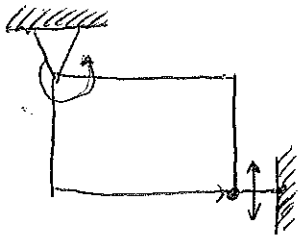
OPTI 421/521

Introductory Opto-Mechanical Engineering

Homework 5

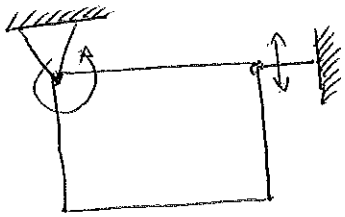
Part 1: Static Equilibrium Problems

1) 1)



Static Determinancy

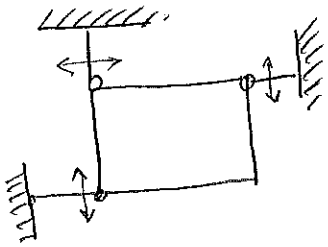
2)



Overconstrained x-direction

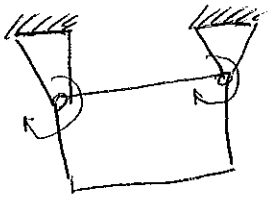
Under constrained in θ_z

3)



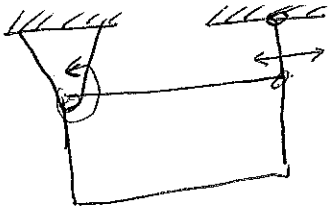
Static Determinancy

4)



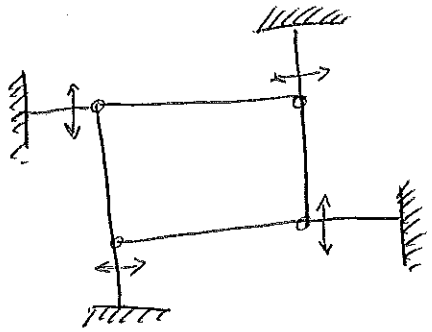
Over constrained x and y direction

5)



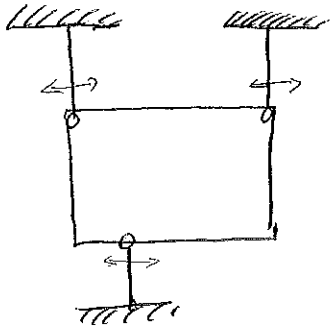
static Determinancy

6)



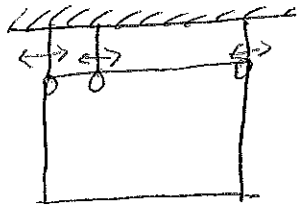
Overconstrained in x and y

7)



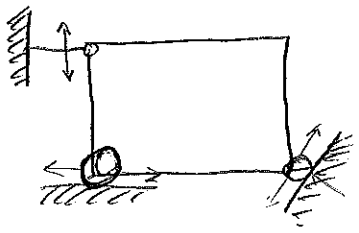
Overconstrained in y
Underconstrained in X

8)



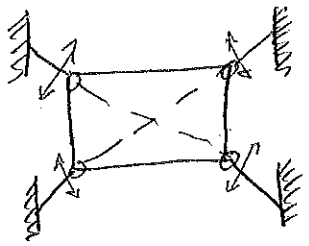
Overconstrained in y
Underconstrained in X

9)



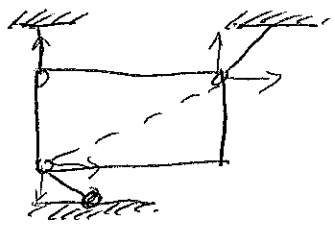
Overconstrained in X and y
Under in θ_z

10)



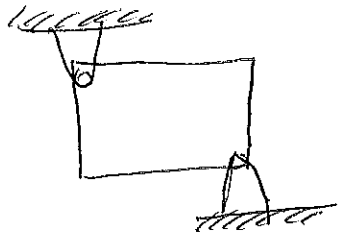
Overconstrained in X and y
under constrained in θ_z

