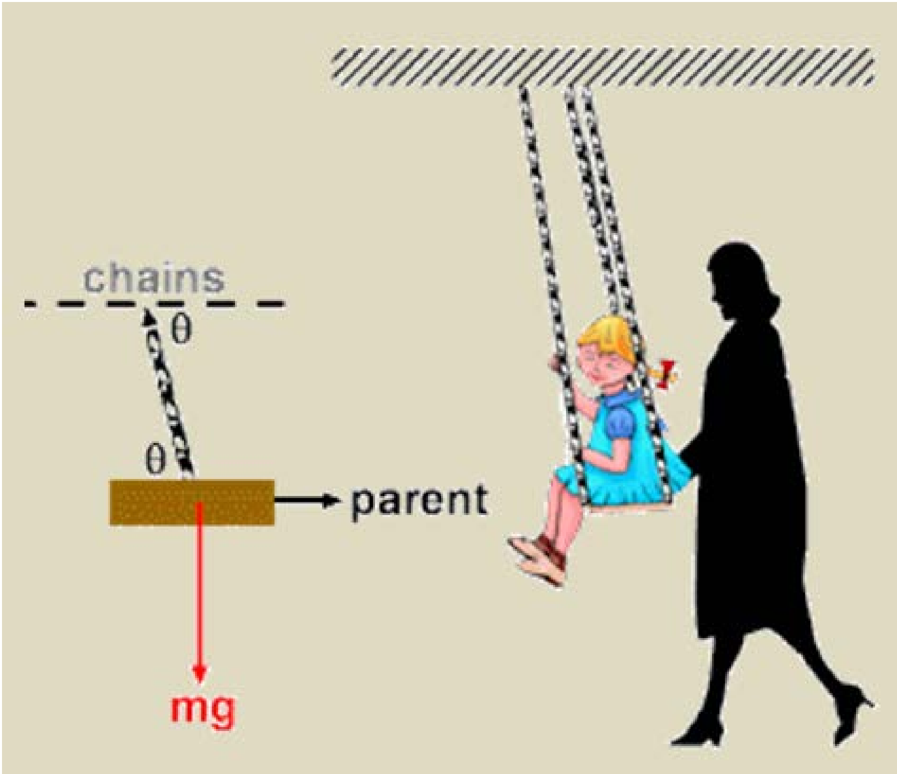
Static Equilibrium

Static Equilibrium Definition:
When forces acting on an object which is at rest are balanced, then the object is in a state of **static equilibrium**.

- No translations
- No rotations

In a state of **static equilibrium,** the resultant of the forces and moments equals zero. That is, the vector sum of the forces and moments adds to zero.



Tolerances for optics are very tight. We need to support them so they are accurately located.

If forces are applied, we want to determine:

Motion

Distortion

In order to do this, we need to evaluate the system, including the applied forces and the reaction forces.

In this section, we define forces and moments, develop the free body diagram, and use the equations of static equilibrium to solve for reaction forces and moments.

Forces are vectors:

They have a magnitude and direction.

What does a force do?

Can accelerate an object F = m a

Can stretch a spring scale

Forces can be applied:

Units of Pounds on Newtons

1 pound (lb_F)= 4.45 N : 1 N = 0.22 lb

Or they can come for gravity

W = m g (g = 9.8 m/s^2 = 386 in/s^2)

1 kg has weigh of 9.8 N or 2.2 lbs

1 lb_M is the mass that weighs 1 pound

1 slug weighs 32.2 lbs

The moment is defined as

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F}_B$$
$$= r_{AB} F_B \sin\theta$$
$$= r_{AB} \cdot F_{\perp}$$
$$= r_{\perp} \cdot F_B$$

Also called "torque"

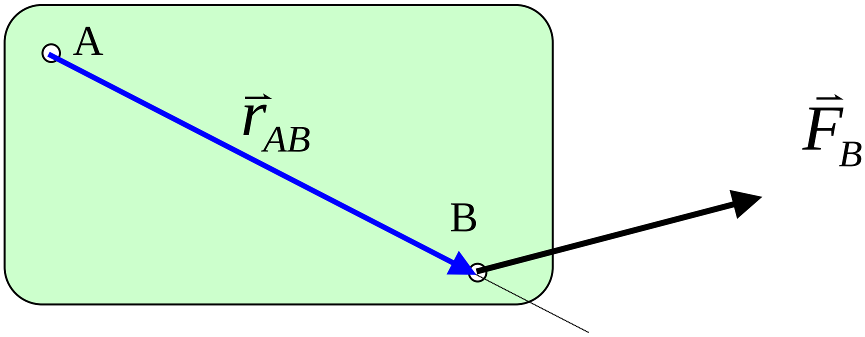
Units are in-lb or N-m

1 N-m = 8.84 in-Lb

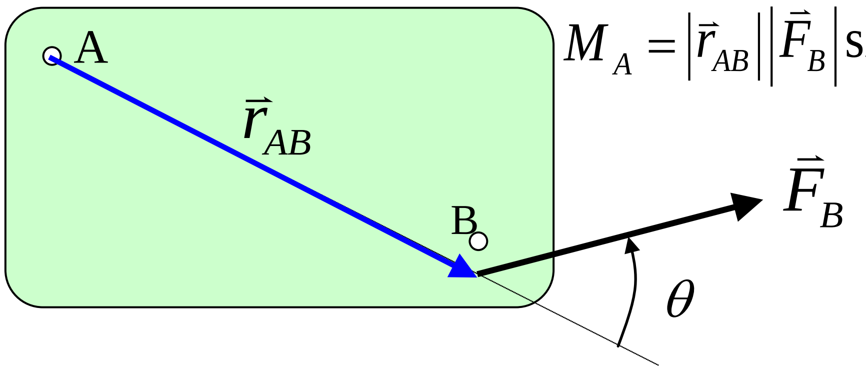
Moments are "twisting forces". They make things rotate

Defining moment from applied force

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F}_B$$

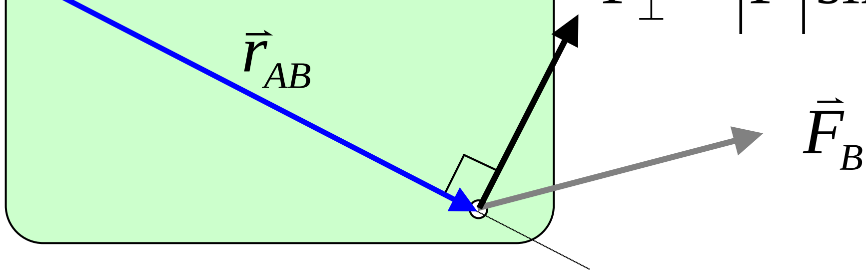


$$M_A = \left|\vec{r}_{AB}\right|\left|\vec{F}_B\right| \sin\theta$$



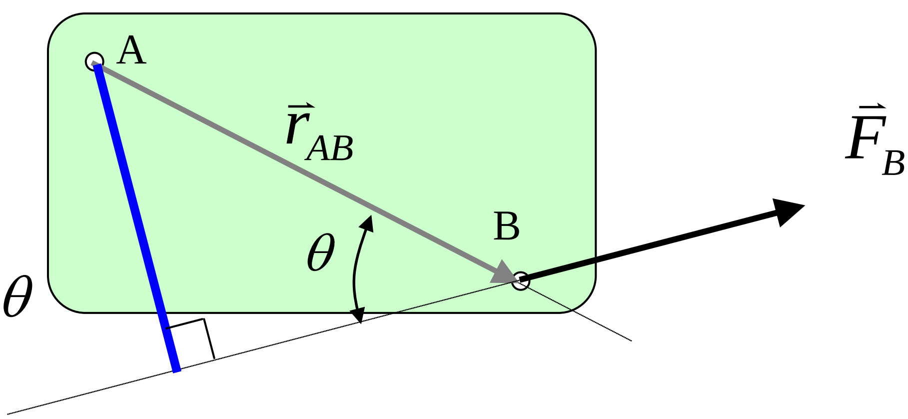
$$F_{\perp} = \left|\vec{F}\right| \sin$$

