Strength of Glass from Hertzian Line Contact

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ABSTRACT

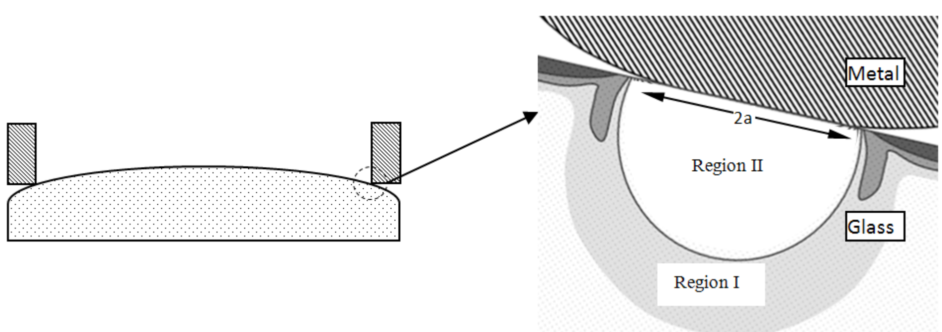
In optical lens assembly, metal retaining rings are often used to hold the lens in place. If we mount a lens to a sharp metal edge using normal retention force, high compressive stress is loaded to the interface and the calculated tensile stress near the contact area from Hertzian contact appears higher than allowable. Therefore, conservative designs are used to ensure that glass will not fracture during assembly and operation. We demonstrate glass survival with very high levels of stress. This paper analyzes the high contact stress between glass lenses and metal mounts using finite element model and to predict its effect on the glass strength with experimental data. We show that even though contact damage may occur under high surface tensile stress, the stress region is shallow compared to the existing flaw depth. So that glass strength will not be degraded and the component can survive subsequent applied stresses.

Keywords: glass lenses, Hertzian contact, tensile stress, strength of glass

1. INTRODUCTION

The general rule of thumb for mounting lenses is that polished glass can withstand tensile stresses of about 1,000 psi (6.9 MPa) before failure[1].Tensile stress will occur in the glass near the contact, which can be calculated using Hertzian contact theory (shown in Fig. 1). The darkness of the contour color indicates the relative amount of tensile stress. There is more tensile stress in the darker region, while the white region inside the tensile stress region is the compressive stress region. While this stress may indeed be present, we were unable to create an effect large enough to break glass samples. This highly concentrated tensile stress may form cracks in the subsurface of the glass. However, shallow damage to the glass may not lead to failure. We need to answer the question: If damage does occur, will the component still survive subsequent applied stresses? How does contact damage affect the strength of glass?

This paper analyzes this phenomenon using the finite element analysis (FEA) and predicts its effect on glass strength with experimental data. More specifically, we exerted a load on B270 flat windows (1.15 mm thick, 50 mm in diameter) using a metal ring that simulates the lens mount. The objective here was to show that with common sharp corner radii and large loads (R=0.01 in = 254 um and R≈50 um, F = 200 lb_f= 890 N), the strength of the glass will not degrade (via double ring strength test [5, 6]).



(a) (b)

Figure 1. (a) Retaining ring mount, (b) Principle normal stress field[3]. Region I is in tension, while region II is in compression. The maximum tensile stress locates just outside the contact area on the glass surface.

2. BACKGROUND KNOWLEDGE

2.1 Hertzian Contact for Cylinder[2, 3]

In sharp edge ring-mounted lenses, contact loading with large stresses is applied over a highly localized region. This type of configuration in the elastic range is called Hertzian contact. When the radius of the mount edge is small compared to the radius of the contact surface. The glass-mount contact can be approximated by Hertzian contact of a cylinder on a flat glass surface[2]. The equation for contact pressure is:

$$\sigma_c = \frac{2P}{\pi a}, \tag{1}$$

where P (lb_f/in) is Loading force, a is the contact half-width, which can be expressed as:

