Fall 2016

OPTI 421/521
Homework 2.

## Part 1

1) BK7 Risley prism $n_{BK7} = 1.517$

$2°$ wedge $\alpha_1 = \alpha_2 = 2° = \alpha$ $\delta_x = 1°$ ; $\delta y = 0°$

$$\delta = \delta_1 = \delta_2 = \alpha(n-1) = 2°(1.517-1) = 1.034$$

$$\delta_y = \delta(\sin\phi_1 + \sin\phi_2) = 0° \Rightarrow \phi_1 = -\phi_2$$

$$\delta_x = \delta(\cos\phi_1 + \cos\phi_2) = 1°$$

$$\cos\phi_1 + \cos(-\phi_1) = 1°/1.034°$$

$$2\cos\phi_1 = \frac{1}{1.034} \Rightarrow \cos\phi_1 = \frac{1}{2.034}$$

$$\phi_1 = \cos^{-1}(0.746) = 60.55°$$

$$\phi_1 = 60.55°$$
$$\phi_2 = -60.55°$$

Maximum deviation possible for this pair

$$\delta x = \delta(\cos\phi_1 + \cos\phi_2)$$ max when $\cos\phi_1 = \cos\phi_2 = 1$

$$\delta x_{max} = \delta(2) = (1.034°)(2) = 2.068$$

2) $D_{EP} = 5\,mm$

$EFL = 20\,mm$

$BFD = 25\,mm$

$FOV = 31^\circ$

$$F_n = \frac{EFL}{D_{EP}} = \frac{20\,mm}{5\,mm} = 4$$

a) $s = 1\,\mu m$

$$\varepsilon = F_n B_i \frac{s}{F_i}$$

where $B_i$ is Beam footprint

| | F(mm) | $B_i$(mm) | $B_i/F_i$ | $\varepsilon$ ($\mu m$) |
|---|---|---|---|---|
| $L_1$ | -30 | 5 | -0.16 | -0.64 |
| $L_2$ | -35 | ~5 | -0.14 | -0.56 |
| $L_3$ | 28 | ~7 | 0.25 | 1 |
| $L_4$ | 28 | ~6 | 0.21 | 0.84 |
| $L_5$ | -12 | ~5 | -0.42 | -1.68 |
| $L_6$ | 20 | ~5 | 0.25 | 1 |

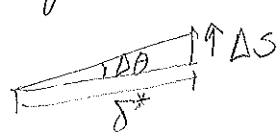
b) Position of the nodal point for the objective

$$\delta^* = BFD - EFL = 25\,mm - 20\,mm = 5\,mm$$

5 mm to the right of C

c) Image motion for 1 mrad rotation of the objective about point C

$$\Delta\theta \approx \frac{\Delta s}{\delta^*} \Rightarrow \Delta s = \Delta\theta\, \delta^* = (1\,mrad)(5\,mm) = 5\,\mu m$$




d)

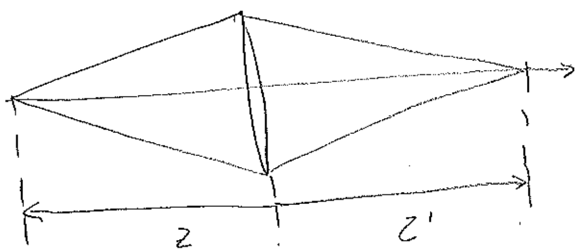
$$\varepsilon_{LSS} = \sqrt{\varepsilon_{L1}^2 + \varepsilon_{L2}^2 + \varepsilon_{L3}^2 + \varepsilon_{L4}^2 + \varepsilon_{L5}^2 + \varepsilon_{L6}^2 + \Delta_s^2}$$

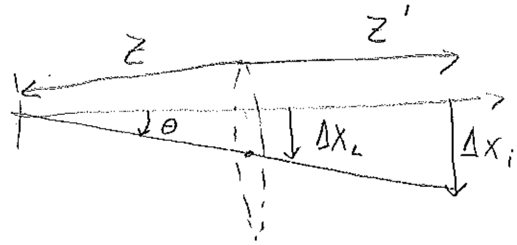
$$\varepsilon_{LSS} = \sqrt{(-0.64)^2 + (-0.56)^2 + 1^2 + (0.84)^2 + (-1.68)^2 + 1^2 + (5)^2}$$

