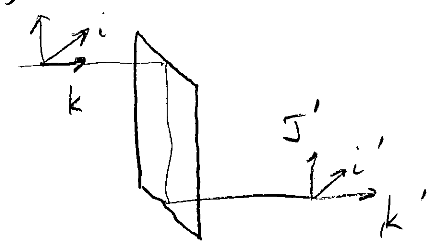
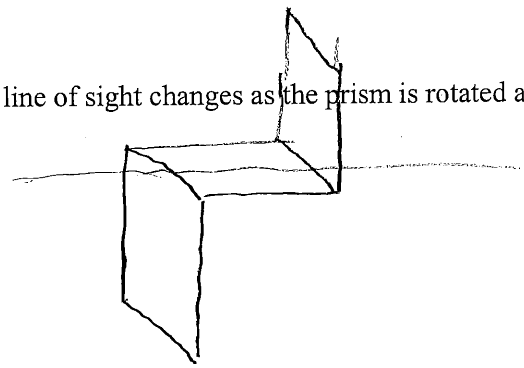
Introductory Optomechanical Engineering 421/521 – Fall 2014: Final Exam

Closed Book, closed notes, One page of notes and calculators are permitted.
110 minutes.

1.) (6) Consider a rhomboid prism
  a) Sketch the prism
  b) Draw a tunnel diagram
  c) Write the mirror matrix
  Use the matrix to show how the line of sight changes as the prism is rotated about each axis



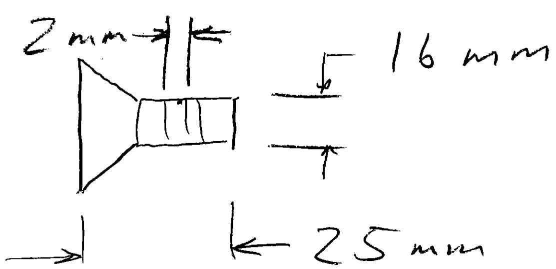


$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \leftarrow \text{Identity matrix } I$$

$$I \cdot \vec{r} = \vec{r} \quad \forall \vec{r}$$

this never changes LOS

2.) (4) Consider an M16 x 2 x 25 flat head screw
  a) Sketch the screw, giving critical dimensions



Outer Diameter = 16 mm
Root Diam = 16 - 2 = 14 mm

  b) Assume standard Property Class 5.6 with 500 MPa material strength, calculate the tensile strength of the screw

$$A = \frac{\pi D_r^2}{4} = \frac{\pi\, 14^2}{4} = 154\ mm^2$$

$$F = \sigma \cdot A = 500 \frac{N}{mm^2} \cdot 154\ mm^2$$

$$\cong 77\ kN$$

3.) (15) Consider a 10 mm thick plano-plano BK7 window with a lateral thermal gradient of 10°C across the 50 mm diameter.

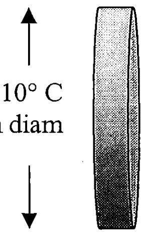
a) Give the values for thermal conductivity, dn/dT and CTE for BK7
b) Determine the heat flux in W/m^2 required to maintain this gradient
c) Calculate the wedge angle in the prism due to the thermal gradient
d) Calculate the deviation of light due to the refractive index gradient
e) Calculate the total light deviation caused by the thermal gradient

λ: 1.1 W/m·k

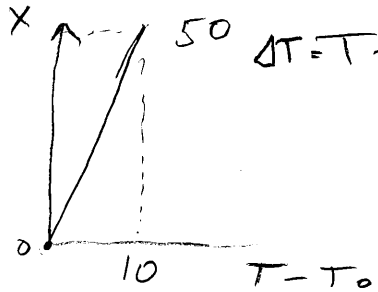
dn/dT: 3 ppm/°C

CTE: 7 ppm/°C

$\Delta T = 10°$ C
50 mm diam



10 mm thick

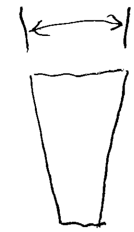


$\Delta T = T - T_o = 10 \cdot \frac{x}{50}$
$= 0.2x$ (x in mm)

b: $Q = \lambda \frac{\Delta T}{L} = \frac{1.1\ W/m \cdot K \cdot 10\ K}{0.05\ m} = 220\ W/m^2$

c: Δthickness $= \alpha\, t\, \Delta T(x) = 7E\text{-}6 \cdot 10 \cdot 0.2 \cdot x$