$$a = \sqrt{\frac{4PR^*}{\pi E^*}}, \tag{2}$$

where $R^* = \left(\frac{1}{R_m} + \frac{1}{R_g}\right)^{-1}$ is the contact radii and $E^* = \left(\frac{1-\nu_m^{\,2}}{E_m} + \frac{1-\nu_g^{\,2}}{E_g}\right)^{-1}$ is the effective modulus, which is comprised of Young's moduli E_m and E_g ,and Poisson ratios ν_m and ν_g for metal and glass, respectively. The maximum tensile stress of Hertzian point contact is[1]:

$$\sigma_t = \frac{1-2\nu_g}{3} \cdot \sigma_c \tag{3}$$

The general form of the contact stress field, especially the tensile stress, is shown in gray colors in Figure 1b. The important feature of the indentation stress field for the initiation of a conical fracture is the tensile region near the specimen surface just outside the area of contact.

Hatheway[4] also developed a set of closed-form equations for the state of stress over the surface of the lens with ring contact. He stated that if radius of the ring contact increase to infinity, the tensile stress will become zero. While if the radius of the ring contact decrease to from a point contact, the equation will agree with equation (3).

There are some limitations to these equations. First of all, the contact is assumed to be purely elastic, but in the real case, metals have plastic properties, which reduce the stress concentration. Secondly, friction is absent at the contact interface, but friction is an important factor affecting the amount of tensile stress inside the glass, this will be discussed in the simulation section following.

2.2 Strength of Glass[5]

Glass does not possess a single characteristic strength. The strength of the material is dependent on the distribution of cracks or surface flaws. The strength of a particular element can be estimated using the following two methods.

2.2.1 Fracture Mechanics Approach

The maximum bending strength of a glass sample depends on the size and geometry of the surface flaws. In the case of a flaw with a small depth in a thick plate with tensile forces acting normal to the crack plane, one can define a stress intensity factor K_I as:

$$K_I \approx 2\sigma_0\sqrt{a} \tag{4}$$

σ_0 nominal tensile stress perpendicular to the stress plane

a depth of the flaw.

A flaw will result in a fracture if K_I is larger than a critical value called the fracture toughness K_{IC}. If exposed to constant stress over time, a surface flaw can grow to critical size with the rate of growth depending on K_I and the atmospheric moisture content. For example, the maximum crack depth is about 0.05 inch (1.27 mm) for a polished BK7 glass that can withstand tensile stresses of about 1,000 psi (6.9 MPa).

2.2.2 Statistic Approach

For a particular type of glass, it is reasonable to suppose that surface fractures can be statistically characterized by a function, which can be related to the probability of failure as a two-parameter Weibull distribution. Based on laboratory test results obtained under well-defined conditions one can calculate design strengths for loads and conditions posed by special application requirements. Equation (5) gives the probability that the sample will fail if it is loaded to stress σ.

The quantities σ_0 and m are model parameters that must be experimentally determined. These parameters for several glasses are given in reference 5.

$$F(\sigma) = 1 - \exp(-(\sigma / \sigma_0)^m) \tag{5}$$

$F(\sigma_0)$	Probability of failure at tensile stress σ
σ_0	Characteristic strength ($F(\sigma_0)$ = 63,21 %)
m	Weibull factor (scatter of the distribution.)

3. ANSYS FEA SIMULATION

Since the ring-glass contact is plane symmetric, a 2D model instead of a 3D one was used in ANSYS®. In figure. 2, the left edge of the grid is the center line of the contact area. Just half of the stress field is shown because of the symmetry. The contour is the tensile stress field under a 50 lb_f /in (8.7k N/m) line force load with a 0.01 in (254um) contact radius. From figure 1 and figure 2, we can see the high tensile stress field is just outside the contact area. The depth of the stress field will not change when the density of mesh grid is changed.