$$M_A = \left|r_{AB}\right| \cdot F_{\perp}$$



$$M_A = r_{\perp} \cdot \left|F\right|$$

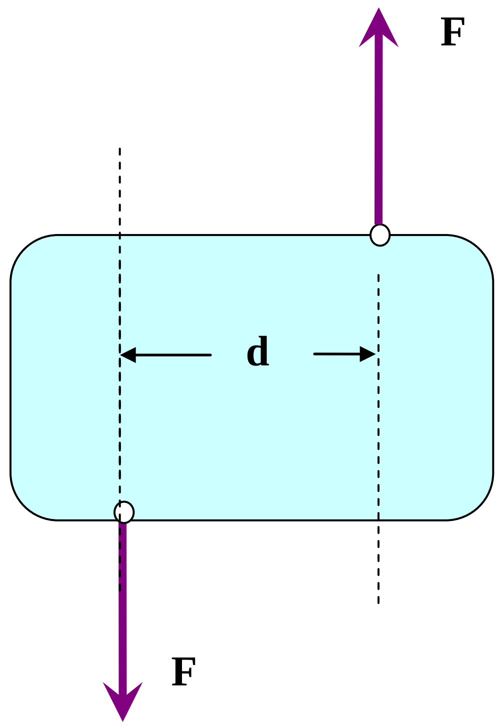
$$r_{\perp} = \left|\vec{r}_{AB}\right| \sin\theta$$



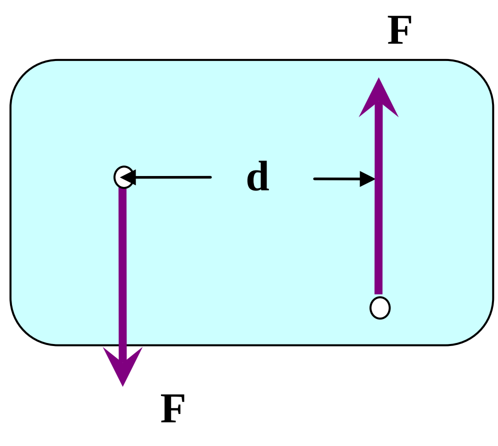
Force couples

Two forces, equal and opposite in direction, which do not act in the same line cause a pure moment

M = F d



M = F d



Simple cases

Cable

Can only transmit tension along direction of cable

No compression

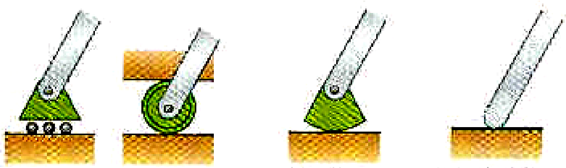
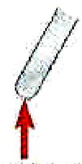
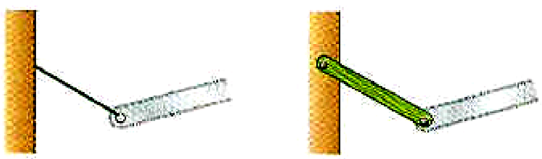
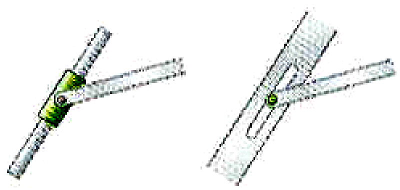
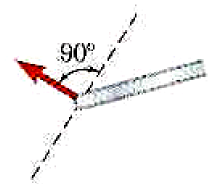
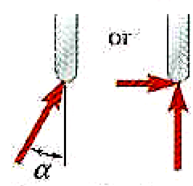
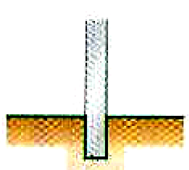
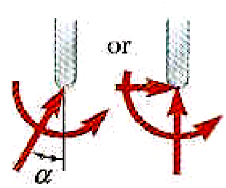
No moment