$$\varepsilon_{LSS} = 5.59\,\mu m$$

# Part 2
## Lens Motion

a) - Transverse Motion $\Delta x_i$



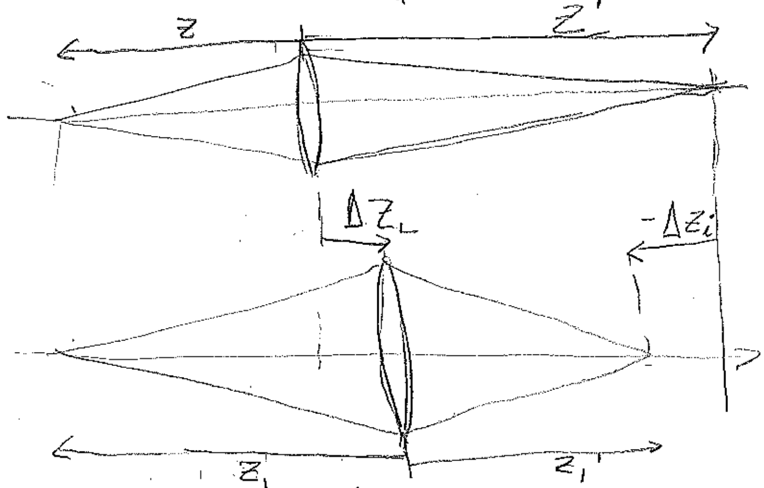




$$\theta = \frac{\Delta x_i}{z' + z} = \frac{\Delta x_L}{z} \Rightarrow \Delta x_i = \Delta x_L \left( \frac{z' + z}{z} \right)$$

$$\Rightarrow \Delta x_i = \Delta x_L \left( 1 - \left( - \frac{z'}{z} \right) \right)$$

$$m = -\frac{z'}{z} \qquad \Delta x_i = \Delta x_L (1 - m)$$

- Axial Motion $\Delta z_i$




$$z_1' = z' + \Delta z_i - \Delta z_L$$

$$z_1 = z + \Delta z_L$$

$$\Delta z = z - z_1 = -\Delta z_L$$

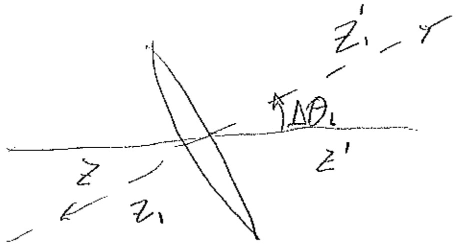
$$\Delta z' = z' - z_1' = \Delta z_L - \Delta z_i$$

Using longitudinal magnification

$$-m^2 = \frac{\Delta z'}{\Delta z} = \frac{\Delta z_L - \Delta z_i}{-\Delta z_L} = -1 + \frac{\Delta z_i}{\Delta z_L}$$

$$\Rightarrow \frac{\Delta z_i}{\Delta z_L} = 1 - m^2 \Rightarrow \Delta z_i = \Delta z_L (1 - m^2)$$

* Rotation $\Delta\theta_L$




$$z_1 = z\cos(\Delta\theta_L)$$
$$z_1' = z'\cos(\Delta\theta_L)$$

If $\Delta\theta_L$ is small

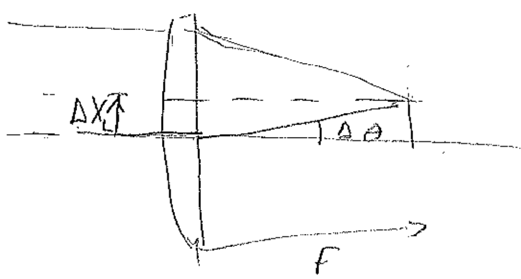
$$z_1 \simeq z$$
$$z_1' \simeq z'$$

No significant effect

b) $\Delta X_i \simeq F_n D_i \Delta\theta_i$

Using working $F_n$

$$F_n D_i \Delta\theta_i = \frac{z'}{\cancel{D_{ep}}} \cancel{D_i} \Delta\theta_i = z'\Delta\theta$$




$$\Delta\theta = \frac{\Delta X_L}{f}$$

$$F_n D_i \Delta\theta_i = z'\frac{\Delta X_L}{f} \quad \text{but} \quad f = \frac{z'z}{z'+z}$$