11)



Overconstrained in X and y
Underconstrained in θ_z

12)

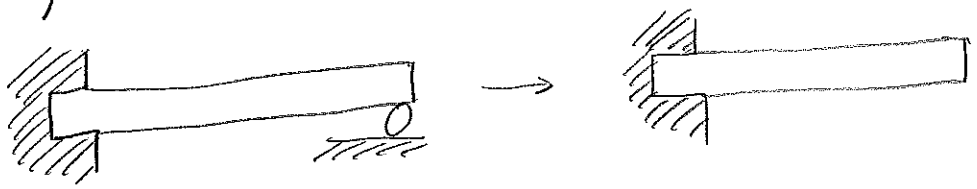


Over constrained in all DoF

2)

1)

2)



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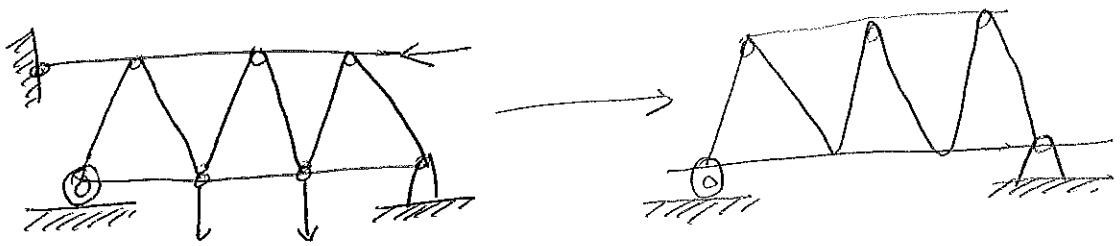
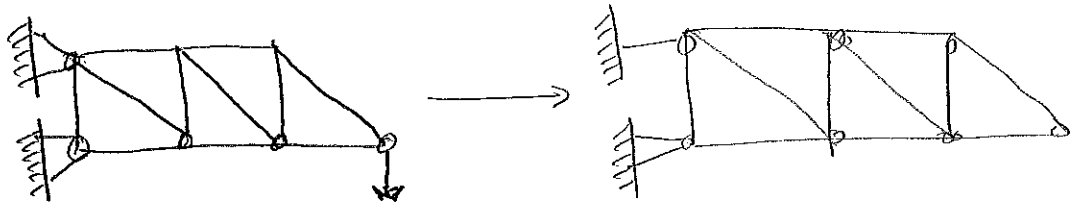
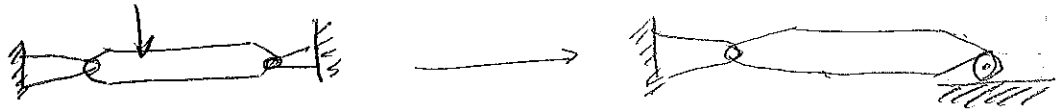


4)



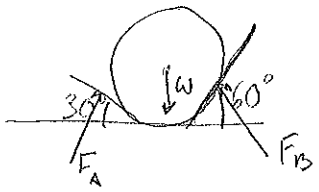
5)

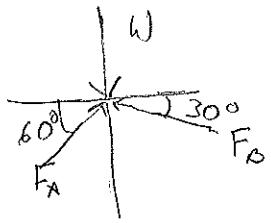


6)

3)

$m = 40\ kg$





$$W = mg = (40\,kg)(9.8\,m/s) = 392$$

$$\Sigma F_x = 0$$

$$F_A \cos 60^\circ - F_B \cos 30^\circ = 0$$

$$F_A = \frac{F_B \cos 30^\circ}{\cos 60^\circ}$$

$$\Sigma F_y = 0$$

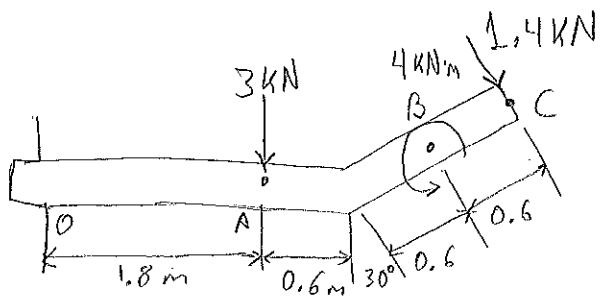
$$-W + F_A \sin 60^\circ + F_B \sin 30^\circ = 0$$

