $= 2 \times \Delta\theta_b = 0.05$ mrad

5) (30) A 6 inch diameter, 1 inch thick Zerodur mirror is bonded to a thick aluminum plate using 0.1" thick RTV elastomeric adhesive. Three small 0.5" diameter bonds are made at the edge of the mirror.

This adhesive has
- 100 psi shear modulus
- 100,000psi bulk modulus
- 100 psi adhesive shear strength

$\rho \cong 0.1 \text{ lb/in}^2$



$A_b = \dfrac{\pi (0.5)^2}{4} \cong 0.2 \text{ in}^2$

a) Calculate the weight of the mirror and use this to calculate the shear force per bond when the mirror is supported with the optical axis horizontal (on edge).

$W = \rho \cdot A \cdot t = 0.1 \cdot (\pi 3^2)(1)$

$= 2.8 \text{ lbs}$ $\quad \tau = F/A_b$

$\text{Force/Bond} = 2.8/3 = .9.4 \text{ lbs}$ $\quad = \dfrac{0.9.4}{0.2} = 4.7 \text{ psi}$

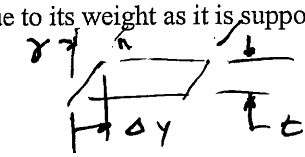
b) Determine the shear stress in the bonds for a 10 G shock load in the shear direction. Compare with the strength.

τ at $10G = 10 \times \tau$ for $1G$

47 psi

Strength is 100 psi $\quad SF = \dfrac{100}{47} \cong 2$

c) Determine the lateral deflection of the mirror due to its weight as it is supported as above.



$\gamma = \tau / G = \dfrac{5}{100}$

$\Delta y = \gamma \cdot t = (.05)(.1)$

$= 0.005''$

d) Calculate the resonant frequency of the mirror for this mode.

$f_n = \omega_n / 2\pi = \dfrac{1}{2\pi}\sqrt{k/m} = \dfrac{1}{2\pi}\sqrt{g/\delta_{sw}}$

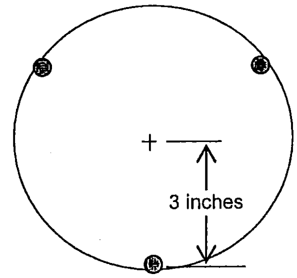
$\delta_{sw} = 0.005''$ from c

$f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{386 \text{ in/s}^2}{.005''}} = 44 \text{ Hz}$

Continued from problem 5.) The 6 inch diameter, 1 inch thick Zerodur mirror is bonded to a thick aluminum plate using 0.1" thick RTV elastomeric adhesive. Three small 0.5" diameter bonds are made at the edge of the mirror.

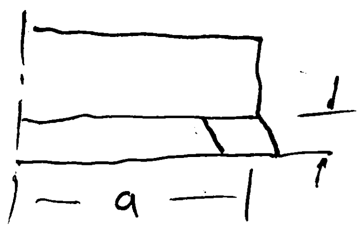
This adhesive has

- 100 psi shear modulus
- 100,000psi bulk modulus
- 100 psi adhesive shear strength



$\alpha_{AL} = 23 \text{ ppm}/°C$

e) Calculate the adhesive shear strain for 33°C change in temperature, coupled with the expansion of the mirror and the aluminum mounting interface.



$$\gamma = \frac{\Delta a_{AL} - \Delta a_{glass}}{t} \qquad \Delta a = a \alpha \Delta T$$

$$= \frac{3(23E{-}6)\cdot 33}{0.1}$$

$$= 23E{-}3 = 0.023$$

f) Calculate the shear stress in the adhesive for the above 33°C temperature change. Compare this to the adhesive strength

