Principles of Kinematic Constraint

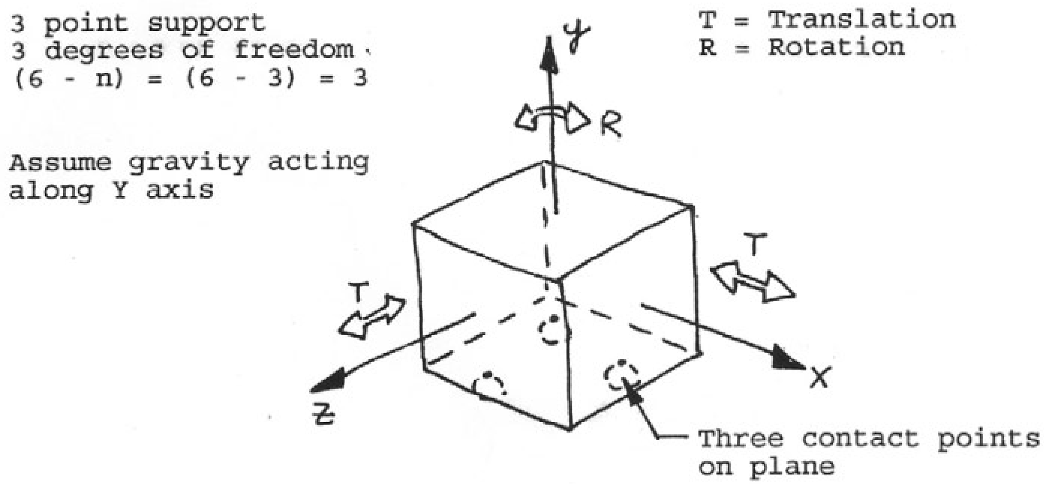
For holding a body (rigid thing) with the highest precision, we require:

- Full 6 DoF constraint
  - If 6 DoFs not fully constrained, then one is loose.

- No overconstraint
  - Any overconstraint can cause problems:
    - constraints can push against each other, resulting in stress and deformation.
    - constraints pushing against each other will “lurch” when forces exceed threshold

Kinematic constraint : All DoFs are constrained, and very strictly, none are overconstrained

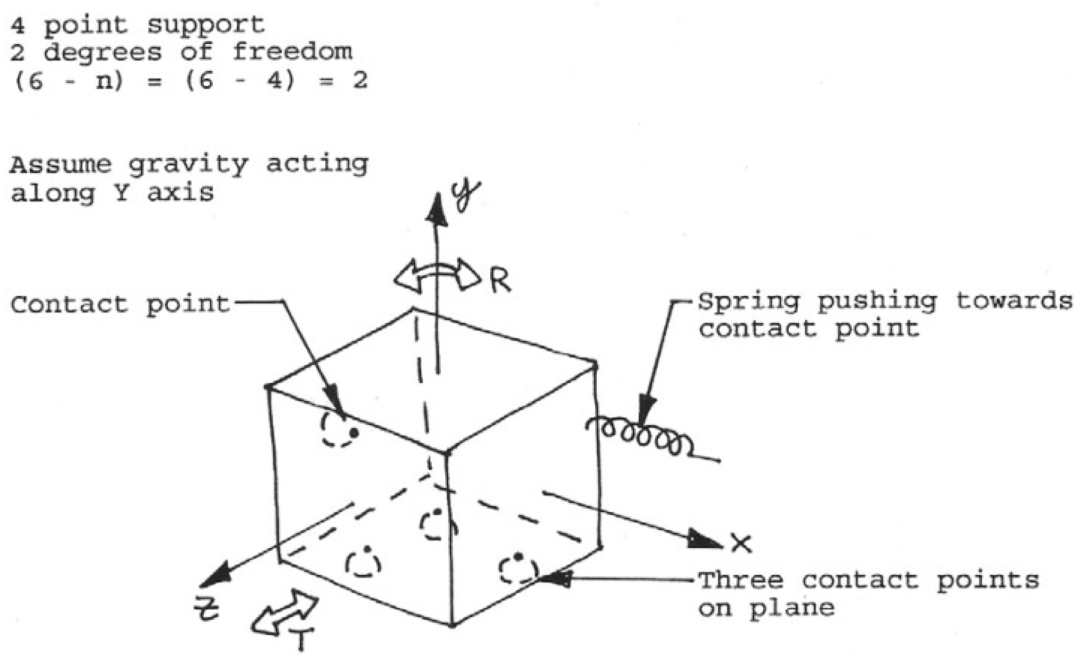
Semi-Kinematic : Slight overconstraint is allowed



What if you use 4 points in the plane?

What about 2 points?

What about 3 points in a line?



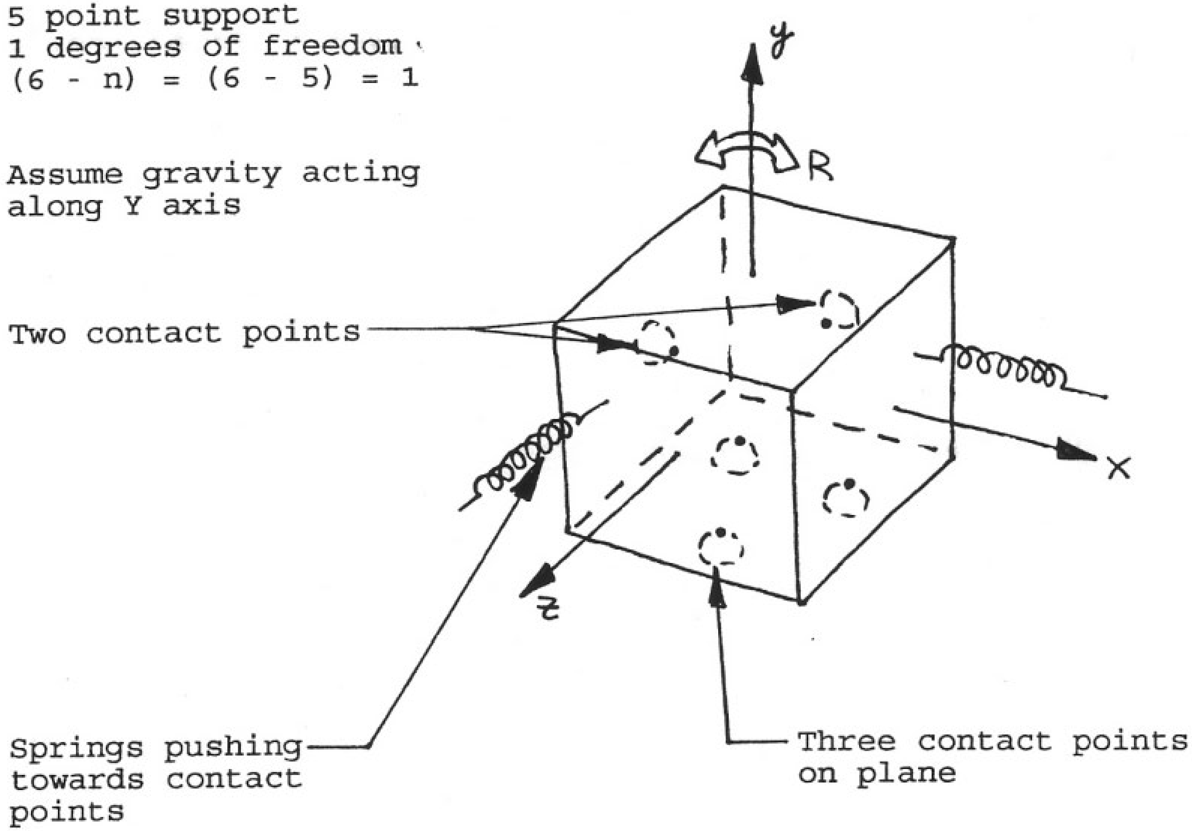
Balls provide position constraint.