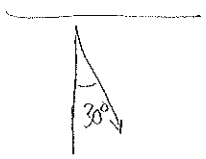
$$-W + F_B\left[\left(\frac{\cos 30^\circ}{\cos 60^\circ}\right)\sin 60^\circ + \sin 30^\circ\right] = 0$$

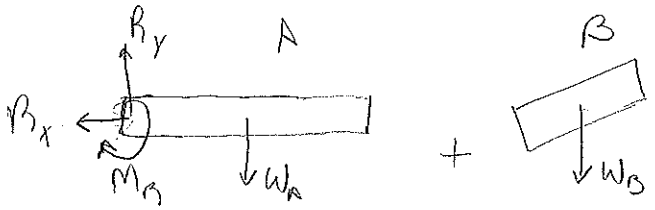
$$F_B = \frac{W}{(1.5+2)} = \frac{392}{2} = 196\ N$$

$$F_A = (196\ N)\frac{\cos 30^\circ}{\cos 60^\circ} = 339.48\ N$$

4)







$$W_A = (1.8_m + 0.6_m)(50\,kg)(9.8\,m/s) = 1176\ N = 1.176\ KN$$

$$W_B = (0.6 + 0.6)(50)(9.8) = 588\ N = 0,588\ KN$$

$$\Sigma F_x = 0$$

$$R_x + (1.4\ KN)\sin(30) = 0$$

$$-R_x + 0.7\ KN = 0$$

$$\boxed{R_x = 0.7\ KN}$$

$$\Sigma F_y = 0$$

$$R_y - (1.4\,KN)\cos(30) - 3\,KN - 1.47\,KN = 0$$

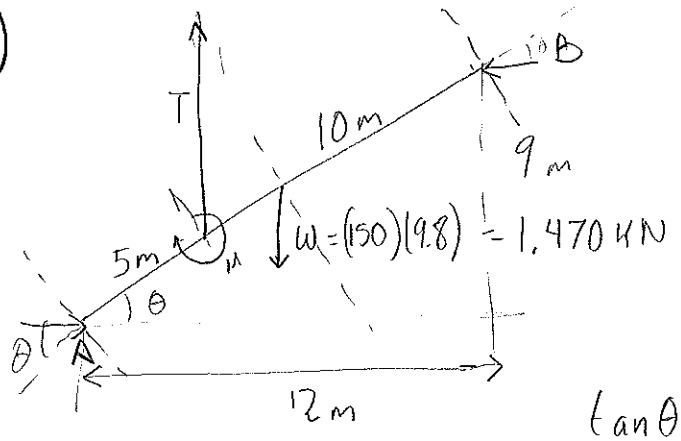
$$\boxed{R_y = 5.68\ KN}$$

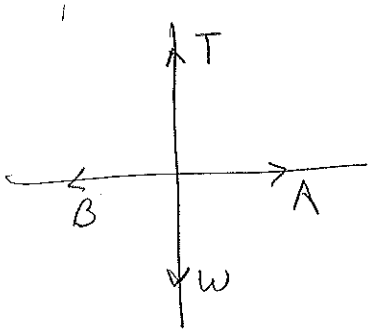
$$\Sigma M_o = 0.$$

$$M_{Ro} + 4\,KNm = (3)(1.8) - (1.176)(1.2) - (2.4 + 0.6\sin 30)(0.588)$$
$$- 3.44(1.4\sin 60) - 0.6(1.4\cos(60)) = 0$$

$$M_{Ro} = 9.42\ KN \cdot m$$

5)





