Tolerancing Optical Systems

• Why are tolerances important?
  – Somebody is going to make it (hopefully)
  – It must meet some performance requirement
  – Cost (and schedule) are always important

• Why is it difficult?
  – Involves complex relationships across disciplines
    • System engineering
    • Optical design and analysis
    • Optical fabrication
    • Opto-mechanical design
    • Mechanical fabrication

• If you can tolerance effectively, then you can be a good designer, otherwise you can not.
Process of optical system tolerancing

1. Define quantitative figures of merit for requirements
2. Estimate component tolerances
3. Define assembly/alignment procedure and estimate tolerances
4. Calculate sensitivities
5. Estimate performance
6. Adjust tolerances, balance cost and schedule with performance
7. Iterate with system engineer, fabricators, management
System Figure of Merit

• Keep this as simple as possible
• Must propagate all performance specs through assembly
• Typical requirements
  – RMSWE (root mean square wavefront error)
  – MTF at particular spatial frequencies
  – Distortion
  – Fractional encircled energy
  – Beam divergence
  – Geometric RMS image size
  – Dimensional limits
  – Boresight
Parameters to tolerance

Allowable errors are called tolerances.

What needs to be tolerated?

• General parts (usually machined metal)
• Physical dimensions of optical elements
• Optical surfaces
• Material imperfections for optics
• Optical assembly
Estimate system performance

For a merit function that uses RSS to combine independent contributions:

$$\Phi = \sqrt{\Phi_0^2 + (\Delta\Phi_1)^2 + (\Delta\Phi_2)^2 + ...}$$

$\Phi_0$ is from design residual – simulation of system with no manufacturing errors

$\Delta\Phi_1$ is effect from a single parameter having an error equal to its tolerance
Calculate sensitivities

• Define merit function $\Phi$

• Make list of parameters to tolerance, $x_1, x_2, x_3, \ldots$ all of the things that will go wrong.

• Use simulation to calculate the effect of each of these on the system performance.
  – For each $x_i$, use perturbation to find sensitivity

$$\frac{\partial \Phi}{\partial x_i} \approx \frac{\Delta \Phi}{\Delta x_i} = \frac{\text{(change in merit function)}}{\text{(change in parameter)}}$$

• So the contribution from a tolerance $\Delta x_i$ on parameter $x_i$ is

$$\Delta \Phi_i = \frac{\partial \Phi}{\partial x_i} \Delta x_i = \text{(sensitivity)} \times \text{(tolerance)}$$
Sensitivity calculation

• If the nominal merit function $\Phi_0$ is small (residual aberrations) calculate sensitivity directly

$$\frac{\partial \Phi}{\partial x_i} \approx \frac{\Phi(x_i + \Delta x_i) - \Phi_0}{\Delta x_i} \approx \frac{\Phi(x_i + \Delta x_i)}{\Delta x_i}$$

$\Delta x_i$ is perturbation (by the expected tolerance)
$\Phi(x_i + \Delta x_i)$ is the system merit function of the perturbed system

• To include the nominal merit function $\Phi_0$

This can be tricky

Evaluate $\Phi_0$ and $\Phi(x_i + \Delta x_i)$

– If $\Phi_0 << \Phi(x_i + \Delta x_i)$ then $\Delta \Phi = \Phi(x_i + \Delta x_i)$ as above
– If $\Phi_0$ is correlated with $\Phi(x_i + \Delta x_i)$ then $\Delta \Phi = \Phi(x_i + \Delta x_i) - \Phi_0$
– Else $\Phi_0$ and $\Delta \Phi$ combine in RSS, so

$$\Delta \Phi = \sqrt{\left(\Phi(x_i + \Delta x_i)\right)^2 - \Phi_0^2}$$
Using compensators

For most optical systems, a final focus adjustment will be made after the system is assembled. The tolerance analysis must take this into account.