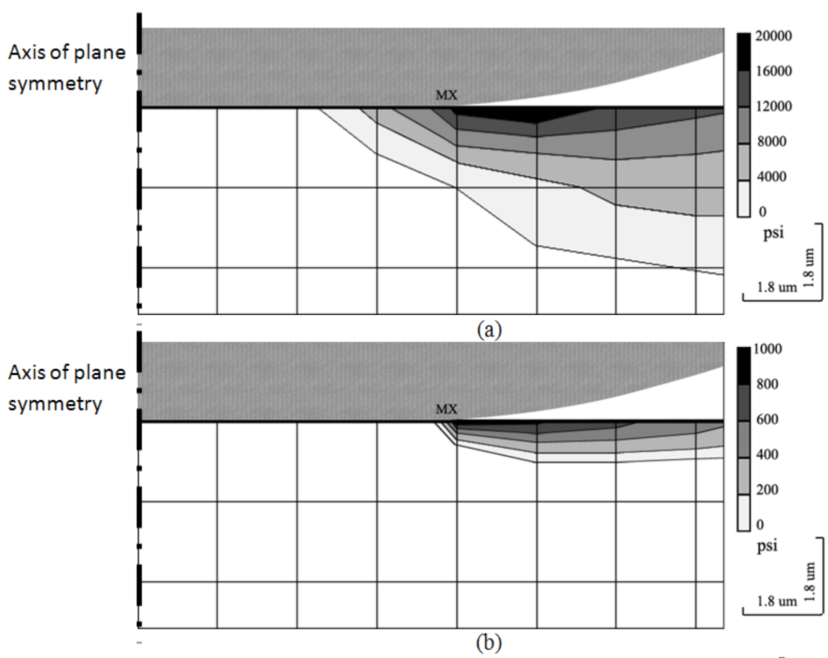


Figure 2. Half-plane tensile stress field on the glass cross section using FEA in ANSYS®.
(a)Frictionless contact, (b) Contact with 0.5 friction coefficient

Fig. 2a is the half cross-section for a frictionless contact, while Fig. 2b is a contact with a friction coefficient of 0.5. The maximum tensile stress is at the surface at the location marked 'MX'. The maximum stresses are 19332 psi (133 MPa) for Fig. 2a and 966 psi (6.8 MPa) for Fig. 2b, respectively. The depths of stresses are about 4 um for Fig. 2a and 1 um for Fig. 2b, respectively. These depths are 2 to 3 orders of magnitude less than the allowable flaw depth for a polished BK7 glass that can withstand tensile stresses of about 1,000 psi (6.9 MPa).

Herztian contact (eq.1 and 2) assumes two materials contact without any friction[3], which is not the real case[7]. A statically loaded model with a friction coefficient was evaluated. The tensile stress decreased when the friction coefficient was increased which is shown in the comparison between Fig 2a and 2b. This is due to different Young's moduli and Poisson ratios between glass and steel. In this case, stresses were reduced because the contact zone is larger. But this may not hold true for cases where the steel component is replaced by a lower modulus component.

In addition, more loads applied to the glass may yield the steel, so the sharp corner will be flattened and stress will be decreased. Because of that, more loads will not necessarily make more degradation to the strength of glass. If we use aluminum instead of steel, we will have a smaller minimum load to yield the mount and flatten the sharp corner.

In another case, when we applied an additional shear force to the indenter (sliding), tensile stress increased on one side. Sliding may happen during both assembly and operation. We need sufficient torque on the retaining ring to hold the lens in place. In the final turn, beside the perpendicular load, a tangential force will be applied to the glass surface. Also temperature change will result in a shear force between two materials having different CTEs (coefficients of thermal expansion).

4. EXPERIMENTS

A piece of glass breaks when two conditions coincide. The first is the presence of enough tensile stress at the surface and the second is the presence of a flaw in the region of the tensile stress. In the experimental procedure, we first made some flaws on the glass using sharp edge ring contact. We then establish allowable load levels that applied tensile stress on the glass surface and test the glass sample to failure.

4.1 Applied stress via line contact

4.1.1 Static load

The setup is shown in Fig. 3. The INSTRON hardness testing machine provides a controllable vertical load force (manually) and a platform. A ball tip was used against the load cell to prevent a side force. A Loadstar iLoad Mini load cell[11]was attachment to the indenter. A clamp fork and bamboo forks were used to concentrically align the indenter, glass sample and the supporting ring. A piece of rubber was place between the glass plate and the supporting ring to prevent the irregular edges damaging the lower surface of the plate. We exerted a load on B270 flat windows (1.15 mm thick, 50 mm in diameter). The load value was displayed instantaneously on a computer screen via a user interface. The maximum indenting load was held for 5 seconds before release.