Springs, gravity provide preload. NO CONSTRAINT!

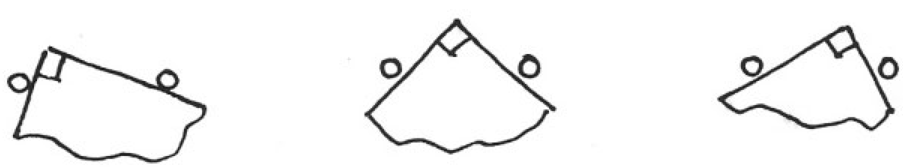
(Vukobratovich p. 68)

5 point support
1 degrees of freedom
$(6 - n) = (6 - 5) = 1$

Assume gravity acting
along Y axis

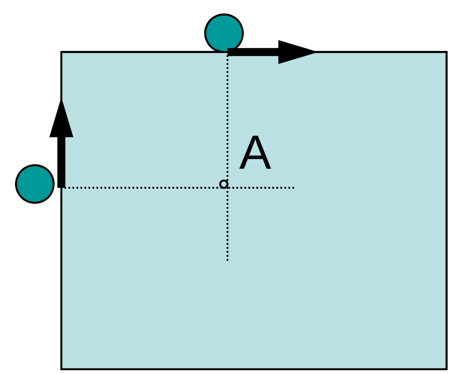


Shows indeterminacy of upper two points.



One DoF left

Small motion : Rotation about point A



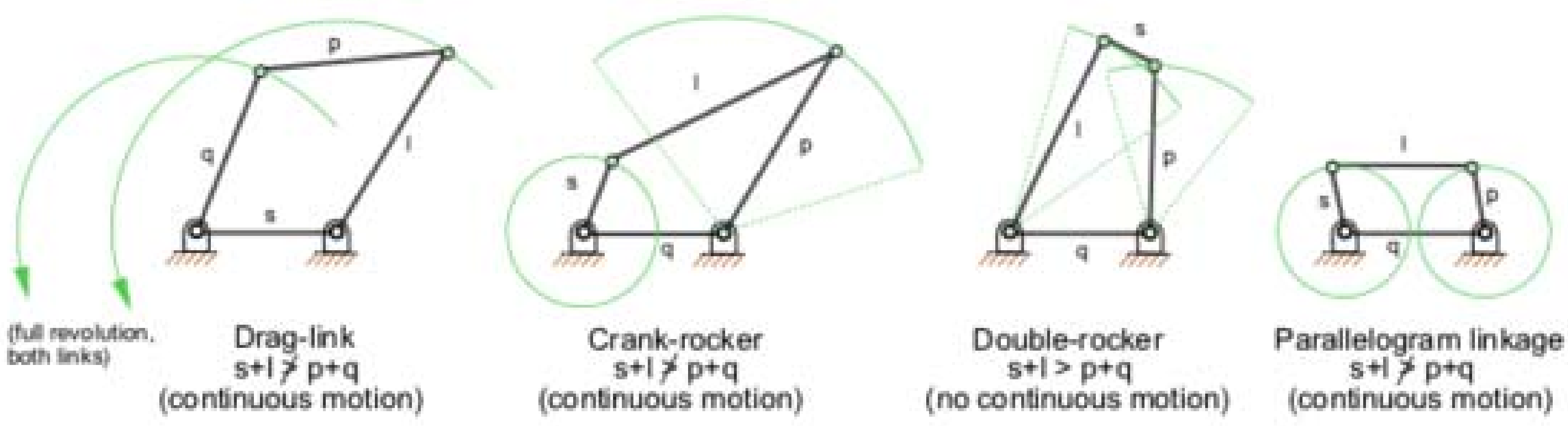
(Vukobratovich p. 69)

Instantaneous center of rotation

Same concept as four-bar linkage.

Instantaneous degree of freedom is rotation about a well defined point
– for small motions

For large motions, the geometry changes and the position of this instantaneous center of rotation moves.



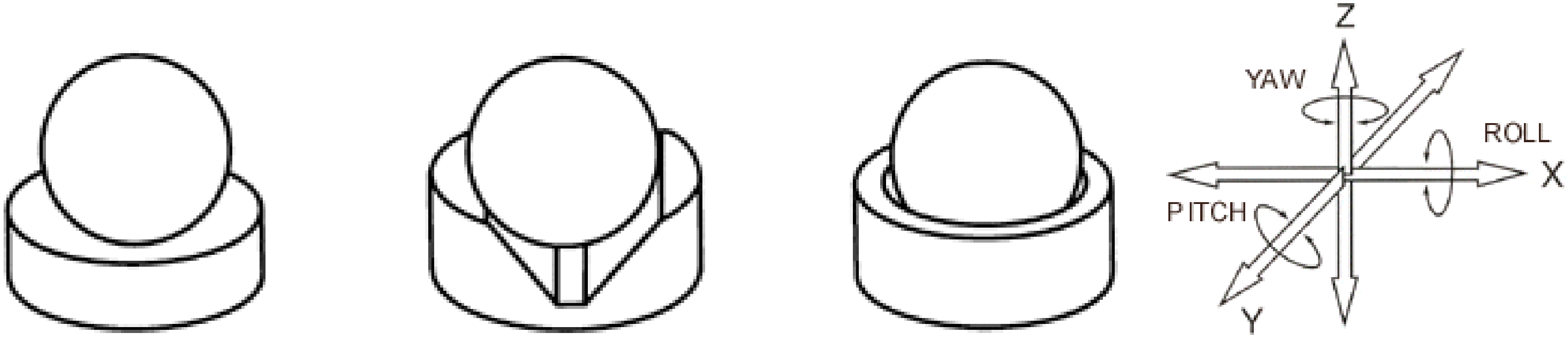
http://kmoddl.library.cornell.edu/resources.php?id=125