No lateral force

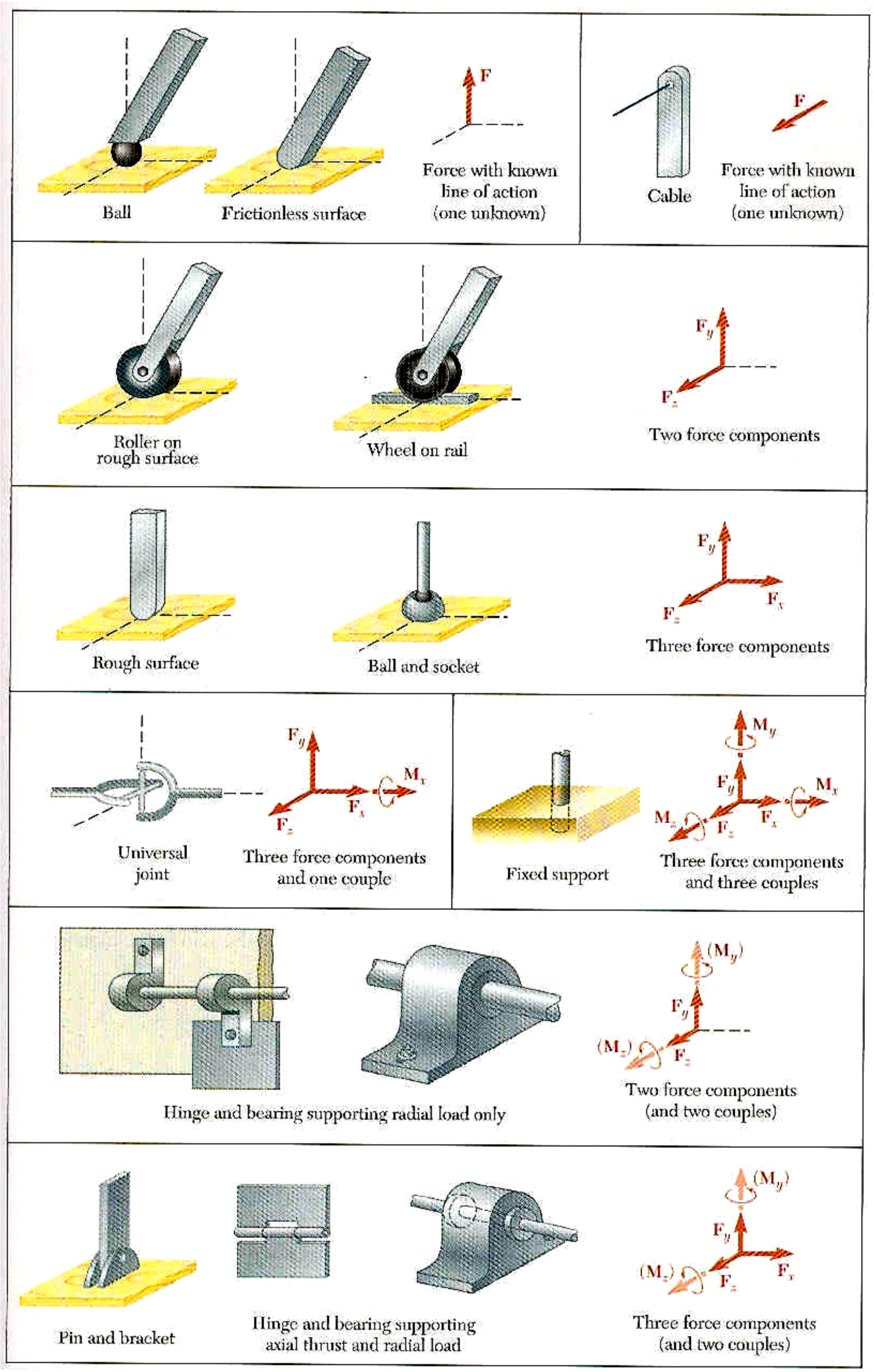
Constraints

Constraints are attachment points that will maintain their position.

<u>Idealization of 2D supports</u>

Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action	1
 Frictionless pin or hinge, Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

Idealization of 3D supports



Free Body Diagrams

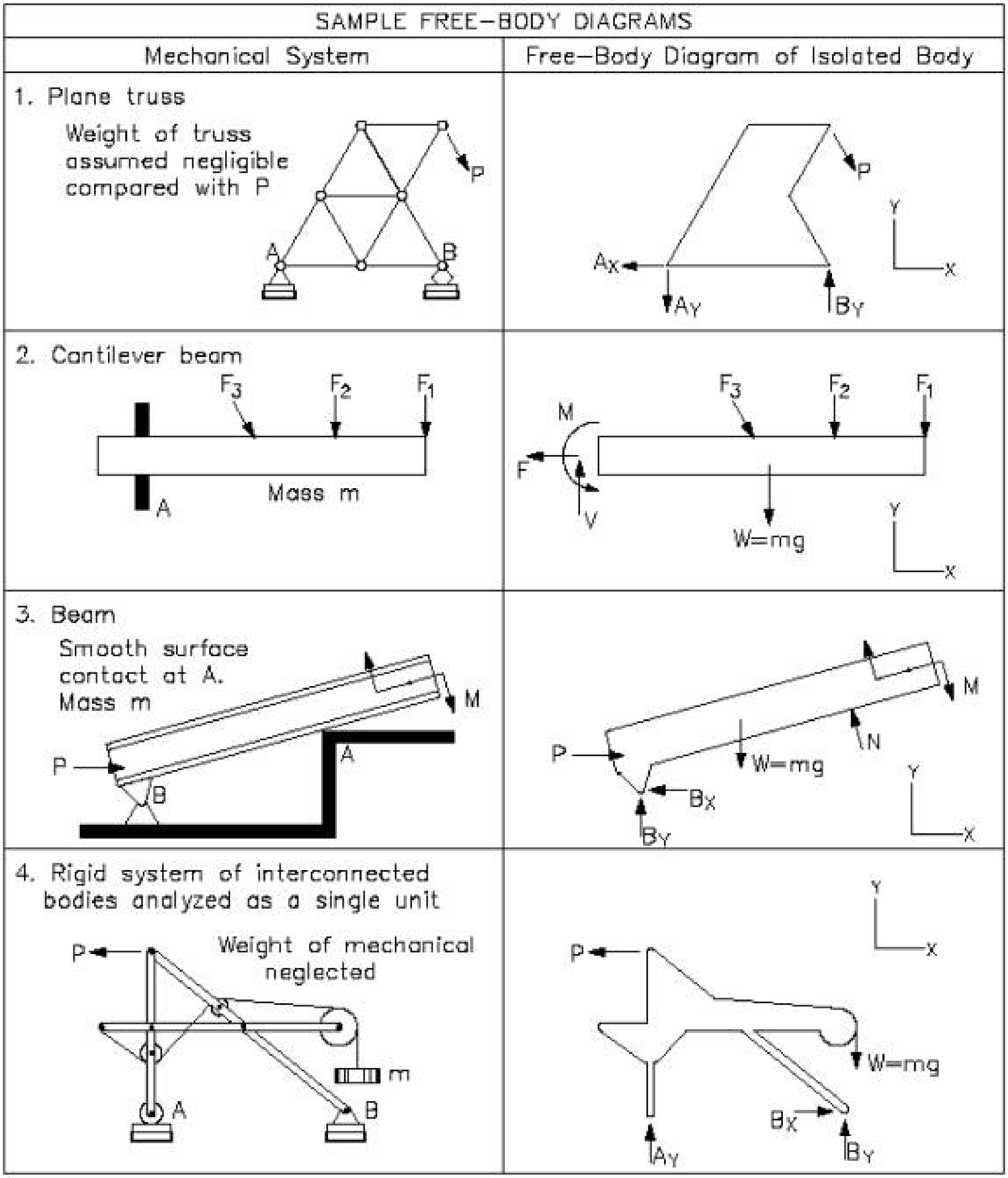
Step 1. **Determine which body or combination of bodies is to be isolated.** The body chosen will usually involve one or more of the desired unknown quantities.

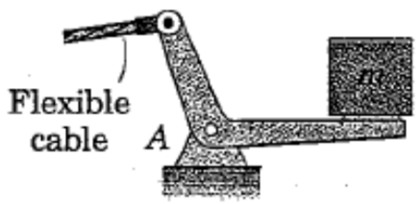
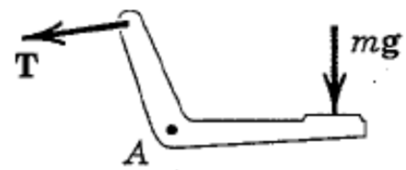
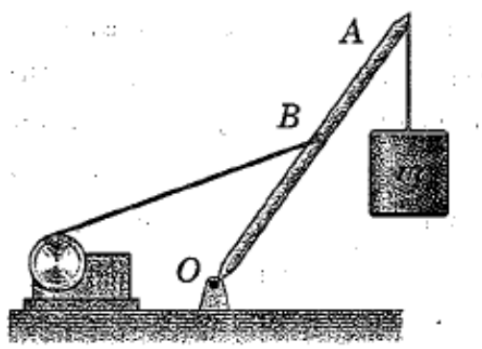
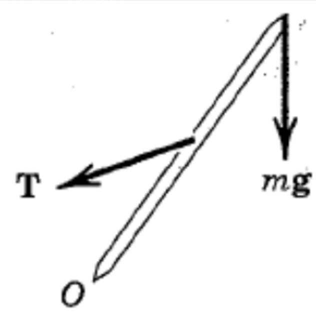
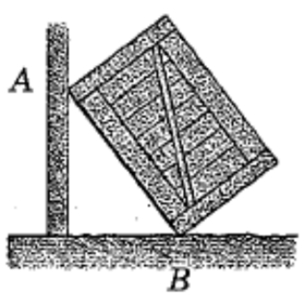
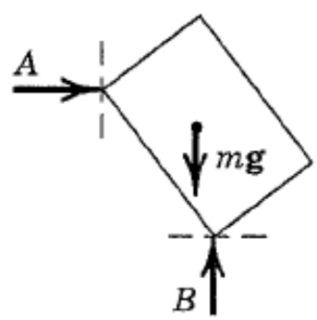
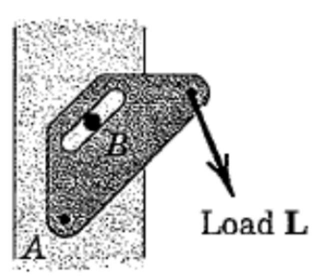
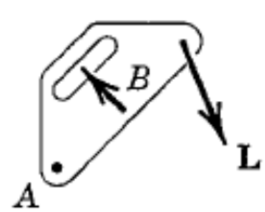
Step 2. Next, **isolate the body** or combination of bodies chosen with a diagram that represents its complete external boundaries.