We loaded up to 17.4k N / m (100 lb_f / in), which indicates about 160 ksi maximum contact pressure and 26 ksi maximum tensile stress. Even at such high loads, we did not observe any failure.

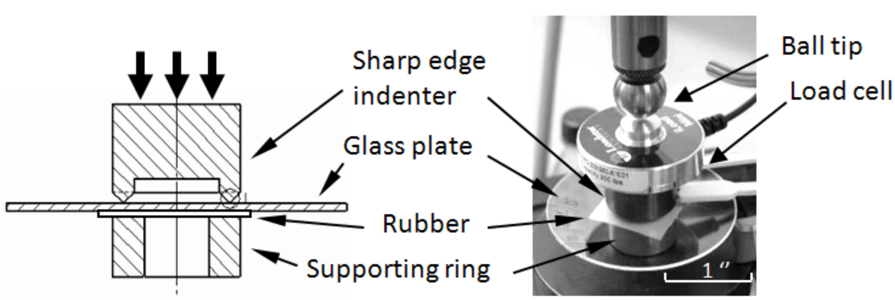


Figure 3. Load the glass with a sharp edge indenter. Drawing is on the left side and the actual set up on the right side.

4.1.2 Shock Load[12]

We used the bench handling procedure from MIL-STD 810D to simulate the shock load. Tape was used to clamp the indenter sample and an aluminum substrate together. To avoid damaging the sample due to irregularities on the metal surface, a piece of paper was placed between the glass and the aluminum block. Using one edge as a pivot, we lifted the opposite edge and let, then let go the whole package with the lifted edge just below the point of perfect balance. The procedure was repeated, using the other edges for a total of four drops.

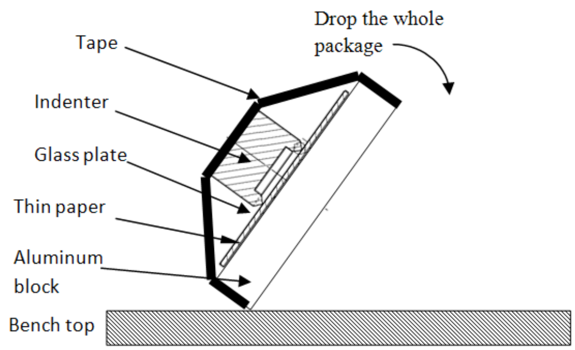


Figure 4. Shock load

4.2 Double Ring Test of Strength of the Glass

The test procedure was according to DIN 52292-1 Double ring method[5]. In this ring-on-ring bending test, the vertical load applied to the sample is read by a mini load cell[11], shown in figure 6. The load will transfer via moment to the glass, and then the tensile stress appears on the surface. From Roark's[10], for the solid circular plate under uniform annular line load with simply supported edge restraint (shown in Fig. 5), we have the bending moment:

$$M_c = waL_9 \,, \tag{6}$$

where w is the amount of annular line load ($\mathrm{lb_f}$/ in),
a is the outer radius where the support force exert,
L_9 is one of the general plate functions: $L_9 = \frac{r_0}{a}\left\{\frac{1+v}{2}\ln\frac{a}{r_0} + \frac{1-v}{4}\left[1-\left(\frac{r_0}{a}\right)^2\right]\right\}$,
r_0 is the radius of the annular line load from the center line,
v is the Poisson ratio of the plate,

And the bending tensile stresses can be found on the convex side of the plate from M_c by the expression:

$$\sigma = \frac{6M_c}{t^2} \,, \tag{7}$$

where t is the thickness of the annular plate.