$\theta = \frac{\Delta thickness}{x}$ $= 14E\text{-}6 \cdot x$



$= 14E\text{-}6 = 14\mu rad$

deviation is $(n-1)\alpha \doteq 7\mu rad$

x in mm

d) $\Delta n = \frac{dn}{dT} \Delta T = 3E\text{-}6 \cdot 0.2x = 0.6E\text{-}6 \cdot x$

$OPD = n \cdot t = 0.6E\text{-}6 \cdot x \cdot 10mm$

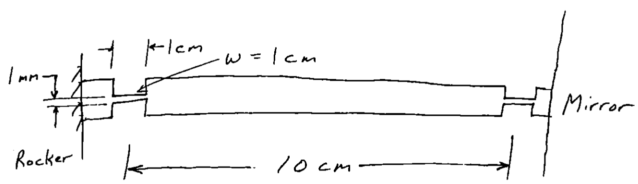
$\frac{\Delta OPD}{x} = \delta = 6\mu rad$

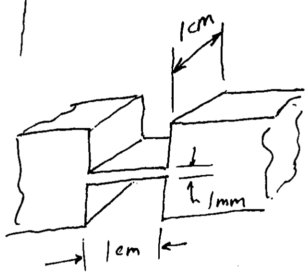
e) These effects add

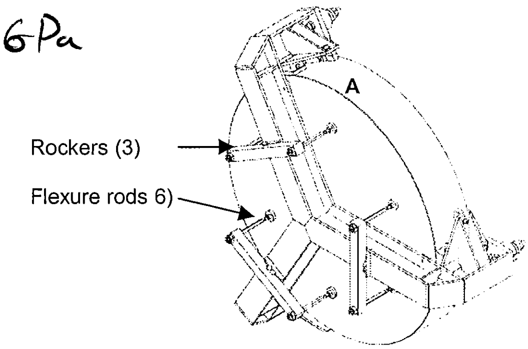
$\delta = 6 + 7 = 13\mu rad$

4) (20) Consider the case of a 50 cm diameter, 5 cm thick Zerodur mirror, supported axially using six rods with blade flexures at the ends, connected by three rocker arms.
The axial flexures are made of 17-4PH stainless steel, 10 cm long with 1 cm long, 1 cm wide, 1 mm thick blades at each end, as shown below:

$E \cong 200\,GPa$







$I = \frac{1}{12} b h^3 = 0.83\, mm^4$

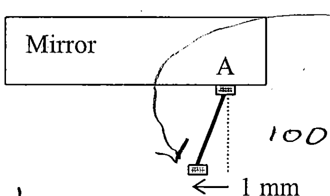
A) Calculate the angular stiffness for each 1 mm thick blade in the soft direction (in N-mm/mrad).

$$K = \frac{dM}{d\theta} = \frac{EI}{\ell} = \frac{200E3\ N/mm^2 \cdot 0.83\, mm^4}{10\, mm}$$

$$= 16.6E3\ \frac{N\cdot mm}{rad} \cong 17\ \frac{N\cdot mm}{mrad}$$

B) Calculate the moment imparted into the mirror due to a 1 mm alignment error in one end of the flexure rod, coupled with the flexure stiffness. Also, determine the maximum stress in the flexure blade due to this moment.

Ignore other flexure (mostly causes shear force at mirror)



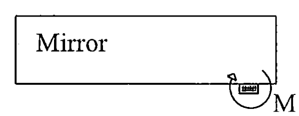
$\theta = \frac{1}{100} = 10\, mrad$

$$M = K \cdot \theta = 170\ N\text{-}mm$$

$$\sigma = \frac{My}{I} = \frac{170 \cdot 0.5\, mm}{0.83\, mm^4} \cong 100\ \frac{N}{mm^2}$$

($y_{max} = 0.5$ mm)

A finite element analysis shows 12 nm rms surface deflection when 1 N-mm moment is applied to the mirror at point A at the attachment point of an axial flexure.



C) Calculate mirror distortion due to the 1 mm flexure alignment error above.