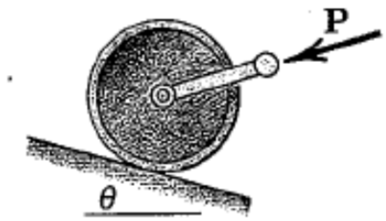
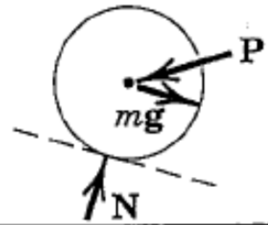
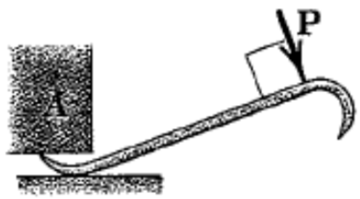
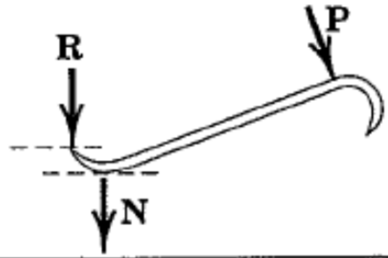
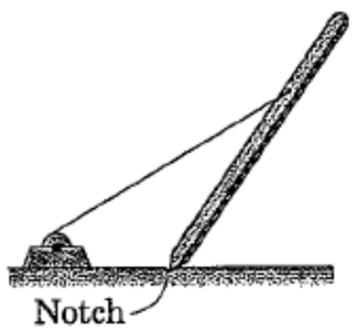
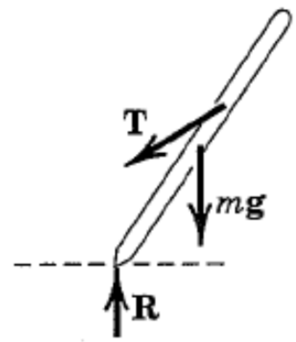
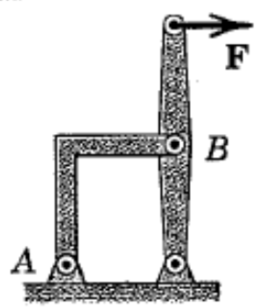
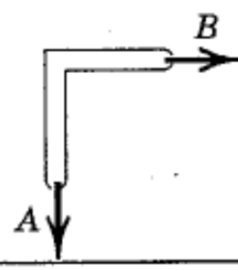
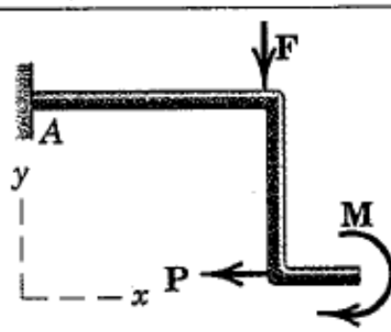
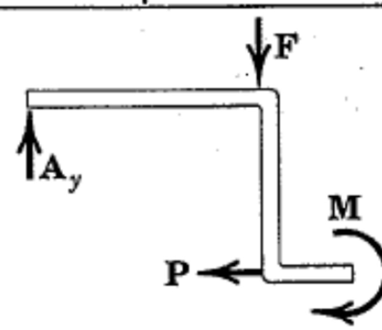
Step 3. **Represent all forces that act on the isolated body** as applied by the removed contacting bodies in their proper positions in the diagram of the isolated body. Do not show the forces that the object exerts on anything else, since these forces do not affect the object itself.

Step 4. **Indicate the choice of coordinate axes** directly on the diagram. Pertinent dimensions may also be represented for convenience. Note, however, that the free-body diagram serves the purpose of focusing accurate attention on the action of the external forces; therefore, the diagram should not be cluttered with excessive information. Force arrows should be clearly distinguished from other arrows to avoid confusion.

When these steps are completed a correct free-body diagram will result. Now, the appropriate equations of equilibrium may be utilized to find the proper solution.

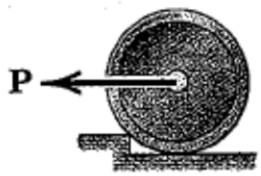

SAMPLE FREE–BODY DIAGRAMS
Mechanical System
Free–Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
A
B
P
Ax
Ay
By
Y
X
2. Cantilever beam
F3
F2
F1
A
Mass m
M
F
V
W=mg
3. Beam
Smooth surface contact at A.
Mass m
P
B
N
W=mg
Bx
By
4. Rigid system of interconnected bodies analyzed as a single unit
Weight of mechanical neglected
m
W=mg
Ay

	Body	Incomplete *FBD*
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

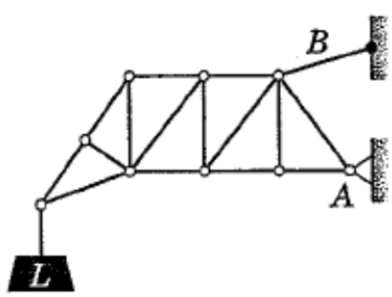
	Body	Wrong or Incomplete *FBD*
1. Lawn roller of mass *m* being pushed up incline *θ*.		
2. Prybar lifting body *A* having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass *m* being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; Pin joints.		
5. Bent rod welded to support at *A* and subjected to two forces and couple.		

1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.

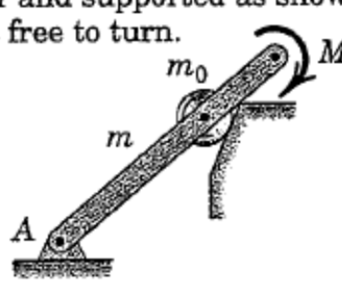
2. Wheel of mass m on verge of being rolled over curb by pull **P**.



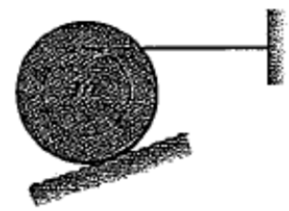
3. Loaded truss supported by pin joint at A and by cable at B.