http://pergatory.mit.edu/2.007/lectures/2002/Lectures/Topic_04_Linkages.pdf

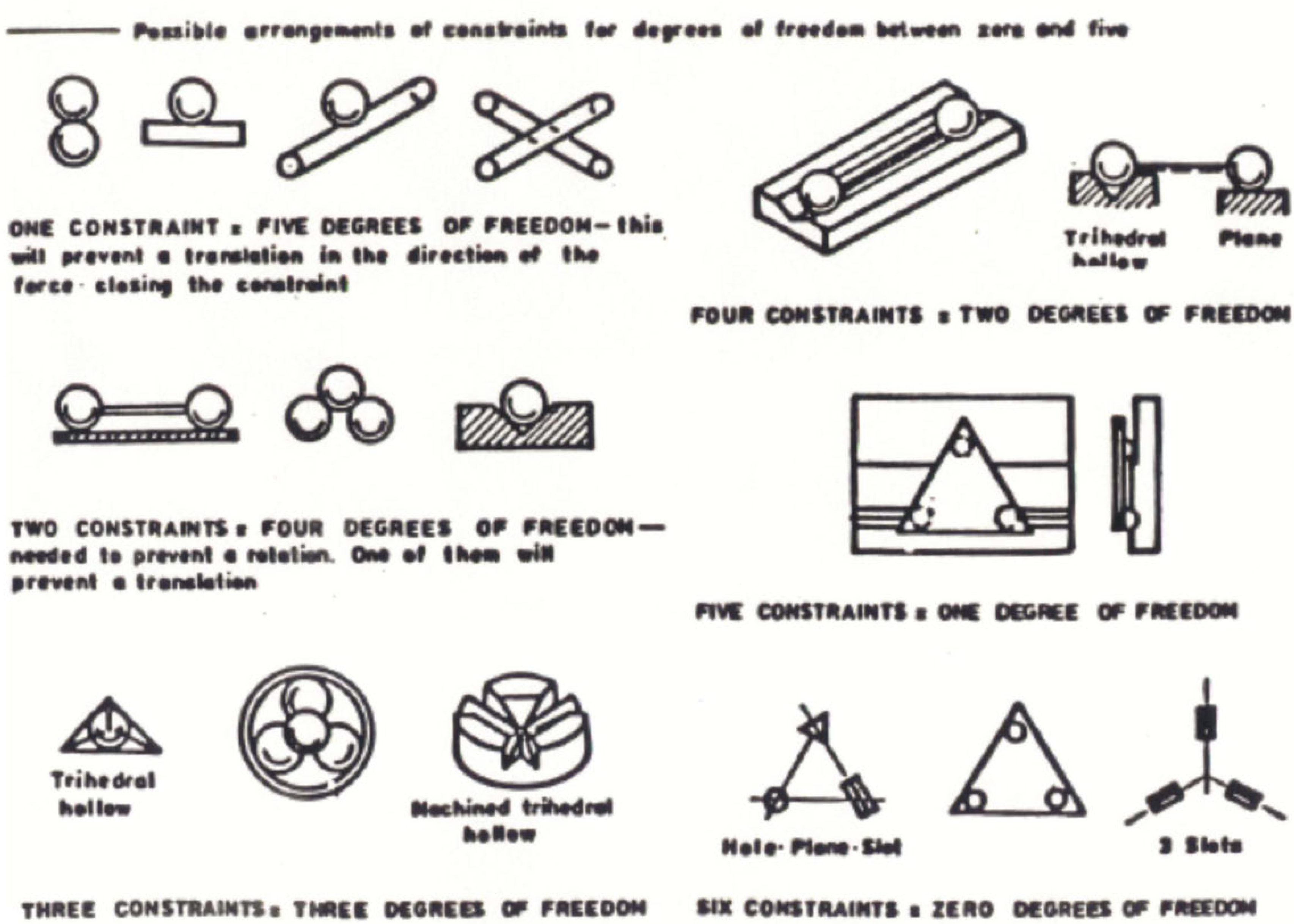
Use of balls

- Use symmetry of balls
- Material: Stainless steel, tungsten carbide, silicon nitride, diamond

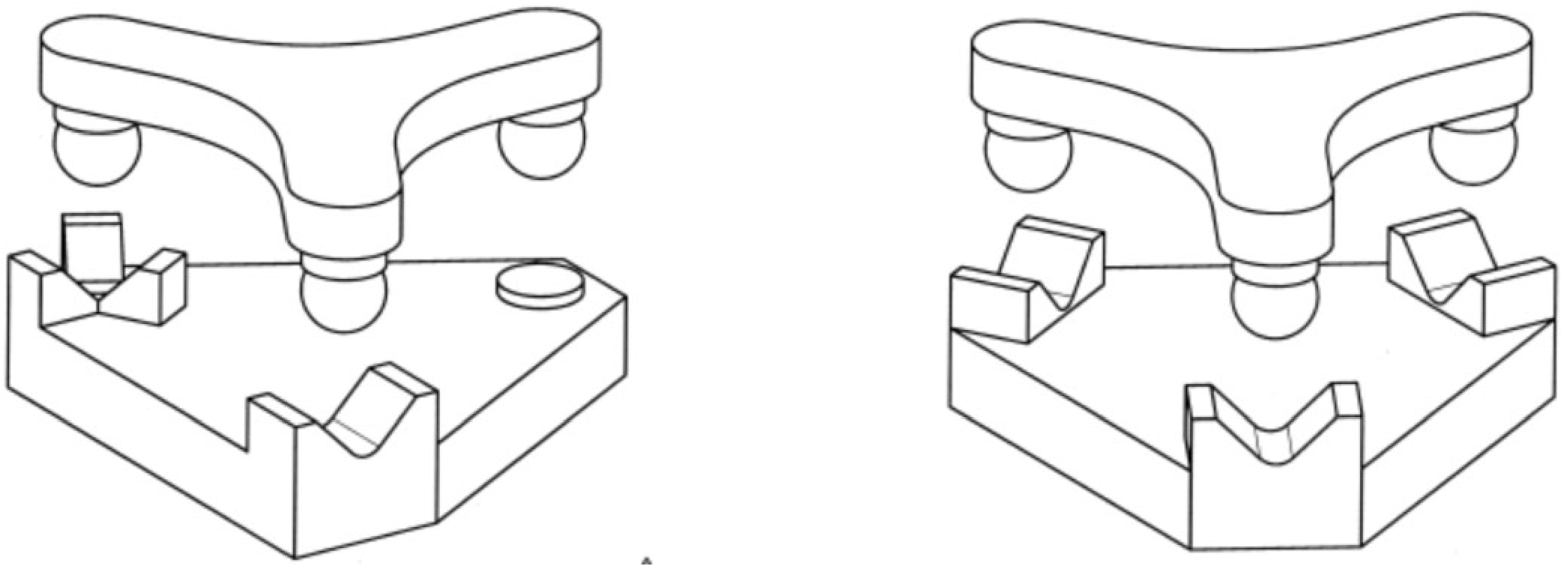
- Constrain position in 1, 2, or 3 DoF



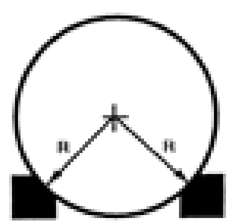
- Always leaves rotation about 3 axes about center of curvature, *If the ball is smooth*


Possible arrangements of constraints for degrees of freedom between zero and five
ONE CONSTRAINT = FIVE DEGREES OF FREEDOM—this will prevent a translation in the direction of the force closing the constraint
Trihedral hollow
Plane
FOUR CONSTRAINTS = TWO DEGREES OF FREEDOM
TWO CONSTRAINTS = FOUR DEGREES OF FREEDOM—needed to prevent a rotation. One of them will prevent a translation
FIVE CONSTRAINTS = ONE DEGREE OF FREEDOM
Trihedral hollow
Machined trihedral hollow
Hole-Plane-Slot
3 Slots
THREE CONSTRAINTS = THREE DEGREES OF FREEDOM
SIX CONSTRAINTS = ZERO DEGREES OF FREEDOM

Kinematic interface



(a)

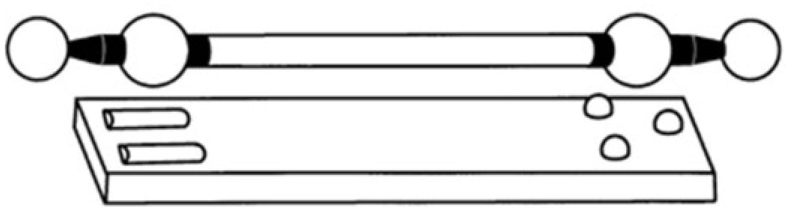


split Vee Block with
a large sphere

small spherical buttons
of a large spherical radius
and split Vee Block

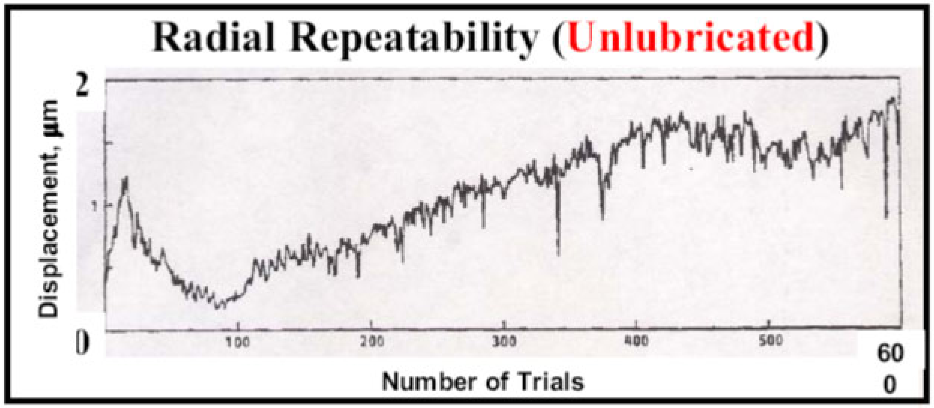
(Hale and Slocum 2001)