Scale $12\ \frac{nm}{N\cdot mm} \cdot 170\ N\cdot mm \cong 2000\ nm\ rms$

$= 2\ \mu m\ rms$

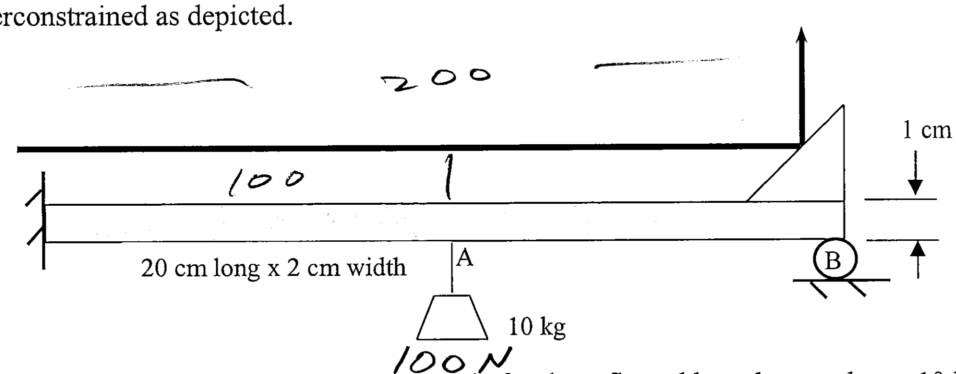
D) Assume isotropic behavior and estimate the mirror surface deflection for the case where the system assembly includes random errors of 1 mm in each end of the 6 flexure rods.

Each end, $6 \times 2 = 12$ contributions

$$\sigma = 2\,\mu m \times \sqrt{12} = 6.9\ \mu m\ rms$$

$E \doteq 70\ GPa$

5) (20) Consider a flat mirror mounted at the end of a 20 cm long rail made from 7075 aluminum alloy. The mirror is used to redirect a collimated laser beam by 90°. The rail has 1 cm x 2 cm cross section. The rail is overconstrained as depicted.



$I = \frac{1}{12} bh^3 = \frac{1}{12} 20 \cdot 10^3 = 1700\ mm^4$

Use superposition to determine the change in pointing angle for the reflected laser beam when a 10 kg weight is attached at the middle of the beam.

$\delta_B = 0.28 + \theta \cdot 100 = 0.7\ mm$

① Cantilever, ignore B

$$\delta_A = \frac{F_A l_A^3}{3EI} = \frac{100 \cdot 100^3}{3\ 70E3(1700)} = 0.28\ mm$$

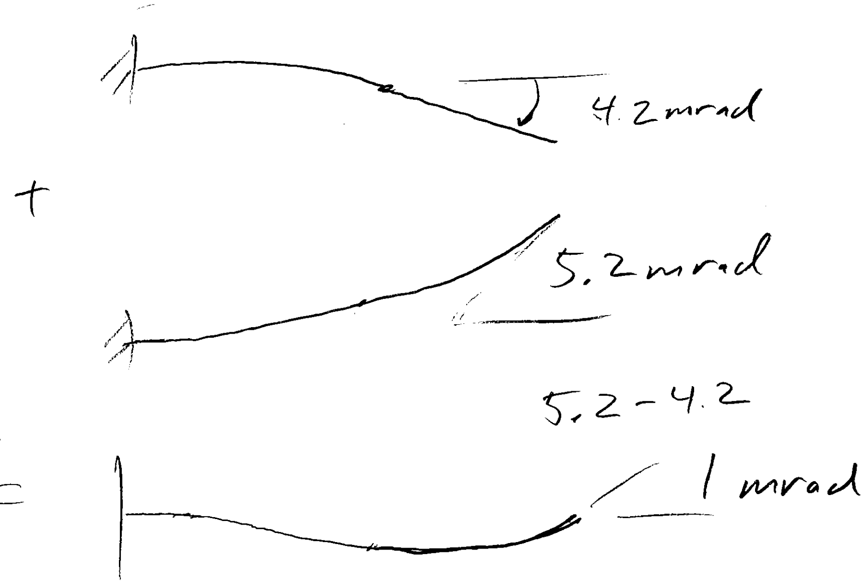
$$\theta_A = \frac{F_A l_A^2}{2EI} = \frac{100 \cdot 100^2}{2 \cdot 70E3\ 1700} = 0.0042\ rad$$