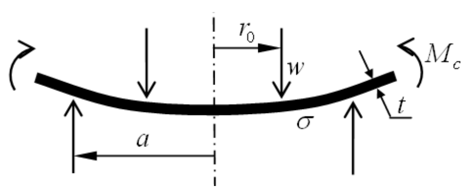


Figure 5. Plate under uniform annular line load

We used both COSMOSWork in SolidWorks® modeling and formulas from Roark [10] to verify the tensile stress calculation. The results agreed well with each other.

The setup is shown in figure 6. Three clamping forks were used to align the double rings and the glass sample. Rubber films were added between the glass and metal rings. This helped to prevent the sharp edge on the metal adding more high local stress to the glass. Gently apply the load until the glass breaks. The load cell software automatically records the maximum load.

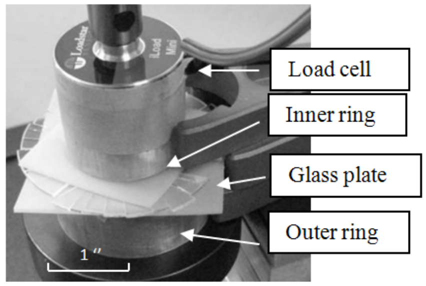


Figure 6. Double ring strength test. Using the same load cell in a different set up, we tested the samples to failure in order to get the strength of the glass compared to the control group.

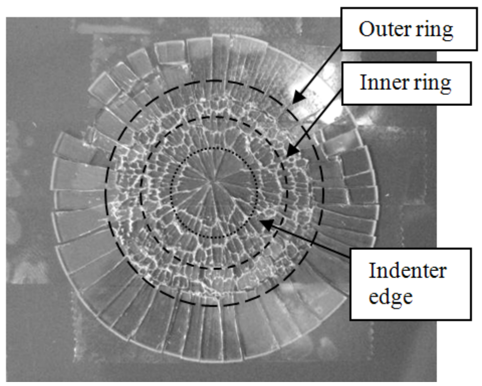


Figure 7. A crack pattern of a sample from a double-ring-test. The dotted rings represent the relative radii among different contact rings on the glass.

From the cracking pattern in figure 7, we can see that the initial crack was from the central region of the sample, where the tensile stress was applied while bending. Because the tensile stress is uniform inside the smaller ring, the initial crack will occur at the location where the deepest existing flaw was. The dotted rings shown in figure. 7 represent the relative radii among different contact rings on the glass. Since there was no tensile force outside the outer ring, the crack paths in this region maybe from the crack propagation from the glass inside.