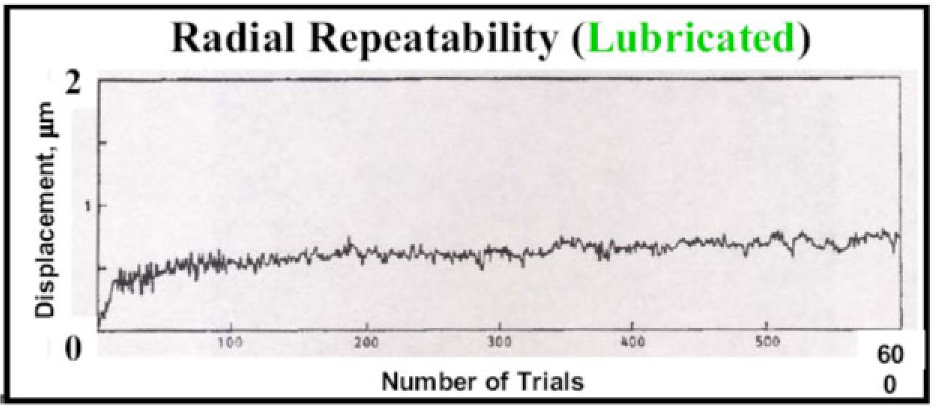
Kinematic hardware



Kinematic location

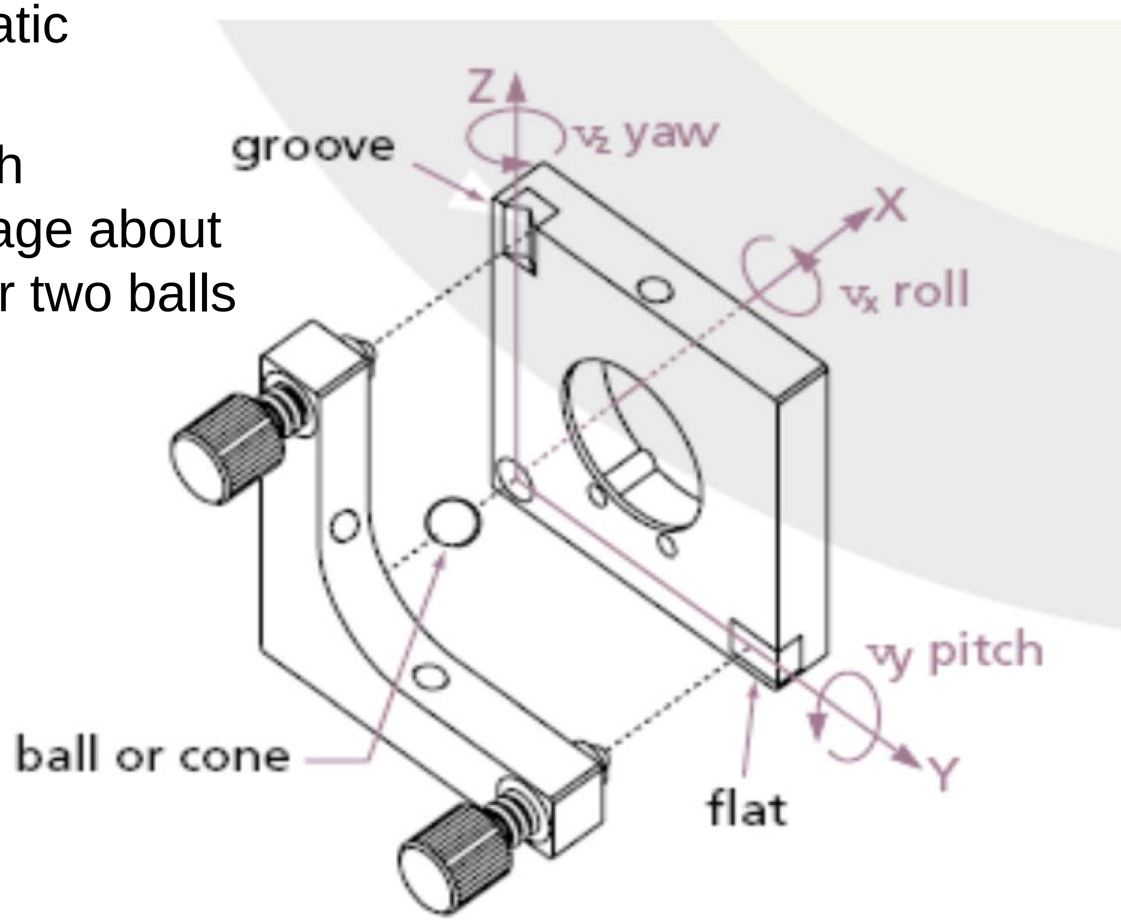
- Since the point contacts are well defined, the location is repeatable to sub-micron.
- Depends on friction, surface finish, loads.





Application of kinematic constraint for precision motion

- For three balls fixed, kinematic constraint
- Move one ball at a time (with micrometer) to rotate the stage about the axis defined by the other two balls
- Very stable
- Smooth motion



(Not shown, springs that hold this together)

Application of kinematic concepts for motion control

5 DoFs constrained using kinematic principles

Remaining DoF is used for the motion

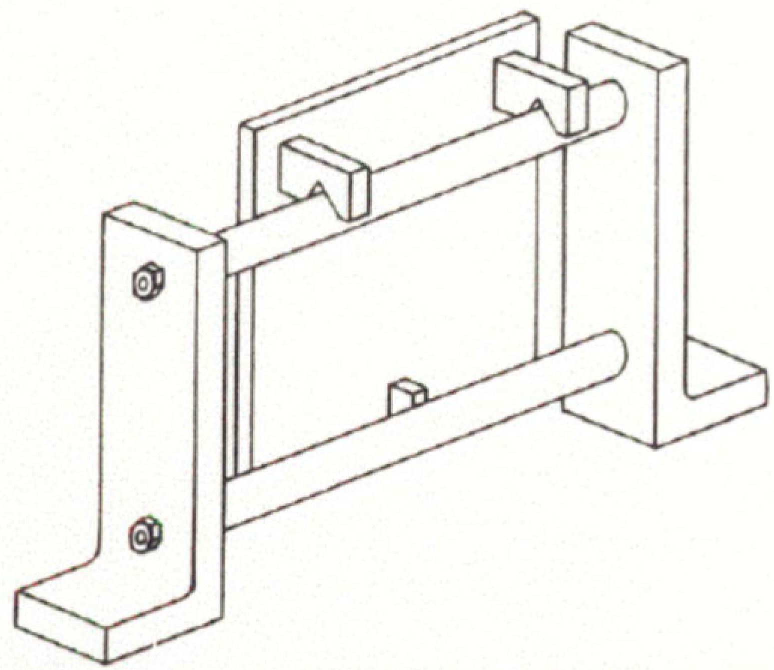
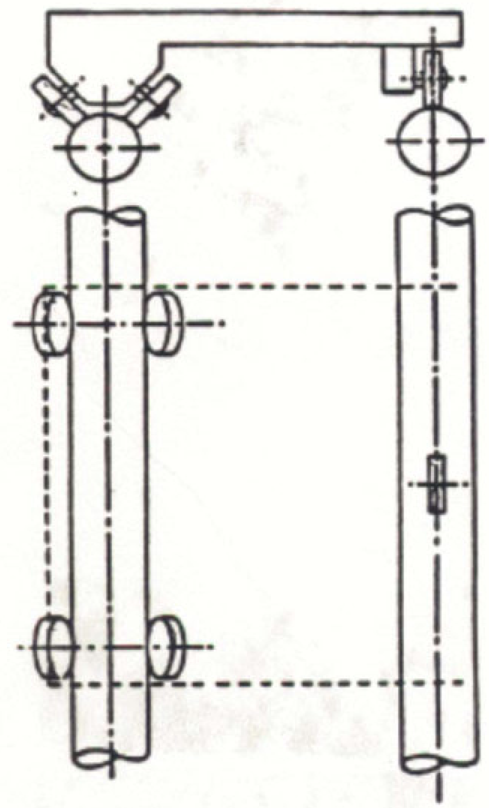