$\tan\theta = \frac{9}{12}$

$\theta = 36.89°$

$\sum F_x = 0$

$A - B = 0 \Rightarrow A = B$

$\sum F_y = 0$

$T - W = 0 \Rightarrow T = W = 1470N$ $\quad W = mg = (150)(9.8) = 1470\ N$

$\sum M_A = 0$

$5[A\sin(36.89°)] - 2.5W\cos(36.89°) + 10B\sin(36.89°) = 0$

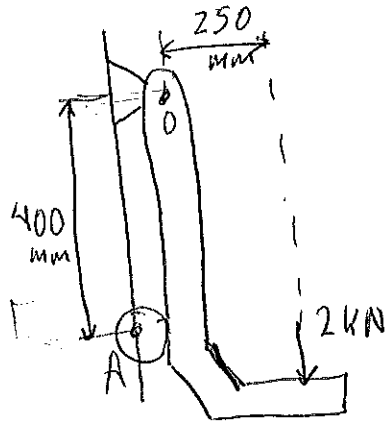
$A(5\sin(36.89) + 10\sin(36.89)) = 2.5(1470)\cos(36.89)$

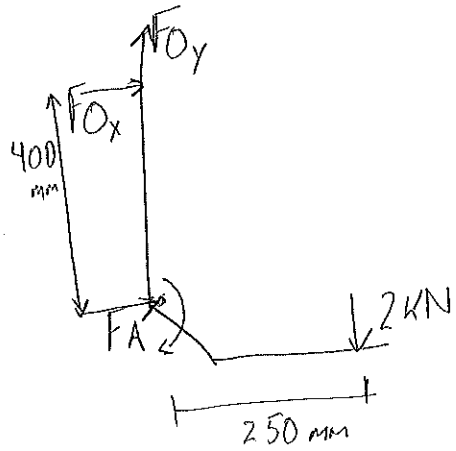
$A(3+6) = 2939.226$

$A = 326.58N$

$B = 326.58\ N$

6)





$$\Sigma F_x = 0$$

$$F_A + F_{Ox} = 0 \implies F_A = -F_{Ox}$$

$$\Sigma F_y = 0$$

$$F_{Oy} - 2\,kN = 0 \implies \boxed{F_{Oy} = 2\,kN}$$

$$\Sigma M_A = 0$$

$$(2\,kN)(250\,mm) - F_{Ox}\,400\,mm = 0$$

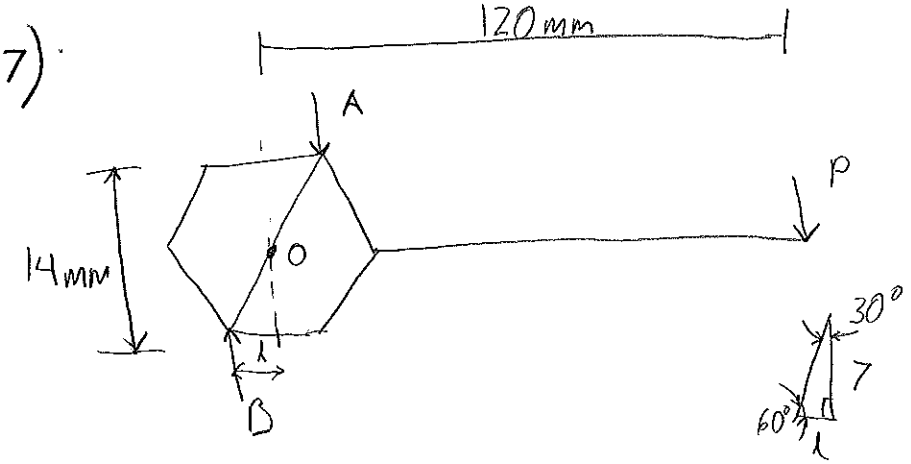
$$F_{Ox} = \frac{(2000)(250)}{400} = 1250\,N$$

$$\boxed{F_{Ox} = 1.25\,kN}$$

$$\boxed{F_A = -1.25\,kN}$$

$$F_{pin} = \sqrt{(1.25)^2 + (2)^2} = 2.36\,kN$$

7)