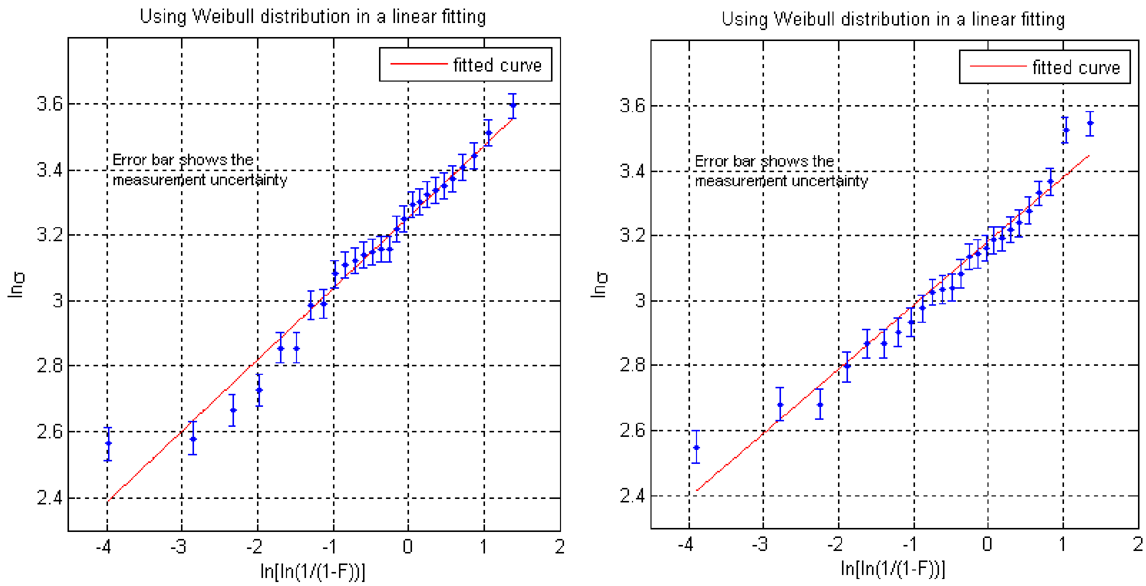
5. RESULTS AND ANALYSIS

5.1 Fit the Weibull distribution

Once we obtained a set of tensile stress data, a probability was assigned to each data point using Harris' method[1] and then fit the Weibull distribution[8]. In order to fit a straight line, we rewrite eq. 4 into:

$$\ln\left\{\ln\left[(1-F(\sigma))^{-1}\right]\right\} = m\ln\sigma - m\ln\sigma_0 \tag{8}$$

We applied a linear fitting to $\ln\left\{\ln\left[(1-F)^{-1}\right]\right\}$ vs. $\ln\sigma$, obtained modulus m as the inverse of the slope and the characteristic strength σ_0 from the intercept $-m\ln\sigma_0$. Matlab® is used to fit load data. Two examples of data sets are shown in figure 8. The major uncertainties were from the thickness variation of the glass plates and the annular line load force we exerted on the samples. We can see from the eq. 7, the sample thickness is inverse square proportional to the tensile stress value and the load is linear.



(a) (b)

Figure 8. Weibull distribution in a linear fitting. (a) Before exposing to contact stress, (b) After 100 lb_f/in loading force. The error bars are the uncertainties in the calculated tensile stresses.

To compare the strength before and after exposing to contact stress, we need a group of 25 samples to test the strength with any damage. Using this method we can get the scatter of the distribution *m* and characteristic strength σ_0 for different sample groups, which were summarized in Table 1. There is no data for optically polished BK7, so we scale between optically polished Zerodur, D64 etched Zerodur, and D64 etched BK7 to obtain an estimated for optically polished BK7. The strength we obtained from our experiment is in the proper range.

Table 1 characteristic strength σ_0 and scatter of the distribution m

Situation	Quantity	Characteristic strength σ_0 ksi	Scatter of the distribution m
1. Control group	25	25.9	4.4
2. 17.4k N/m, R=254 um	25	24.2	4.9
3. 17.4k N/m, R≈50 um	10	21.3	4.0
4. Shock load	10	27.3	3.8
5. Grind with 25um compound	7	10.5	7.2

5.2 level of confidence in the results

The t-statistics may be used to test the hypothesis that two data sets have the same mean. We can verify that if the two sets of data arose from identical physical causes. We used the table of the student's distribution[13] to determine if the results of two sets of data are the same, except for statistical error.

For example, $\langle x_1 \rangle$ is the average stresses before breaking in the double ring test (control group); and $\langle x_2 \rangle$ is the average stresses after 17.4k N/m (100 lb_f/in) loading force. Their sample quantities and standard deviations are N_1, S_1; N_2, S_2, respectively. Then value of the parameter t in the student distribution is:

$$t = \frac{\left|\langle x_1\rangle - \langle x_2\rangle\right|}{\sqrt{N_1 S_1^2 + N_2 S_2^2}} \sqrt{\frac{N_1 + N_2 - 2}{1/N_1 + 1/N_2}} \qquad (9)$$

Using equation (9), we can calculate the t value for each situation compare with the control group, which is list in table 2. Then we can find the t value in appendix C [13]. The number corresponds to a confidence level. In common situation, we can reject the hypothesis that the two sets of data are from the same cause, when the t value exceed the 0.95 confidence level. The t value for 95% confidence level is about t_0 = 2.06. We can see that all situations listed are well within this range ($t < t_0$), except for situation 5. So we can accept that the situations 1 to 4 are from the same physical cause. The apparent degradation listed in table 1 are due to statistical issues. This means the strength of the glass will not significantly degrade at these levels of load.