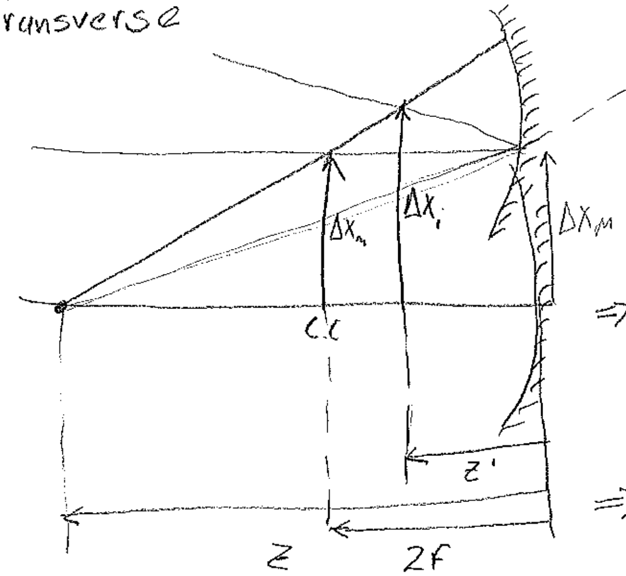
and $m = -\frac{z'}{z}$

$$F_n D_i \Delta\theta_i = \frac{z'\Delta X_L}{\frac{z'z}{z'+z'}} = \frac{z'+z}{z}\Delta X_L = \Delta X_L\left(\frac{z'}{z}+1\right) = \Delta X_L(1-m)$$

From a) $\Delta X_i = \Delta X_L(1-m)$

## 2. Mirror Motion

### a) Transverse




$$\frac{\Delta x_i}{z - z'} = \frac{\Delta x_m}{z' - 2f}$$

$$\frac{\Delta x_i}{\Delta x_m} = \frac{z - z'}{z - 2f} \quad \text{from imaging Eq.} \quad f = \frac{z'z}{z + z'}$$

$$\Rightarrow \frac{\Delta x_i}{\Delta x_m} = \frac{z - z'}{z - 2\left(\frac{z'z}{z + z'}\right)} = \frac{(z - z')(z + z')}{z^2 + zz' - 2z'z}$$

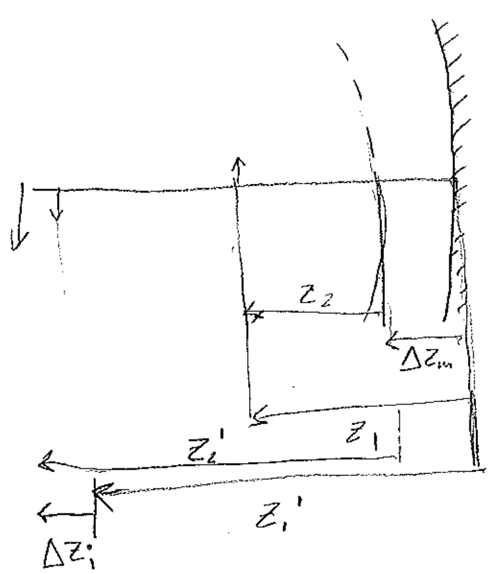
$$\Rightarrow \frac{\Delta x_i}{\Delta x_m} = \frac{(z - z')(z + z')}{z(z - z')} = \frac{z + z'}{z}$$

$$m = -z'/z$$

$$\Delta x_i = \Delta x_m\left(1 + \frac{z'}{z}\right) = \Delta x_m(1 - m)$$

$$\underline{\Delta x_i = \Delta x_m(1 - m)}$$

### b) Axial shift




$$z_2' = z_1' - \Delta z_M + \Delta z_i \Rightarrow z_1' - z_2' = \Delta z_m - \Delta z_i$$

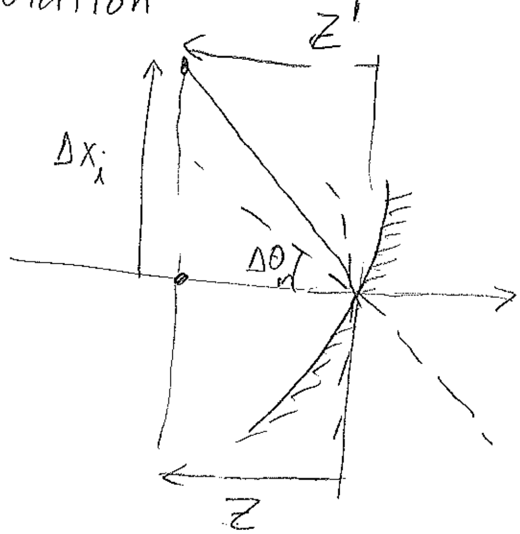
$$z_2 = z_1 - \Delta z_m \Rightarrow z_1 - z_2 = \Delta z_m$$

$$-m^2 = \frac{z_1' - z_2'}{z_1 - z_2} = \frac{\Delta z_m - \Delta z_i}{\Delta z_m} = 1 - \frac{\Delta z_i}{\Delta z_m}$$