② Apply reaction force at B $F_B \uparrow$

$$\delta_B = \frac{F_B l_B^3}{3EI} = 0.7\ mm \qquad F_B = \frac{3 \cdot 70E3 \cdot 1700}{(200)^3} \cdot 0.7 = 31\ N$$

$$\theta_B : \frac{F_B l_B^2}{2EI} = \frac{31(200^2)}{2 \cdot 70E3 \cdot 1700}$$

$= 0.0052$



mirror:

$\delta = 2 \cdot \Delta\theta_M$

$= 2\ mrad$

$$E = 30E6 \frac{lb}{in^2}$$

6 (10) A 4000 lb load is supported by a 0.25" diameter steel cable, 20 inches long. (Use coordinates with +z pointing up.)

$$A = \frac{\pi (.25)^2}{4} \doteq 0.05\, in^2$$

Determine the stress in the cable due to the 4000 lb load.

$$\sigma = F/A = \frac{4000\ lb}{0.05\, in^2} = 80{,}000\, psi$$

Determine the change in the cable length as the 4000 lb load is applied.

$$\delta l = \frac{Fl}{EA} = \sigma \cdot \frac{l}{E} = \frac{80{,}000 \cdot 20}{30E6} \doteq 0.05''$$

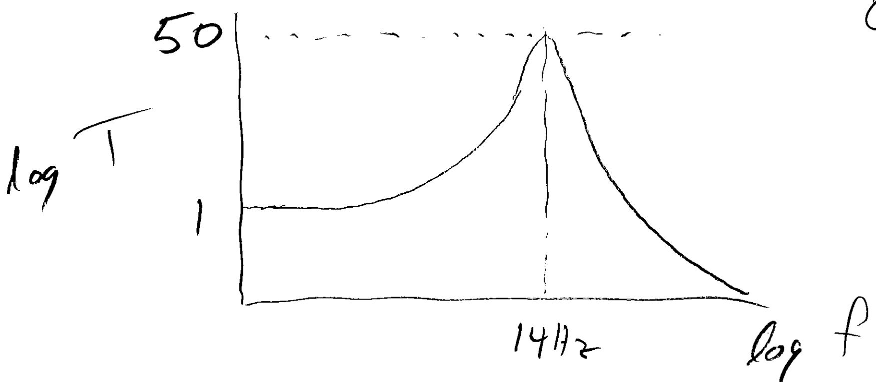
Determine the resonant frequency of the load for the bouncing mode, moving vertically in the z direction.

$$\omega_n = \sqrt{k/m} = \sqrt{G/\delta_{SW}} = \sqrt{\frac{386}{0.05}} = 88\ rad/s$$

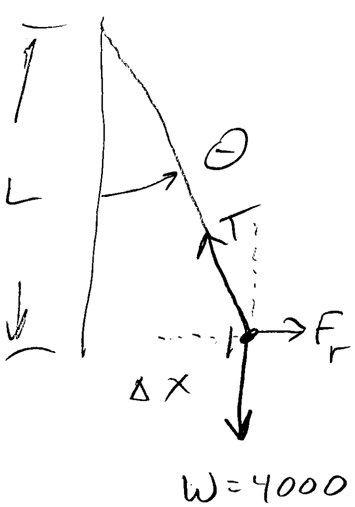
$$f_n = \frac{\omega_n}{2\pi} = 14\, Hz$$

Assuming 1% damping, sketch the transmissibility curve for this mode
. (amplitude of mass motion/base motion vs frequency)

$$Q = \frac{1}{2\zeta} = \frac{1}{.02} = 50$$



Determine the restoring force of the mass is displaced laterally in the x direction by 1 inch. Use this to determine the resonant frequency of the mass swinging as a pendulum.



$$F_r \doteq W \cdot \theta = W \cdot \frac{\Delta x}{L} = \frac{4000 \cdot \Delta x}{20}$$

$$= 200 \cdot \Delta x$$

$$k = 200\ lb/in$$

$$m = W/G$$

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{200}{(4000/386)}} = 4.3\ rad/s$$