FIG. 6.2.—Kinematic design employing cylindrical surfaces as guides. Such surfaces can be accurately made.



Problems with point and line contact

Nominally, the contact area is **zero** for a point or line

Really, the contact area comes from deformations and depends on the geometry and material properties.

More force causes more deformation which increases the contact area.

Non-point contact = not purely kinematic

Stiffness = Force required for displacement is very low for the unloaded case. and very nonlinear. Preload is required.

Increased preloading makes stiffer, more stable interface in normal direction

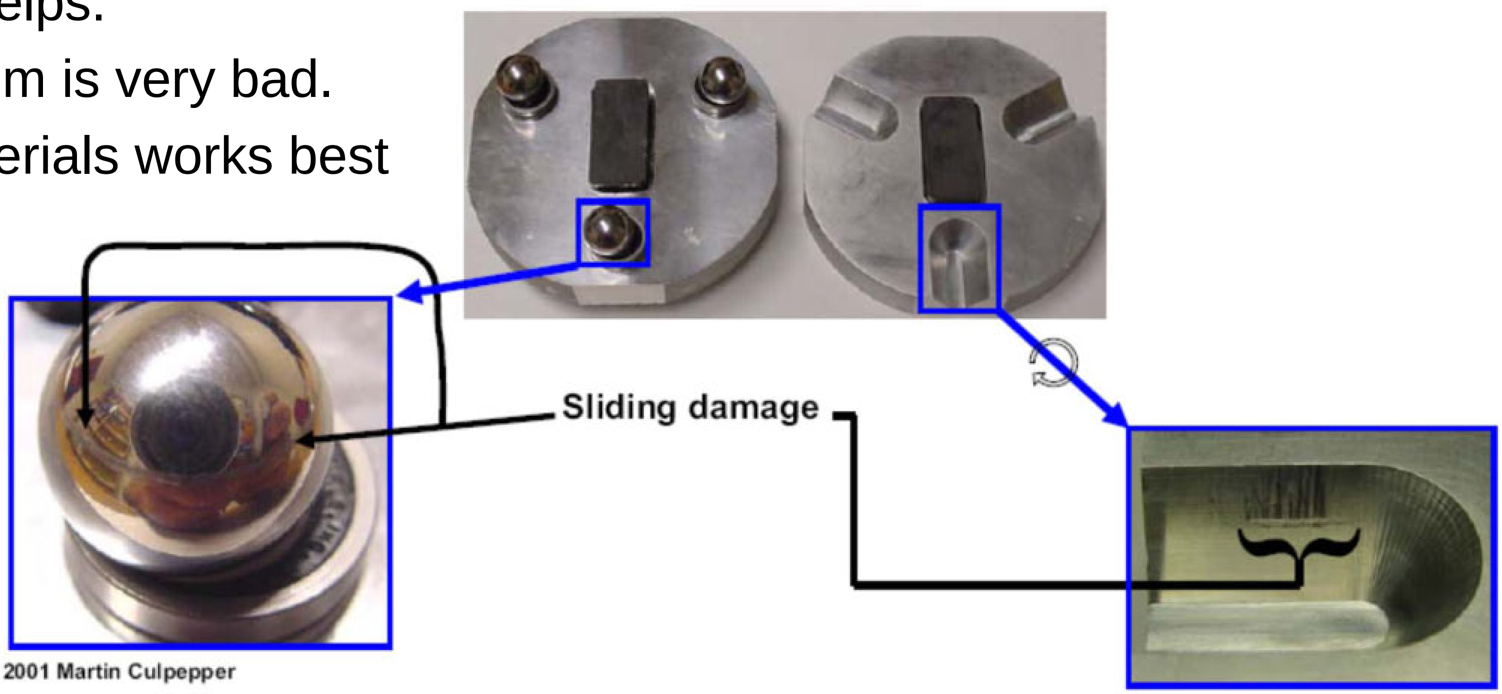
But:

Stress = Force/Area is very high and can damage the materials

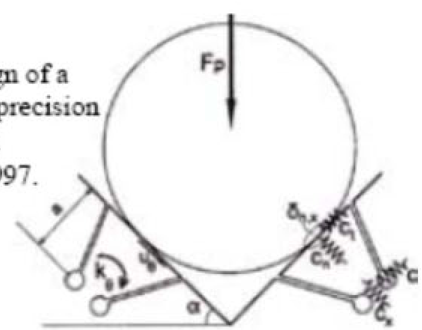
Tangential effects due to friction can be large

Effect of contact stress

- Contact stress can cause fretting of the surface
- Lubrication helps.
- Bare aluminum is very bad.
- Different materials works best



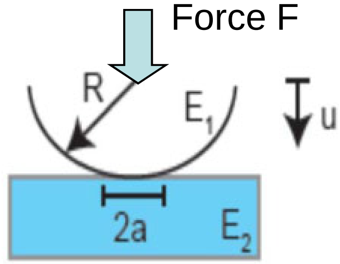
Picture from:
Schouten, et. al., "Design of a kinematic coupling for precision applications", Precision Engineering, vol. 20, 1997.