When calculating the effect of each perturbation, you simulate this adjustment:
  - simulate sensing the error
  - adjust the appropriate parameter

This can be used for other degrees of freedom

*Always make the simulation follow the complete procedure.*

Every compensator requires a real measurement and a real adjustment. The limitations of the measurements and adjustments should show up in your error budget.
Combining different effects

Calculate system merit function by scaling from the sensitivities, and use RSS

\[ \Phi = \sqrt{\Phi_0^2 + \left( \frac{\partial \Phi}{\partial x_1} \cdot \Delta x_1 \right)^2 + \left( \frac{\partial \Phi}{\partial x_2} \cdot \Delta x_2 \right)^2 + \ldots} \]

\( \Delta x_i \) is now the tolerance for \( x_i \) which could be adjusted

\( \frac{\partial \Phi}{\partial x_i} \) is the sensitivity to unit change in parameter \( x_i \)

Put the sensitivities into a spreadsheet to allow easy calculation of the system errors with all effects.
## Spreadsheet for combining tolerances

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tolerance</th>
<th>Sensitivity</th>
<th>Effect on merit function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$\Delta x_1$</td>
<td>$\frac{\partial \Phi}{\partial x_1}$</td>
<td>$= \Delta x_1 \cdot \frac{\partial \Phi}{\partial x_1}$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$\Delta x_2$</td>
<td>$\frac{\partial \Phi}{\partial x_2}$</td>
<td>$= \Delta x_2 \cdot \frac{\partial \Phi}{\partial x_2}$</td>
</tr>
<tr>
<td>$x_3$</td>
<td>$\Delta x_3$</td>
<td>$\frac{\partial \Phi}{\partial x_3}$</td>
<td>$= \Delta x_3 \cdot \frac{\partial \Phi}{\partial x_3}$</td>
</tr>
</tbody>
</table>

Root Sum Square

= sqrt(sumsq(D1:D23))

You can change the tolerance value

Sensitivities calculated from simulation. These do not change

Automatically recalculate effect from each term and RSS
Confidence Levels

We assume Gaussian statistics:
This makes it easy
On average, it is correct
Any one system will NOT follow such

Use tolerance analysis to establish confidence levels

<table>
<thead>
<tr>
<th>Range</th>
<th>± σ</th>
<th>±2 σ</th>
<th>±3 σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence level</td>
<td>68%</td>
<td>95%</td>
<td>99.7%</td>
</tr>
</tbody>
</table>

Common assumptions for tolerancing:
All error terms are handled so that the tolerated value represents ±2σ or 95% confidence
Then the RSS of all of these can be interpreted as ±2σ or 95% confidence
This provides 95% yield (5% rejection)

Monte Carlo analysis can provide similar results
LOS Tolerancing example

A) Determine the focal length of the lens and find its nodal point.

Calculate the following sources of error, consider the effects for both inputs

B) Lateral translation of beamsplitter cube 20 µm
C) Rotation of the beamsplitter cube about point A of 3 µrad
D) Lateral translation of the focusing lens of 0.1 µm
E) Rotation of focusing lens about point B of 20 µrad (decompose motion into rotation about nodal point + translation of nodal point.)
F) Lateral translation of the fiber of 0.1 µm
G) Calculate the combined effect of all of the above. How does this compare to the requirement?

Show how to meet a requirement of 0.1 µm stability
### Example Spreadsheet

#### 1. Sensitivities -- Find these by analysis

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>From analysis</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam #1 (µm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B) Lateral translation of beamsplitter cube (µm)</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>C) Rotation of the beamsplitter cube about point A (µrad)</td>
<td>3</td>
<td>0.09</td>
</tr>
<tr>
<td>D) Lateral translation of the focusing lens of (µm)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>E) Rotation of focusing lens about point B of (µrad)</td>
<td>20</td>
<td>0.1</td>
</tr>
<tr>
<td>F) Lateral translation of the fiber (µm)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Simply scale the effects

#### 2. Scale to see effect of changing tolerances

<table>
<thead>
<tr>
<th>B) Lateral translation of beamsplitter cube (µm)</th>
<th>Tolerance</th>
<th>Sensitivity</th>
<th>Beam #1 (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C) Rotation of the beamsplitter cube about point A (µrad)</td>
<td>20</td>
<td>0.000</td>
<td>0</td>
</tr>
<tr>
<td>D) Lateral translation of the focusing lens of (µm)</td>
<td>3</td>
<td>0.030</td>
<td>0.09</td>
</tr>
<tr>
<td>E) Rotation of focusing lens about point B of (µrad)</td>
<td>0.1</td>
<td>1.000</td>
<td>0.1</td>
</tr>
<tr>
<td>F) Lateral translation of the fiber (µm)</td>
<td>20</td>
<td>0.005</td>
<td>0.1</td>
</tr>
<tr>
<td>RSS</td>
<td>0.1</td>
<td>1.000</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Enter new tolerance value

Excel multiplies by sensitivity to give net contribution
Assigning initial tolerances

• Start with rational, easy to achieve tolerances
• Only tighten these as your analysis requires
• Rules of thumb for element tolerances
• Rules of thumb for assembly tolerances

• *Best -- know the limitations of your fabrication and alignment processes*
Using optical design codes

- Much of the above work can be done entirely within the optical design code.