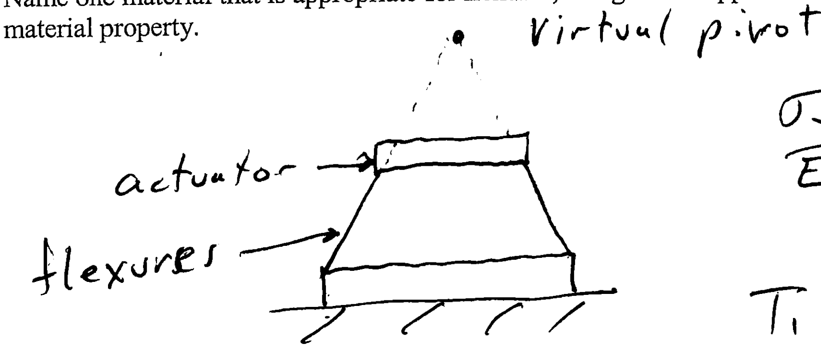
$$\tau = G \cdot \gamma = 100 psi \cdot 0.023$$

$$= 2.3 psi$$

strength is 100 psi

$$SF = 100/2.3 = 43$$

7) (5) Sketch a design that uses flexures to allow a small rotation about a virtual pivot point, yet constrains all other degrees of freedom.
Give the material property (actually a ratio of two properties) that provides a useful figure of merit for flexures.
Name one material that is appropriate for flexures, and give the approximate value for the relevant material property.



$$\frac{\sigma_Y}{E} = \frac{\text{yield strength}}{\text{elastic modulus}}$$

$$\text{Ti}: \frac{\sigma_Y}{E} \doteq \frac{130\text{ ksi}}{16\text{ Msi}} = 0.008$$

4. (5) What determines the strength of glass?

critical flaw size at the surface

How can you determine allowable stresses for glass, assuming a standard ground finish?

Weibull statistics

$$\text{Probability of failure} = 1 - e^{-(\sigma/\sigma_0)^m}$$

σ_0 and m fit to data

5. (12) Consider a 20 cm long aluminum bar with 2 cm x 2cm cross section. Calculate the following:

$\alpha = 23E\text{-}6\ /°C$
$E \doteq 69\ GPa$
$\lambda \doteq 160\ W/m\cdot C$

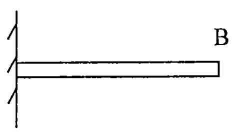
A) Motion of point B and maximum stress if the bar is heated 1°C, and allowed to expand.

B) Stress in the material if the bar is heated 1°C while constrained as shown.

C) Motion of point B for the case where a linear thermal gradient is applied, with the top of the bar 0.01°C warmer than the bottom.

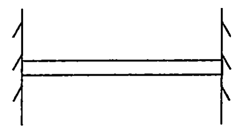
D) Thermal power in Watts required to maintain the above temperature gradient.

A)



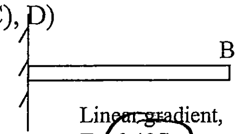
ΔT = 1°C

B)



ΔT = 1°C

C), D)



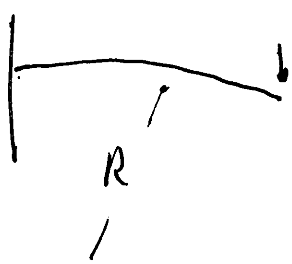
Lin~~ear, gra~~dient,
T = 0.1°C warmer on top than on bottom

.01

A) $\Delta L = L \alpha \Delta T$
$= 20 \cdot 23E\text{-}6 \cdot 1 = 4.6E\text{-}4\ cm = 4.6\ \mu m$: No stress

B) $\Delta L = \frac{FL}{EA} = L \alpha \Delta T$; $\sigma = \frac{F}{A} = E \alpha \Delta T = 69 \cdot 23E\text{-}6 \cdot 1$
$\sigma = 1.6\ MPa$

C) Gradient, beam bends $C = \frac{1}{R} = \alpha \frac{dT}{dx} = 23E\text{-}6 \cdot \frac{.01}{20\,mm}$
$= 1.15E\text{-}8\ mm^{-1}$