Table 2 t values, average and standard deviation of tensile stress to break the sample in each situation

Situation	Quantity	Average tensile stress (ksi)	Standard deviation (ksi)	t value
1. Control group	25	23.7	5.4	0
2. 17.4k N/m, R=254 um	25	22.3	5.6	0.953
3. 17.4k N/m, R≈50 um	10	21.8	6.1	0.879
4. Shock load	10	24.6	6.9	0.398
5. Grind with 25um compound	7	9.9	1.6	6.468

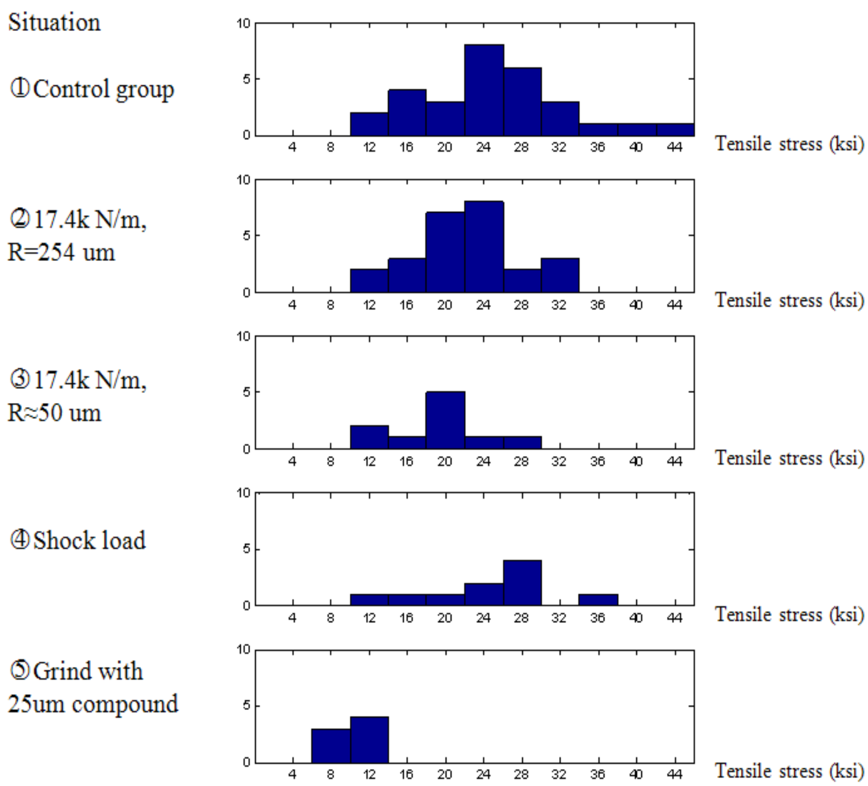


Figure 9. Histograms of the tensile stress values in each situation

Another way to state the t-statistics is that the mean of the tensile stress in the control group can be known no more accurately than $\sigma_{t-avg} = 23.7 \pm 2.1$ ksi for a 95% confidence level. The sample means of situation 2 to 4 are within this range. Again, we can accept that they are from the same cause, which means that exposing to contact stresses in situation 2 to 4 has little effect on the strength of glass. All the tensile stress values in each situation are plotted in histograms in figure 9.

6. CONCLUSION

The strength of glass is highly dependent on its surface finish with optically polished glass being substantially stronger than coarse ground or scratched glass. Although maximum tensile stress on a surface of a optically polished glass high under high static annular line load (100 lb_f/ in) with sharp metal edge contact, the shallow tensile stress region inside the glass helps to prevent deep cracks and maintains the strength of the glass. It is safe to say the assumption, that polished glass can only withstand tensile stresses of about 1,000 psi, is too conservative. Adding a safety factor of 4 in our experiment, we can conclude that at 4.4k N/m (25lb/in) static load with R=254 um (R=0.01 in), the strength of glass will not degrade. Moreover, shock load seems do not have catastrophic effect to the glass contacting with sharp edge.

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