- You can specify tolerances, and the software will calculate sensitivities and derive an RSS

- **Be careful with this!** It is easy to get this wrong.

- The optical design codes also include a useful Monte Carlo type tolerance analysis. This creates numerous simulations of your system with all of the degrees of freedom perturbed by random amounts.
Develop complete set of tolerances

• Simulate system performance
  – Include all compensators

• Check overall magnitudes of the terms
  – Terms with small effects, loosen tolerances
  – Terms with big effects, may need to tighten tolerances

• Revise fabrication, alignment plans as needed the goal is:
  1. Meet performance specifications
  2. Minimize cost
Mechanical tolerancing

- This is a huge, important subject for opto-mechanical engineers.

- Basic types of tolerances for optical systems
  - General position tolerances
    - lens spacing and alignment
  - Surface texture
    - comes from fabrication process
  - Level of constraint
    - overconstrain for stiffness, clearance for motion
    - interference or clearance for optic mounts
Dimensional tolerances for machined parts

- Depends on fabrication methods and equipment so discuss these with your fabricator!
- Rules of thumb for machined parts

<table>
<thead>
<tr>
<th>Machining Level</th>
<th>Metric</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse dimensions (not important)</td>
<td>±1 mm</td>
<td>± 0.040”</td>
</tr>
<tr>
<td>Typical machining (low difficulty)</td>
<td>±0.25 mm</td>
<td>±0.010”</td>
</tr>
<tr>
<td>Precision Machining (readily available)</td>
<td>±0.025 mm</td>
<td>±0.001”</td>
</tr>
<tr>
<td>High Precision (requires special tooling)</td>
<td>&lt; ±0.002 mm</td>
<td>&lt; ±0.0001”</td>
</tr>
</tbody>
</table>
Dimensional tolerancing of optical elements

- Diameter
- Clear aperture
- Thickness
- Wedge
- Angles
  - wedge or optical deviation for lenses
  - angles for prisms
- Bevels
- Mounting surfaces

Start with nominal tolerances from fabricator
Tolerancing surface figure

- Radius of curvature
- Surface irregularity
  - Inspection with test plate.
    Typical spec: 0.5 fringe
  - Measurement with phase shift interferometer.
    Typical spec: 0.05 $\lambda$ rms
- For most diffraction limited systems, rms surface gives good figure of merit
- Special systems require PSD spec
- Geometric systems really need a slope spec, but this is uncommon. Typically, you assume the surface irregularities follow low order forms and simulate them using Zernike polynomials

This topic is covered in detail in OPTI 415/515
Define assembly procedure

- Determine adjustments that will be made in assembly that can compensate other errors
  - Each of these needs a measurement to know how to set it
  - Consider several things --
    - Range of adjustment
    - Resolution required (for motion and for measurement)
    - Required accuracy of motion and measurement
    - Frequency of adjustment
- Other dimensions will be set once (like lenses in cells)
Tolerancing optical assemblies

- Element spacing
- Tilt of elements
- Mounting decenter
- Mounting distortion
- Include stability and thermal errors

Get nominal tolerances from assembly and alignment procedures

Work with the mechanical designer
Example – 2 element null corrector

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lens spacing</td>
<td>50 µm</td>
</tr>
<tr>
<td>Lens thickness</td>
<td>25 or 50 µm</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>1 fringe power or 25 µm (whichever is smaller)</td>
</tr>
<tr>
<td>Flatness</td>
<td>λ/4</td>
</tr>
<tr>
<td>Surface figures</td>
<td>0.008 λ rms interferometer</td>
</tr>
<tr>
<td></td>
<td>0.015 λ rms lenses</td>
</tr>
<tr>
<td>Index of refraction</td>
<td>± 0.0002 (Grade A BK7)</td>
</tr>
<tr>
<td>Index Inhomogeneity</td>
<td>0.25 E-6 rms (H4 grade)</td>
</tr>
<tr>
<td>Wedge in lenses</td>
<td>50 µm</td>
</tr>
<tr>
<td>Decenter in mounting</td>
<td>50 µm</td>
</tr>
<tr>
<td>Tilt in mounting</td>
<td>50 µm</td>
</tr>
<tr>
<td>Primary radius of curvature</td>
<td>2 mm</td>
</tr>
</tbody>
</table>
Table 4. Tolerances for null lens