$$\Rightarrow -(m^2 + 1) = -\frac{\Delta z_i}{\Delta z_m}$$

$$\Rightarrow \underline{\Delta z_i = \Delta z_m(m^2 + 1)}$$

Rotation




$$Z = Z'$$

Law of reflection

$$\theta_i = \theta_r$$

$$\Delta\theta_i = 2\Delta\theta_m$$

$$\Delta\theta_i = \frac{\Delta x_i}{Z'} \Longrightarrow \Delta x_i = Z'\Delta\theta_i$$

$$\Delta x_i = 2Z'\Delta\theta_m$$

→ Flat mirror

Transverse: $\Delta x_i = \Delta x_m(1-m) = \Delta x_m(1-1) = 0$

Axial: $\Delta z_i = \Delta z_m(1+m^2) = \Delta z_m(1+1^2) = 2\Delta z_m$

Tilt: $\Delta x_i = 2z'\Delta\theta_m$

→ Convex

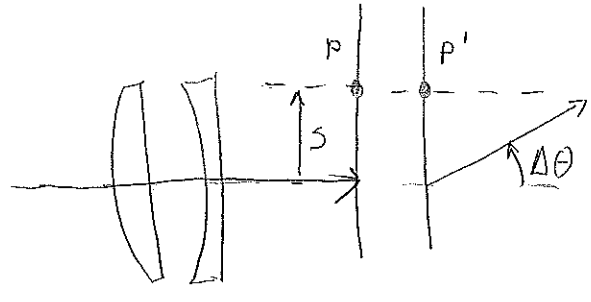
Negative axial shifts

Tilt and transverse the same.

3. Derive the relation that gives the stationary point for the general case, where the object is not at infinity

- Decompose general motions into a combinations of lateral translation and rotation about the front principal point P. Assume small angles.

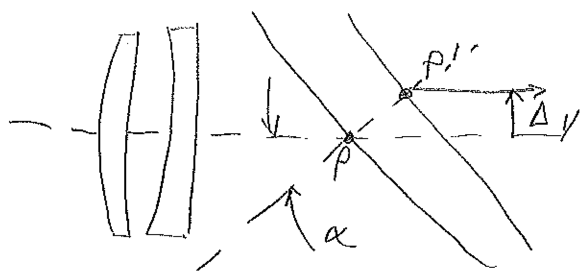
Pure Translation




$\Delta\theta \approx \frac{S}{F}$

$\Delta y = 0$

Pure rotation

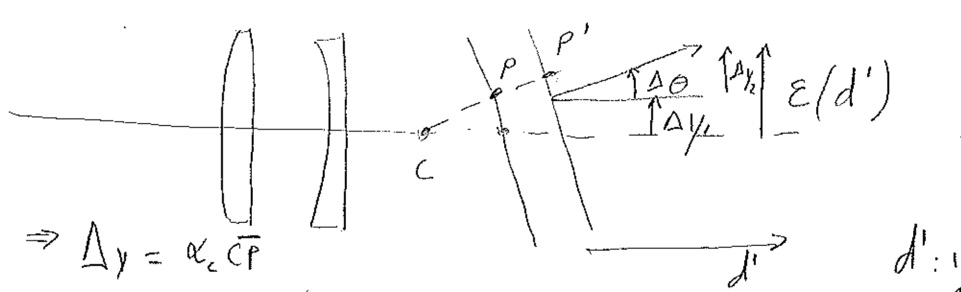



$\alpha \approx \frac{\Delta y}{PP'} \Rightarrow \Delta y = \alpha PP'$

$\Delta\theta = 0$

$PP'$ = distance between principal points

Consider rotating an angle $\alpha_c$ about a general point C, located a distance CP from the front principal point




d': image distance

$\alpha_c \approx \frac{\Delta y}{CP} \Rightarrow \Delta y = \alpha_c \overline{CP}$

$\Delta\theta \approx \frac{\Delta y}{f} = \frac{\alpha_c \overline{CP}}{f}$

and from pure rotation

$\Delta y_1 = \alpha \overline{PP'}$

$\Delta y_2 = \Delta\theta\, d' = \alpha_c \frac{\overline{CP}}{F} d'$

$\Rightarrow \varepsilon(d') = \Delta y_1 + \Delta y_2$

$\varepsilon(d') = \overline{PP'}\alpha_c + \alpha_c \frac{\overline{CP}}{F} d'$

By defintion, a small system rotation about the stationary point does not cause image motion

$$\varepsilon(d') = 0 = \overline{PP'}\alpha_c + \alpha_c \frac{\overline{CP}}{f} d' = \alpha_c\left(\overline{PP'} + \overline{CP}\frac{d'}{f}\right)$$

$$\Longrightarrow \qquad \overline{CP} = -\overline{PP'}\frac{f}{d'}$$