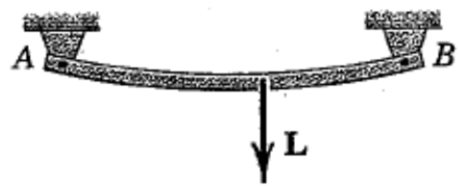
4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.



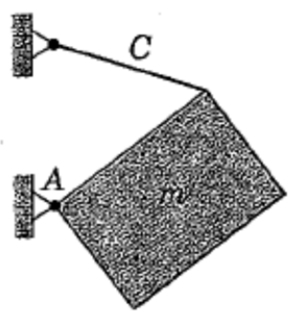
5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.



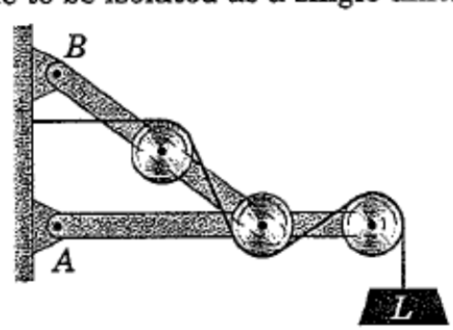
6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.



7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.



For a rigid body to be static, the net sum of forces and moments acting on it must be zero.

$$\sum \vec{F} = 0 \qquad \sum \vec{M} = 0$$

$$\sum F_x = 0 \qquad \sum M_x = 0$$

$$\sum F_y = 0 \qquad \sum M_y = 0$$

$$\sum F_z = 0 \qquad \sum M_z = 0$$

In general six equations, in the plane this reduces to 3

$$\sum F_x = 0$$

$$\sum F_y = 0$$

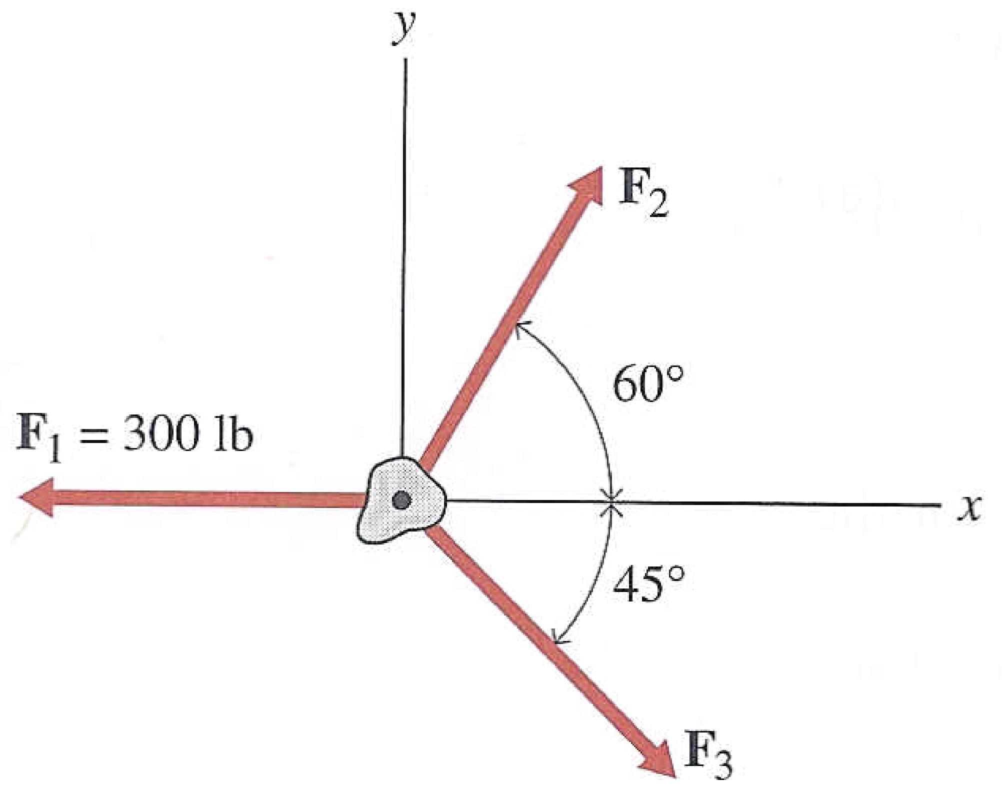
$$\sum M = 0$$

Solving Statics problems

Determine reaction forces for static equilibrium.

1. Draw Free Body Diagram
   Decide if the problem is solvable
   a. How many unknowns?
   b. How many equations can you write?
2. Write equations to sum forces and moments to be 0
   a. Use reaction forces as unknowns
   b. Be smart about coordinates and choice of points for summing moments
3. Solve equations for reaction forces
4. Check your answer and the direction

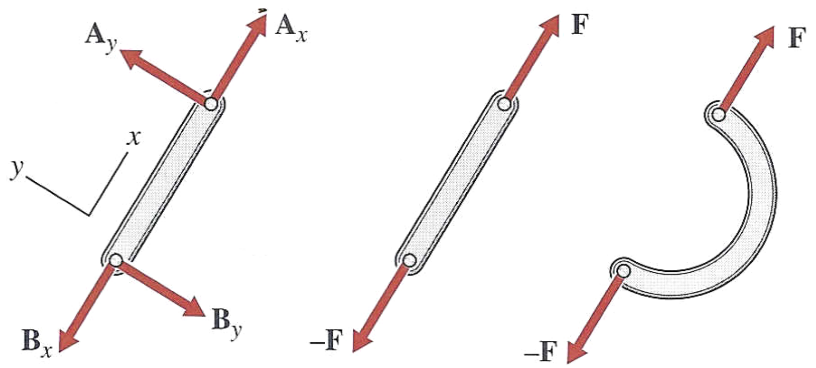
2D Particle Example



- **Determine magnitude of F_2 and F_3**

Link Pin joint at both ends

Equilibrium requires that the forces be equal, opposite and collinear.



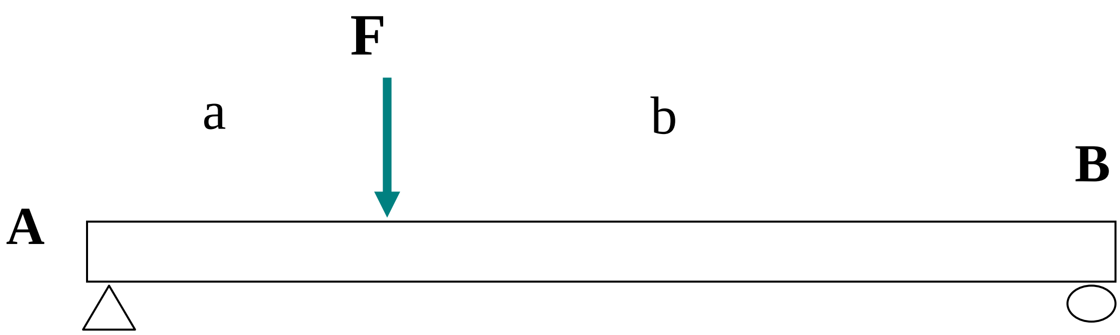
Therefore, for this member $A_y = B_y = 0$