<table>
<thead>
<tr>
<th></th>
<th>Design value</th>
<th>Tolerance (Allowable Error)</th>
<th>Spherical aberration (nm rms)</th>
<th>Figure (nm rms)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interferometer</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregularity (rms) waves</td>
<td>Nil</td>
<td>0.008</td>
<td>5.06</td>
<td></td>
</tr>
<tr>
<td>Decenter µm</td>
<td>Nil</td>
<td>0.050</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Airspace</strong></td>
<td>mm</td>
<td>103.972</td>
<td>0.05</td>
<td>1.36</td>
</tr>
<tr>
<td><strong>Relay Lens:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Curvature 1 /mm</td>
<td>0.00E+00</td>
<td>2E-06</td>
<td>0.22</td>
<td>0.02</td>
</tr>
<tr>
<td>Thickness mm</td>
<td>10.386</td>
<td>0.025</td>
<td>0.47</td>
<td>0.02</td>
</tr>
<tr>
<td>Radius 2 mm</td>
<td>41.595</td>
<td>0.025</td>
<td>0.26</td>
<td>0.03</td>
</tr>
<tr>
<td>Irregularity 1 (rms) waves</td>
<td>Nil</td>
<td>0.015</td>
<td>4.89</td>
<td></td>
</tr>
<tr>
<td>Irregularity 2 (rms) waves</td>
<td>Nil</td>
<td>0.015</td>
<td>4.89</td>
<td></td>
</tr>
<tr>
<td>Index 1.51509</td>
<td>2E-04</td>
<td>1.17</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Inhomogeneity rms</td>
<td>2.5E-7</td>
<td>0.00</td>
<td>0.73</td>
<td>0.07</td>
</tr>
<tr>
<td>Wedge µm</td>
<td>50</td>
<td>0.00</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Decenter µm</td>
<td>50</td>
<td>0.00</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Tilt µm</td>
<td>50</td>
<td>0.00</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td><strong>Airspace</strong></td>
<td>mm</td>
<td>150.418</td>
<td>0.050</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Field Lens:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radius 1 mm</td>
<td>129.681</td>
<td>0.050</td>
<td>0.88</td>
<td>0.04</td>
</tr>
<tr>
<td>Thickness mm</td>
<td>2.924</td>
<td>0.050</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Curvature 2 /mm</td>
<td>0.00E+00</td>
<td>1.4E-06</td>
<td>0.20</td>
<td>0.03</td>
</tr>
<tr>
<td>Irregularity 1 (rms) waves</td>
<td>Nil</td>
<td>0.015</td>
<td>4.89</td>
<td></td>
</tr>
<tr>
<td>Irregularity 2 (rms) waves</td>
<td>Nil</td>
<td>0.015</td>
<td>4.89</td>
<td></td>
</tr>
<tr>
<td>Index 1.51509</td>
<td>2E-04</td>
<td>0.89</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Inhomogeneity rms</td>
<td>2.5E-7</td>
<td>0.00</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>Wedge µm</td>
<td>50</td>
<td>0.00</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Decenter µm</td>
<td>50</td>
<td>0.00</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Tilt µm</td>
<td>50</td>
<td>0.00</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Residual Wavefront waves</td>
<td>0.000182</td>
<td>0.00</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Primary Radius mm</td>
<td>7000</td>
<td>2</td>
<td>0.52</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>RSS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RSS</td>
<td>3</td>
<td>11.34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Error Tree

Optical surface
22.3 nm rms

Membrane
20.4 nm rms

- Membrane fabrication 18 nm rms
- Optical Testing 5 nm rms
- Coating 8.2 nm rms
- ROC 3 mm Corrections 1.8 nm rms
- Parasitic forces from interface 6.4 nm rms
- Actuator resolution 5 nm rms
- Wavefront sensor 3 nm rms

Operational effects
7.0 nm rms

Control system
5.8 nm rms

- 7 nm rms support
- 6 nm rms polishing residual
- 15.8 nm rms annealing strains