Ball in V-Groove with Elastic Hinges

Hertz Contact Stress. Ball on Flat

- Elastic deformation defined by Hertz contact
- Both the ball and the plane deform, increasing the contact area to diameter $2a$



$$a \cong \left(\frac{3}{2} \frac{FR}{E_*} \right)^{1/3}$$

$$u \cong \left(\frac{2F^2}{E_*^{\,2} R} \right)^{1/3}$$

$$\frac{1}{E_*} = \frac{1}{2} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)$$

Stiffness

$$k = \frac{dF}{du} \cong \left(E_*^{\,2} R F \right)^{1/3}$$

Stress

$$\left(\sigma_c\right)_{\max} \cong \frac{3}{2} \frac{F}{\pi a^2} = 0.4 \left(\frac{E_*^{\,2} F}{R^2} \right)^{1/3} = 0.4 \frac{k}{R}$$

$$\tau_{\max} \cong \frac{\left(\sigma_c\right)_{\max}}{3}$$

- Stiffness is zero if there is no preload force
- Decrease stress, maintain stiffness, increase R
- For two convex spheres, radii R_1 and R_2, use

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Hertzian stresses for line contact (cylinder on flat)

- Cylinder radius R, applying force F over length L
- Similar to point contact. width of contact area is 2*b*
- Determine maximum compressive stress and maximum shear stress

$$b \cong \left(\frac{8}{\pi} \frac{R}{E_*} \frac{F}{L} \right)^{1/2}$$

$$\frac{1}{E_*} = \frac{1}{2} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)$$

$$(\sigma_c)_{\max} \cong \frac{2F}{\pi b L}$$

$$\tau_{\max} \cong \frac{(\sigma_c)_{\max}}{3}$$

Just light the point loading case:

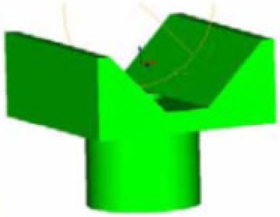
- Stiffness is zero if there is no preload force
- Decrease stress, maintain stiffness, increase R
- For two convex cylinders, radii R_1 and R_2, use

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Use geometry to reduce contact stress

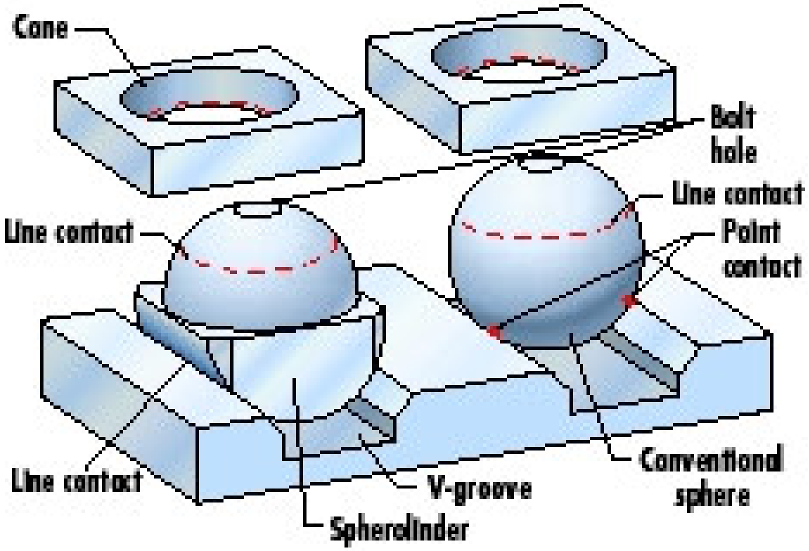
“Canoe ball”
1 meter ROC

(Baltek)



“Spherolinder”

G2 Engineering



Cylinders in V's

- Easy to make to high accuracy
- Leaves axial motion, clocking rotation unconstrained

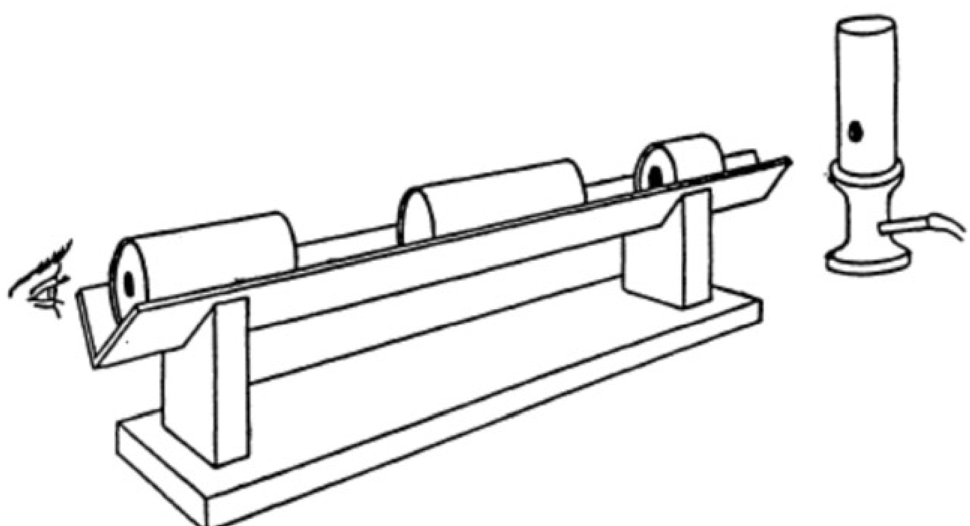
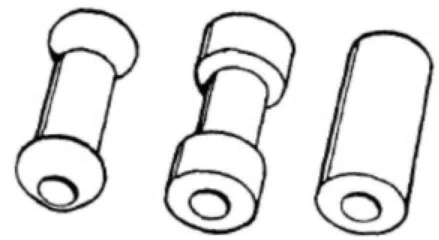


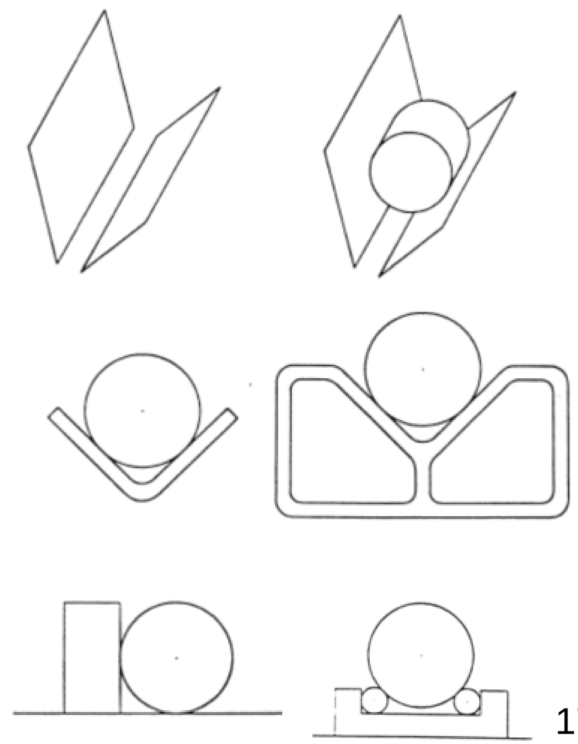
FIG. 59.—V Bench for Lenses of Uniform Diameter.