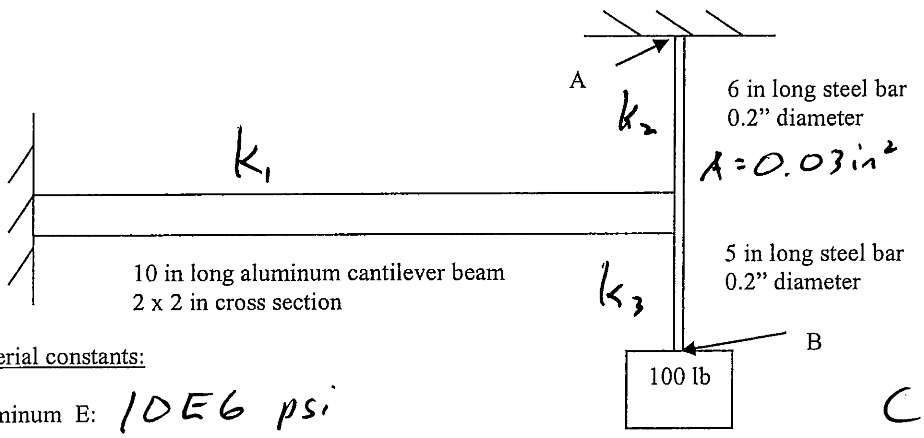
$\delta = \frac{L^2}{2R}$
$= \frac{200^2}{2 \cdot} 1.15E\text{-}8 = 0.23\ \mu m$

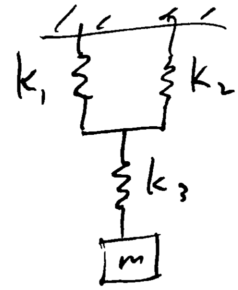
D) $Q = H/A = \lambda \frac{dT}{dx}$: $H = A \cdot \lambda \cdot \frac{\Delta T}{t}$
$= \frac{(.2 \times .02)(160)(.01)}{.02} \quad \frac{m^2 \cdot W\,C}{m \cdot C\ m}$

$= 0.32\ W$

6) (18) Consider the geometry below. A 10-in long aluminum cantilever is supported at the end by a 6" long steel bar. A mass is suspended from the end by another steel bar, 5" long.

$I = \frac{1}{12} 2 \cdot 2^3 = 1.3 \text{ in}^4$





Material constants:

Aluminum E: 10E6 psi

Steel E: 30E6 psi

$C = \frac{1}{k_3} + \frac{1}{k_1 + k_2}$

a) Determine the total deflection at B as a 100 pound weight is suspended.

$k_1 = F/\delta = \frac{3EI}{L^3} \quad \frac{3(10E6)(1.3)}{10^3} = 3.9 \times 10^4 \text{ lb/in} = 39.0 \frac{\text{klb}}{\text{in}}$

$k_2 = \frac{EA}{L} = \frac{(30E6)(.03)}{6} = 157 \times 10^3 \text{ lb/in} = 157 \frac{\text{klb}}{\text{in}}$

$k_3 = \text{''} \quad \frac{(30E6)(.03)}{5} = 180 \times 10^3 \text{ lb/in} = 180 \frac{\text{klb}}{\text{in}}$

$k_{eff} = \left[\frac{1}{k_3} + \frac{1}{k_1 + k_2}\right]^{-1} = \left[0.0056 + \overset{0.0051}{\cancel{.00183}}\right]^{-1} = 93 \text{ klb/in}$

$\delta = F/k = 0.1 \text{ klb}/93 = 0.00107''$

b) Assuming 1% damping, sketch a plot of vibration transmissibility T as a function of frequency for the above system. Label the axes and show the values of T for low frequency and at resonance.

$f_n = \frac{1}{2\pi}\sqrt{g/\delta_{st}} = \frac{1}{2\pi}\sqrt{386/.00107} = 95 \text{ Hz}$

$Q = \frac{1}{2C_R} = 50$