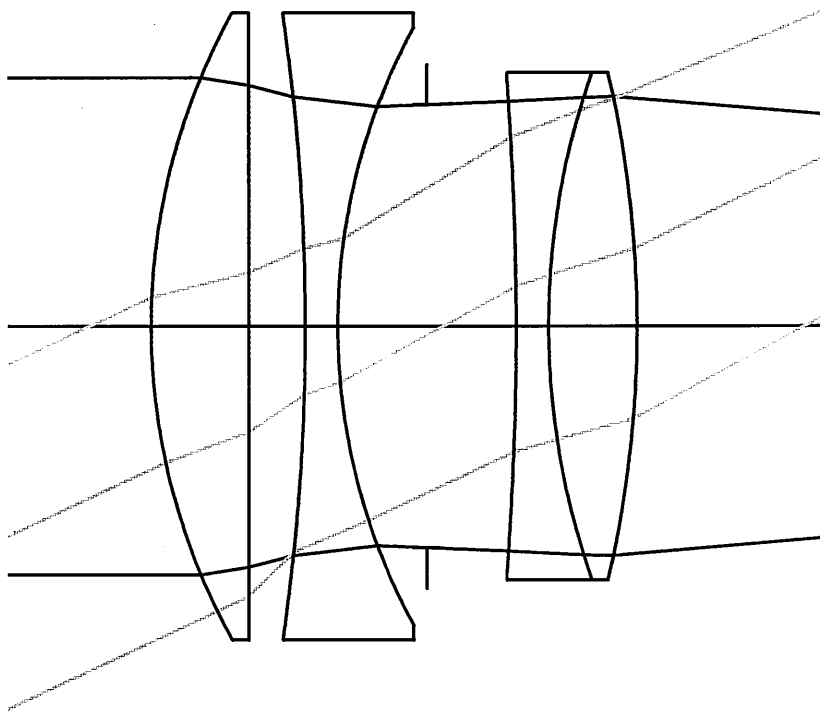
$$f_n = 0.7\ Hz$$ ~~8 sec period~~

1.4 sec period

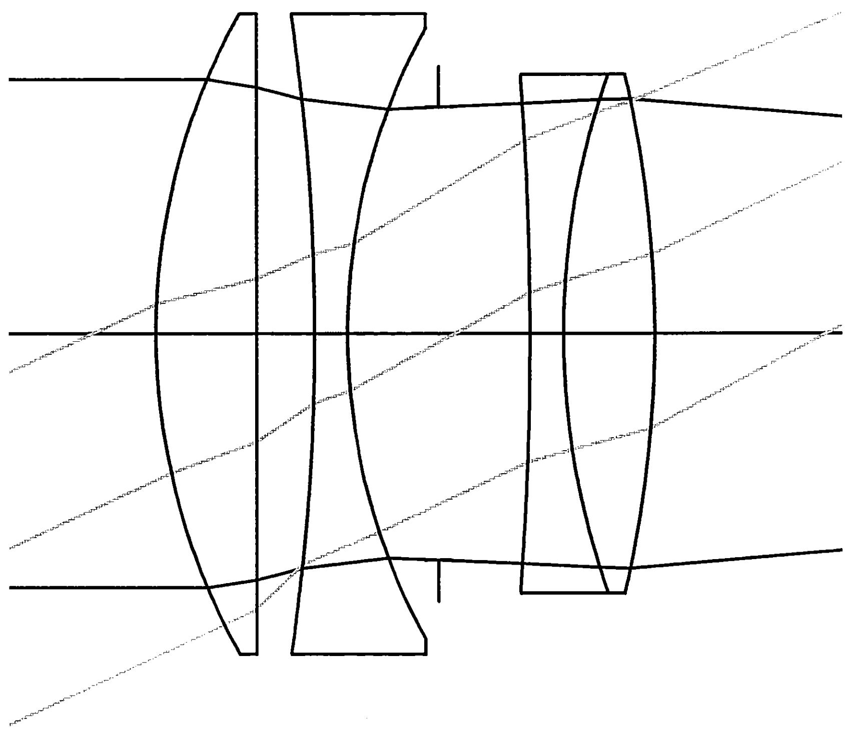
7. (15) Design a method for mounting the following lenses which range from 33 to 24 mm diameter. The tilt, decenter, and axial position must be maintained to 50 μm accuracy. Assume the lenses must survive +/- 30 C temperature and 20 G shock. The lenses are made from common optical glass.