Fig 35 - From *Treatise on Practical Light*, 1911

cylinders



V's

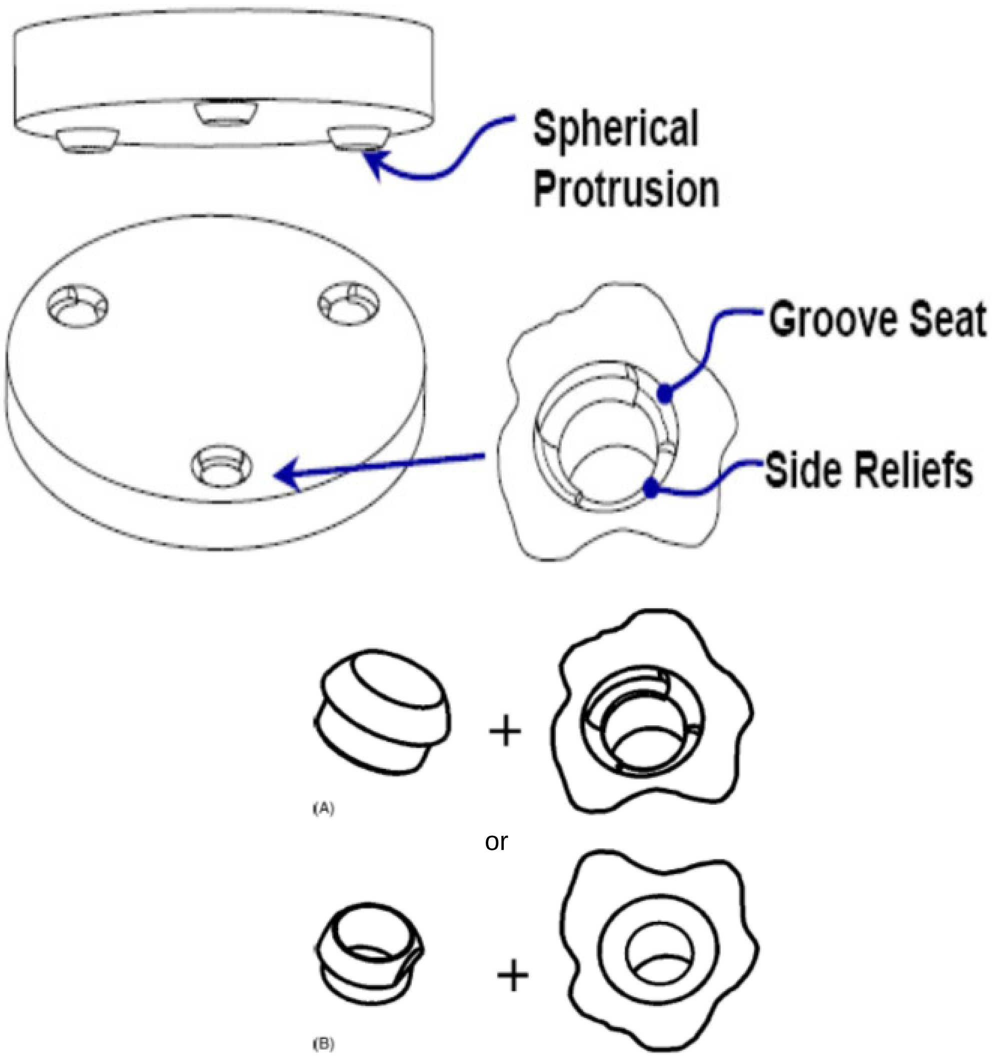


"Cylinders in Vs—An Optomechanical Methodology," Douglas S. Goodman, SPIE Proceedings 3132 *Optomechanical Design and Precision Instruments*, Santa Diego, CA, July, 1997

"More Cylinders in Vs," Douglas S. Goodman, SPIE Proceedings 4198, *Optomechanical Engineering*, Boston, MA, November, 2000

Long radius surfaces to decrease point loading

L. Hale US Patent #6,065,898



Semi-kinematic constraint

Use kinematic concepts, but allow small amount of overconstraint

1. Replace point contacts with “small” contacts
2. Replace idealized constraints with flexures, that use compliance to minimize forces and moments in directions other than the intended constraint.

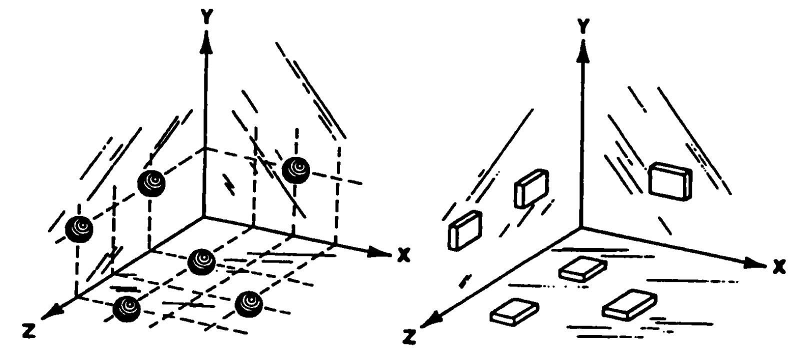


Fig. 2.17 (a) Kinematic and (b) semikinematic position-defining registration surfaces intended for interfacing with a cube-shaped prism (not shown). (Adapted from Smith.[3])

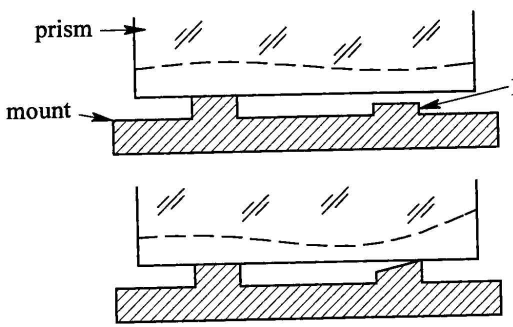


Fig. 2.18 Effects of coplanarity errors on a prism forced into contact with two registration pads of a small area.

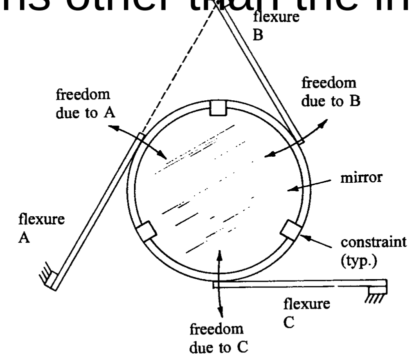


Fig. 9.32 Concept for a flexure mounting for a circular mirror. (From Yoder.[4])

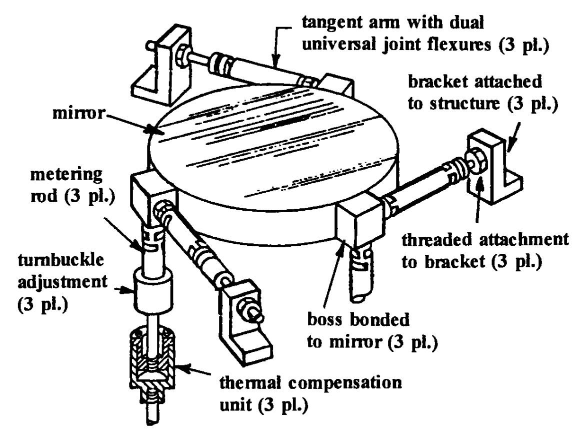
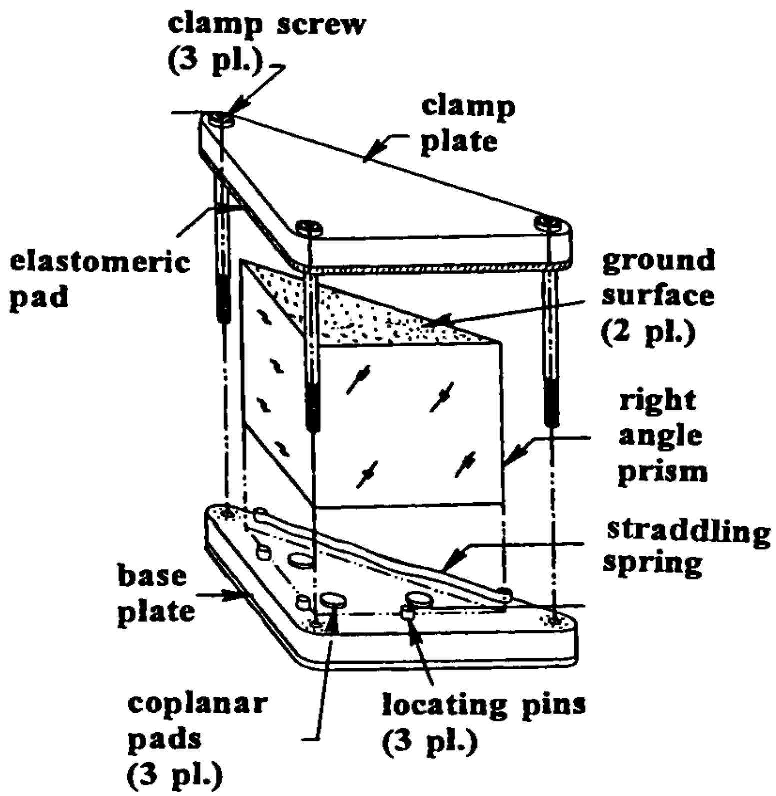


Fig. 9.40 A system of flexures configured to minimize displacement and/or distortion of the optical surface of a modest-sized mirror caused by temperature changes and mounting forces.