Torque of 24 N·m

$\tan 60° = \frac{7}{\ell} \Rightarrow \ell = \frac{0.007\,m}{\tan 60°}$

$\ell = 0.00404\,m$

$M = 24 = P(0.12) \Rightarrow P = \frac{24}{0.12} \Rightarrow \boxed{P = -200\ N}$

$\sum M_o = 0 = B(0.00404) + A(0.00404) - (0.12)(200)$ (1)

$\sum F_y = 0 = -B + A + P \Rightarrow B = P + A = -200 + A$ (2)

$(-200 + A)(0.00404) + A(0.00404) - 2.4 = 0$

$A(0.00808) = 24 + 0.808$

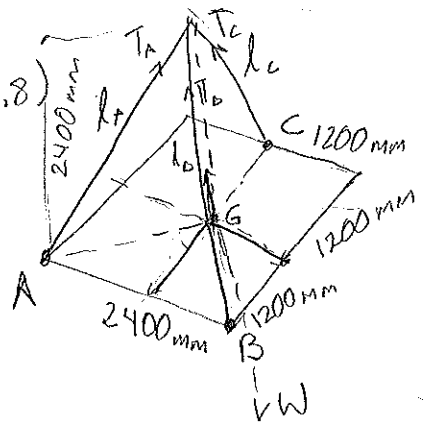
$A = \frac{24.808}{0.00808} = 3070.30\ N$

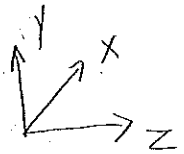
$\boxed{\begin{aligned} B &= 2870.30\ N \\ A &= 3070.30\ N \end{aligned}}$

8) $m = 1800\,kg$

$W = Mg = 1800(9.8)$

$W = 17.64\,KN$





$l_C = \sqrt{2400^2 + 1200^2} = 2683.28\,mm = 2.68\,m$

$l_A = l_B = \sqrt{2400^2 + 1697^2} = 2939.39\,mm = 2.9\,m$

$$\cos\gamma_{xc} = \frac{1200}{l_C} = \frac{1200}{2683.28} = 0.447$$

$$\cos\gamma_{zA} = \cos\gamma_{xA} = \frac{1200}{l_A} = \frac{1200}{2939.39} = 0.408 = \cos\gamma_{zB} = \cos\gamma_{xB}$$

(by symmetry)

$$\cos\gamma_{yc} = \frac{2400}{l_C} = \frac{2400}{2683.28} = 0.8944$$

$$\cos\gamma_{yA} = \cos\gamma_{yB} = \frac{2400}{l_A} = \frac{2400}{2939.39} = 0.8165$$

$$\Sigma F_z = T_{Bz} - T_{Az} = 0 \Rightarrow T_B \cos\gamma_{zA} = T_A \cos\gamma_{zB}$$

$$T_B = T_A$$

$$\Sigma F_x = T_{Cx} - T_{Ax} - T_{Bx} = 0 \Rightarrow T_C \cos\gamma_{xc} - 2T_A\cos\gamma_{xA} = 0$$

$$\Rightarrow T_C = T_A \frac{2\cos\gamma_{xA}}{\cos\gamma_{xc}} = T_A\left[\frac{2(0.408)}{0.447}\right]$$

$$T_C = 1.825\,T_A$$

$$\Sigma F_y = T_{Cy} + T_{Ay} + T_{By} - W = 0 \Rightarrow 0 = T_C\cos\gamma_{yc} + 2T_A\cos\gamma_{yA} - 17.6\,KN$$

$$0.8944\,T_C + 2(0.8165)\,T_A - 17.6\,KN = 0$$

$$(0.8944)(1.825)\,T_A + 2(0.8165)\,T_A = 17.6\ KN$$

$$T_A(1.6323 + 1.633) = 17.6\ KN$$

$$T_A = \frac{17.6\ KN}{3.2653}$$

$$\boxed{T_A = 5.4\ KN}$$

$$\boxed{T_B = 5.4\ KN}$$

$$T_C = (1.825)(5.4\ KN) = 9.84\ KN$$

$$\boxed{T_C = 9.84\ KN}$$