Sketch the layout for the barrel to support the elements. You can specify custom edging of the lens elements as well as the other mechanical components. You must include the following:

- Datum reference surfaces
- Identify all critical surfaces in the barrel and use correct feature control frames and appropriate tolerances for each critical surface
- Show specifics of any retainers, adhesives, or other added parts.
- Provide description of the design and rationale



Another drawing if you mess up and want to start over.



7) (10) How do we choose materials for mirrors? Each of the following materials has some interesting material properties appropriate for mirrors. They also have some important limitations. Provide a key advantage and disadvantage for each. Give an example of a mirror application for this material and discuss the rationale for this choice.

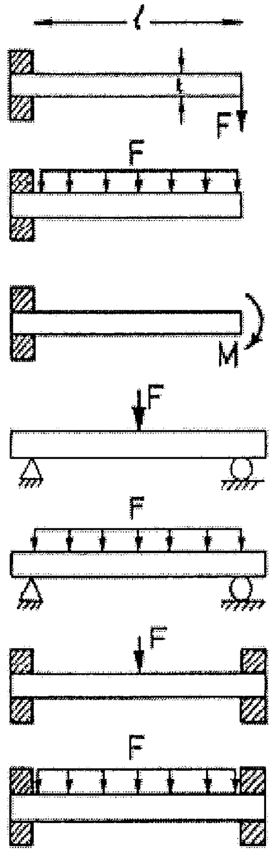
	Zerodur	Silicon Carbide	Beryllium
Key advantage	CTE = 0	High stiffness High Thermal Conductivity	High stiffness low mass
Key disadvantage	Fragile Low E/ρ	Expensive Fragile	Expensive large CTE poor ability to take good polish
Likely application	Most optical mirrors	Laser scan mirror	Cryo space mirror
Rationale			

Extra page

Useful expressions and data

$$\delta_{\max} = C_D \left(\frac{\rho}{E}\right)\left(\frac{r}{h}\right)^2 r^2 \left(1-\upsilon^2\right)$$

SUPPORT CONDITION	C_D
Ring at 68% of diameter	0.028
6 points equal-spaced at 68.1% of diameter	0.041
Edge clamped	0.187
3 points equal-spaced at 64.5% of diameter	0.316
3 points equal-spaced at 66.7% of diameter	0.323
3 points equal-spaced at 70.7% of diameter	0.359
Edge simply supported	0.828
Continuous support along diameter	0.943
"Central support" (mushroom or stalk mount)	1.206
3 points equal-spaced at edge	1.356



Case	C_1	C_2
1	3	2
2	8	6
3	2	1
4	48	16
5	$\frac{384}{5}$	24
6	192	...
7	384	–

$$\delta = \frac{Fl^3}{C_1 E I} = \frac{Ml^2}{C_1 E I}$$

$$\theta = \frac{Fl^2}{C_2 E I} = \frac{Ml}{C_2 E I}$$

- E : YOUNGS MODULUS (N/m^2)
- δ : DEFLECTION (m)
- F : FORCE (N)
- M : MOMENT (N m)
- l : LENGTH (m)
- b : WIDTH (m)
- t : DEPTH (m)
- θ : END SLOPE (-)
- I : SEE TABLE 2 (m^4)

$$I = \frac{1}{12} b h^3$$

$$D = \frac{Et^3}{12(1-\nu^2)}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\beta = \alpha - \frac{1}{n-1}\frac{dn}{dT}$$

$$\sigma = \frac{My}{I}$$

LENS MOUNTING ADHESIVE PROPERTIES

ELASTOMER	E (MPA)	υ	G (MPA)
3M – EC2216	690	0.43	241
SUMMERS MILBOND	159	0.43	55.6
NORLAND 65	140	0.43	49.0
DOW CORNING 93-500	3.45	0.50	1.15

$$T = \frac{isolated\ motion}{base\ motion} = \frac{x}{u} = \sqrt{\frac{1+\left(2\frac{\omega}{\omega_n}C_R\right)^2}{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\frac{\omega}{\omega_n}C_R\right)^2}}$$

$$a_{rms} = \sqrt{\frac{\pi}{2} f_n \cdot Q \cdot PSD_{ISO}}$$