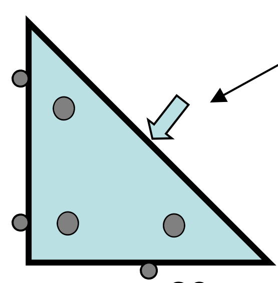
Yoder, *Mounting Optics In Optical Instruments 2nd Ed.*

Semi-kinematic example : Loads vs. constraints



Rigid constraints (6)

These are stiff, so they do not move. The force applied is a *reaction force* necessary to maintain the position



Preload force (2)

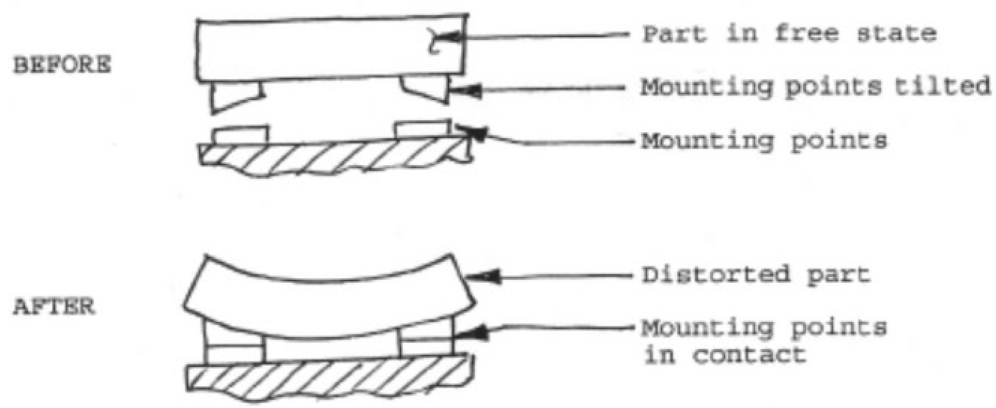
A force is applied, in this case with springs. Ideally, this force is constant. It insures the contacts are in compression, but itself does not constrain position

Yoder, *Mounting Optics In Optical Instruments 2nd Ed.*

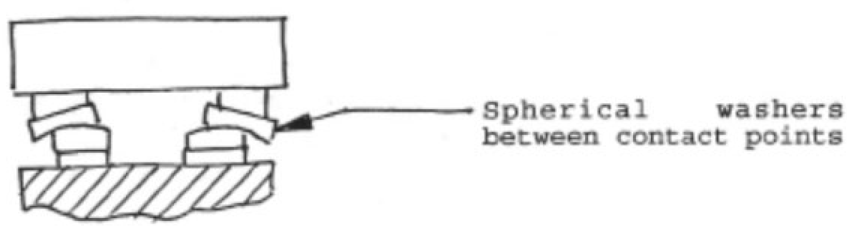
- A major problem with kinematic design is high stress in contact areas. Hertz contact stress theory is used to evaluate this problem.

- If stress is too high, use kinematic principles but replace point contacts with small area contacts. This is known as semi-kinematic design.

- A potential problem with semi-kinematic design is distortion of the part due to non-coplanarity of the mounting points.



- This problem is alleviated by making points very coplanar by introducing rotary compliance in the mounting points.



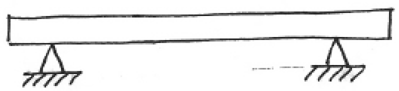
- Assembly procedures for semi-kinematic design are usually critical if part distortion is to be avoided.

(Vukobratovich p. 76)

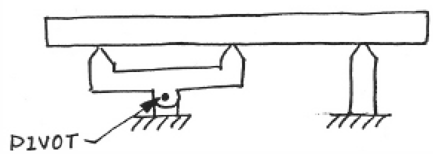
Semi-kinematic using whiffle tree assemblies

- One solution to the multi-point problem is a whiffle tree. This is a cascaded system of support where each level of support is kinematic.
- Consider a simple beam:

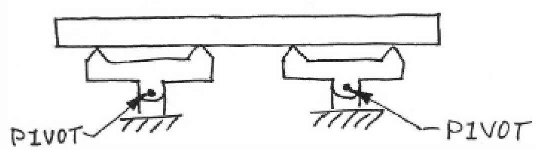
2 point support



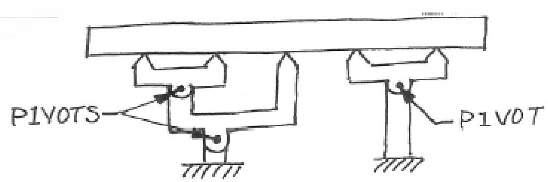
3 point support



4 point support

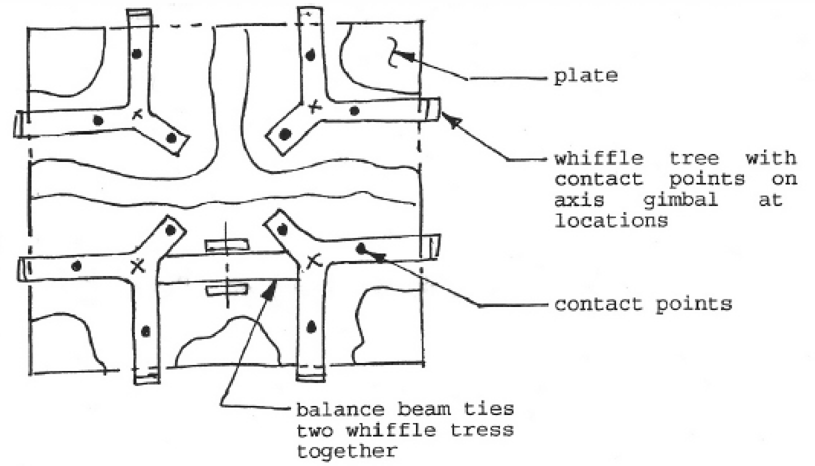


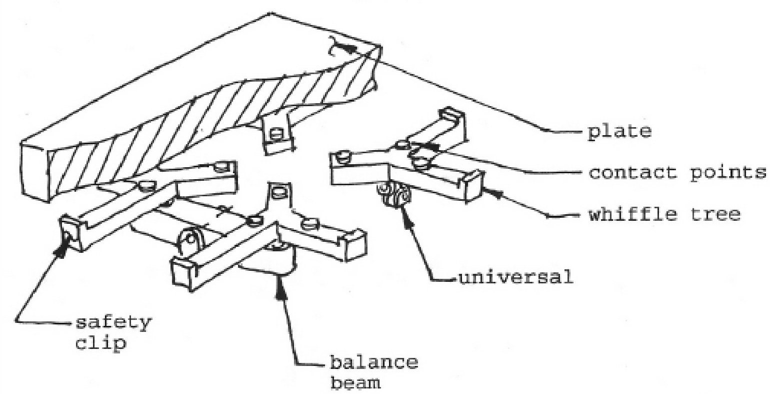
5 point support



NOTE: Pivots insure equal loads on each support.

- The same approach also "floats" plates on whiffle trees.





- Design used on Moore surface plates.

(Vukobratovich)