Pin joint will not transmit a moment

Simple Examples

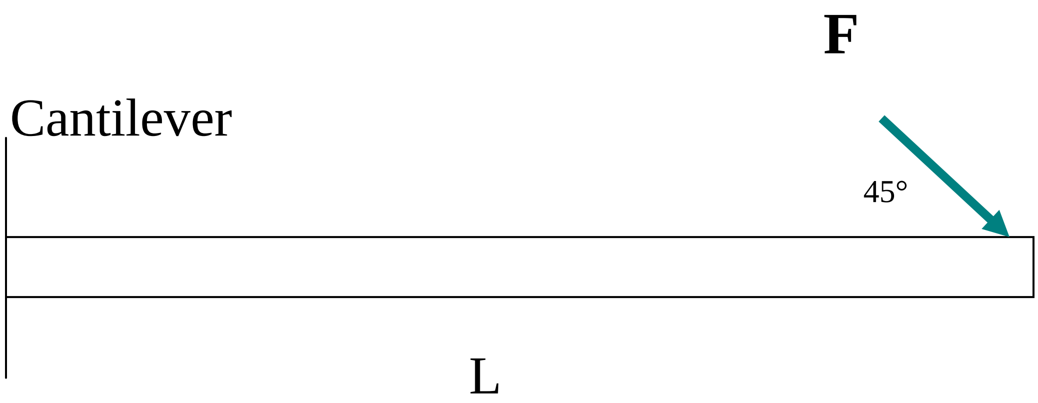
Determine reaction forces and moments:

Simple support

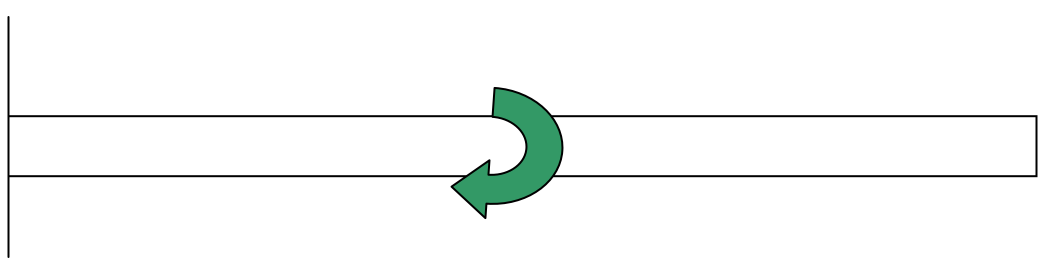


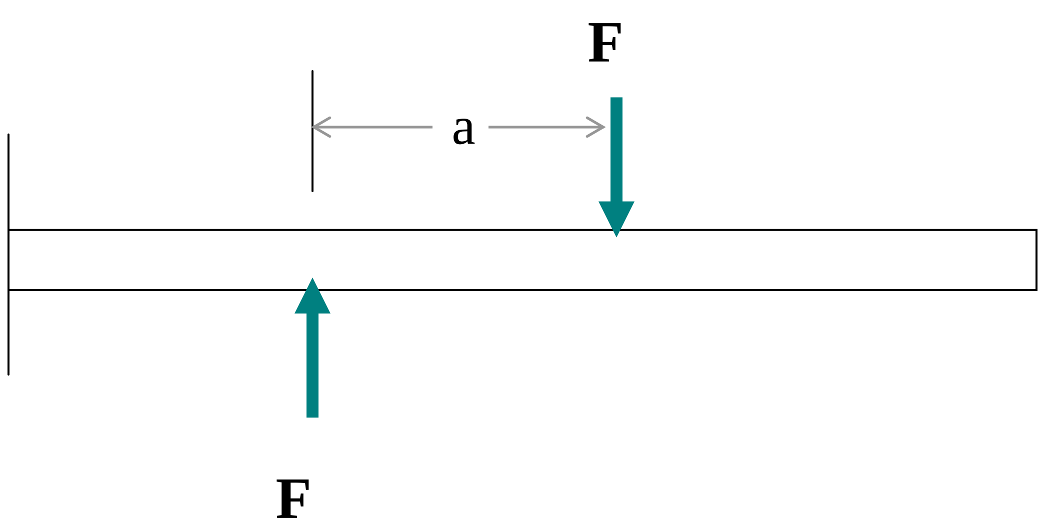
Cantilever

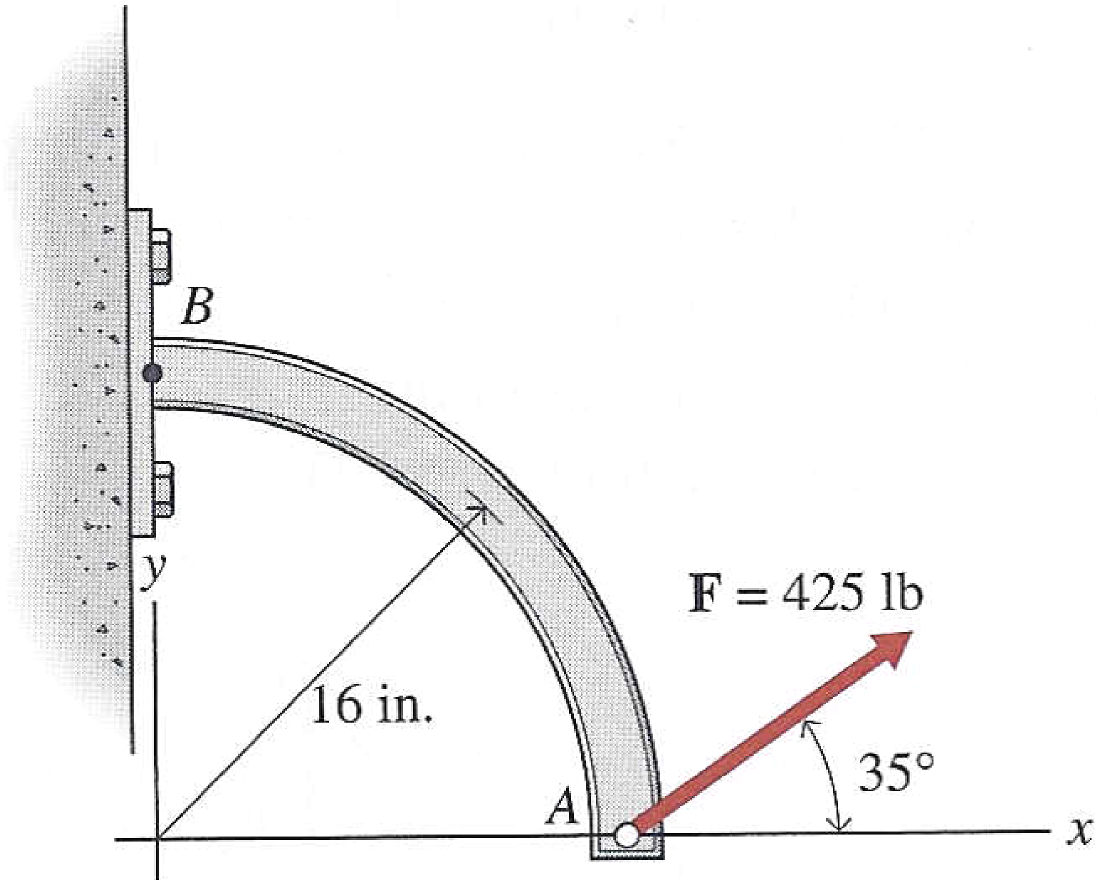
X and Y components



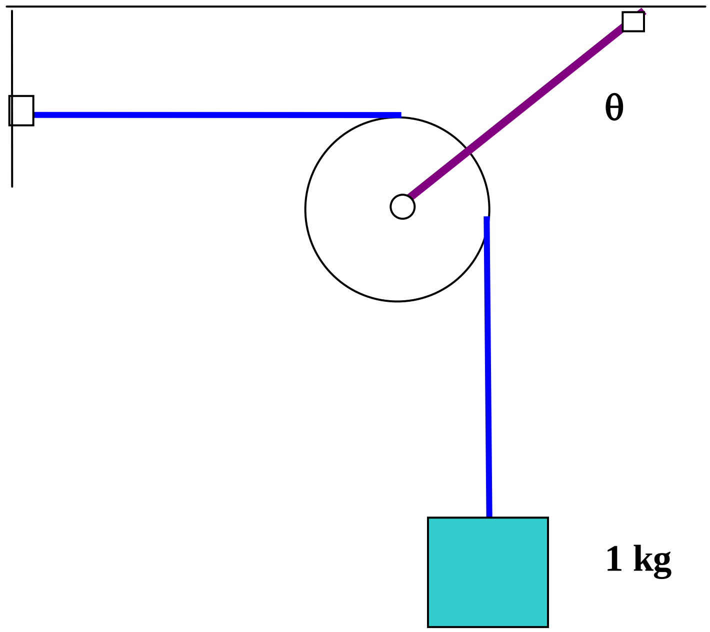
Reaction from moments



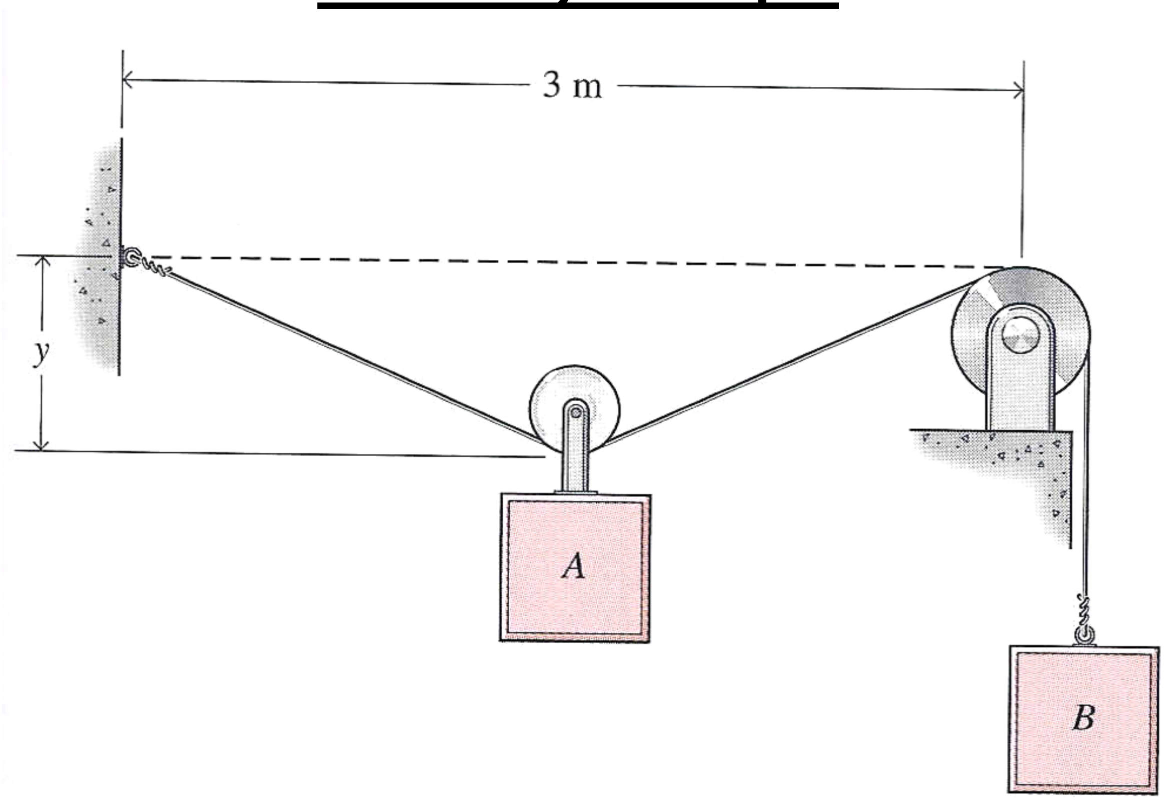



B
y
16 in.
F = 425 lb
35°
A
x

Example: Hanging a mass, using a pulley



2D Pulley Example



Specifications:

- Mass of block A = 22 kg
- Mass of block B = 34 kg

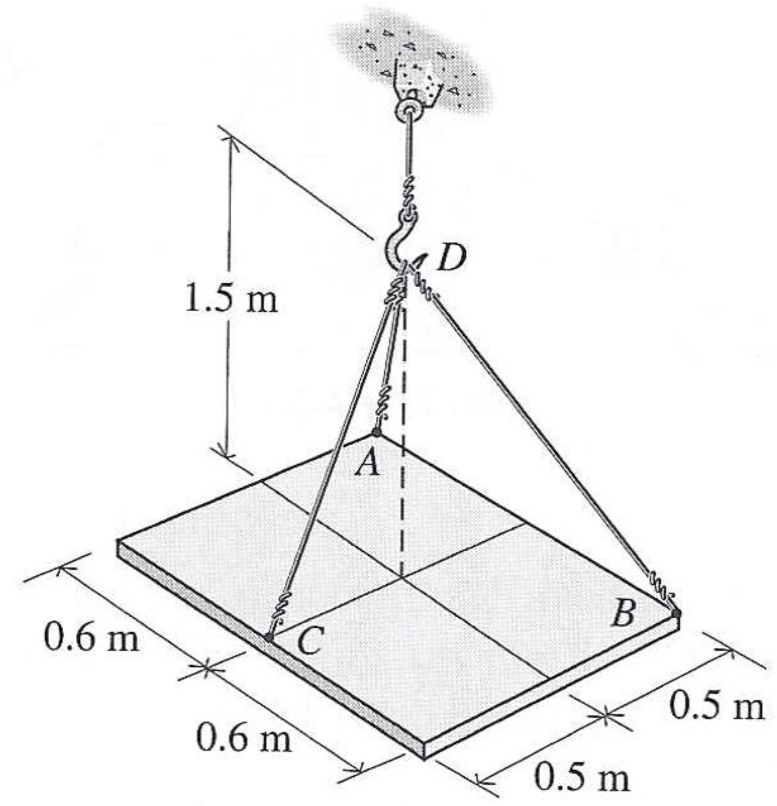
Assumptions:

- Pulleys are frictionless
- Block A is free to roll
- Cable system is continuous

Determine:

- Displacement “y” for equilibrium

3D Cable System Example



Specifications:

- Weight of plate = 250 lb

Assumptions:

- Plate is homogeneous

Determine:

- Force in each supporting cable

Use direction cosines

Overconstraint

Each body has a total of 6 degrees of freedom that define its position

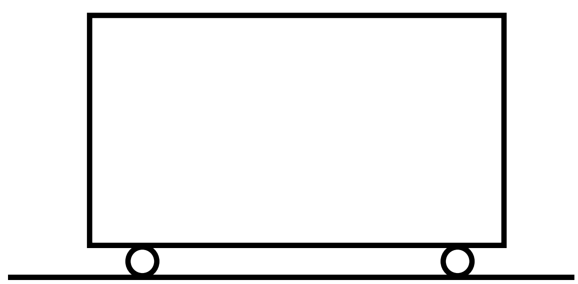
Such as x, y, z, θ_x, θ_y , θ_z

These lead to 6 Equations that can be used to solve for reaction forces:

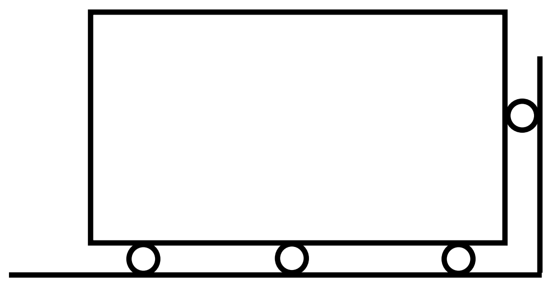
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

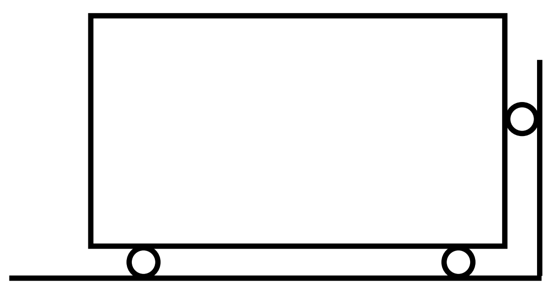
If the mechanical constraints provide an attachment so that one or more degrees of freedom are free, the body is underconstrained



If the mechanical constraints provide an attachment so that there is no unique solution for the reaction forces, the body is overconstrained



A body that is neither overconstrained nor underconstrained is called static determinant



Static equations must have 6 unknowns for 3-space, or 3 unknowns for in-plane

If you are not sure, then try solving for the reaction forces and moments.

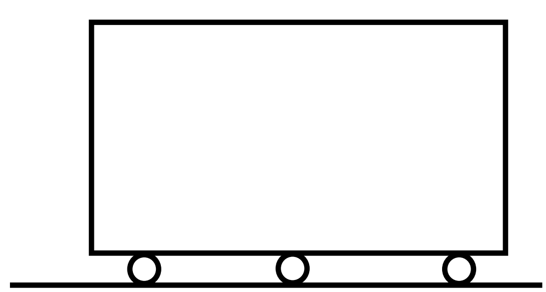
If you have a unique solution
static determinant

If you have multiple solutions (more unknowns than equations)
Overconstrained
reaction forces can be pushing against each other

If you have more equations than unknowns
Underconstrained
Some degree of freedom is not constrained and could move

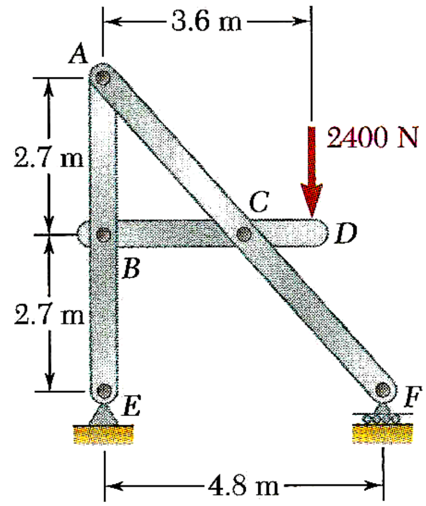
Try to figure out what degree of freedom has not been constrained.

You can be overconstrained and underconstrained at the same time!



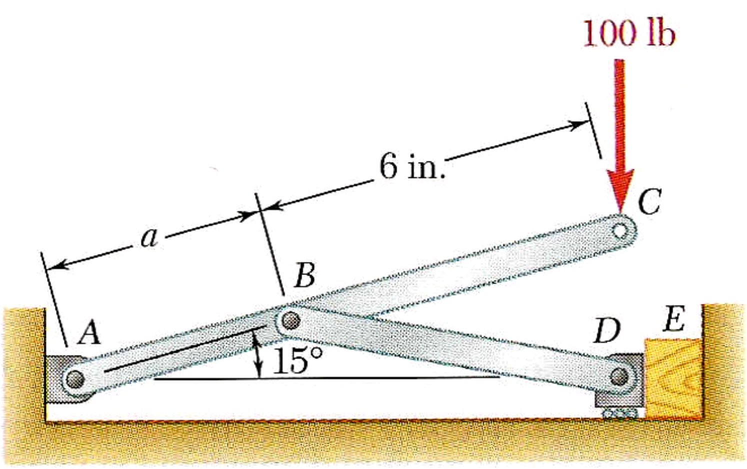
Frames

- Designed to support loads.
- Typically rigid, stationary and fully constrained.
- Contains at least one multi-force member.



Machines

- Designed to transmit or modify forces.
- Contain moving parts.
- Contains at least one multi-force member.



Analysis of Structures – Method of joints

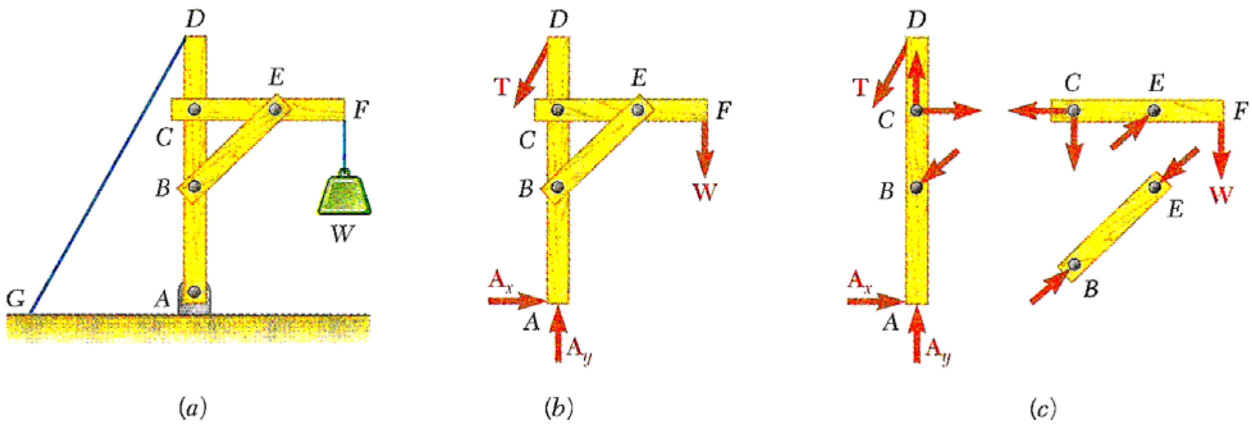


Figure (a) – Crane example

Figure (b) – Free body diagram of crane showing external forces.

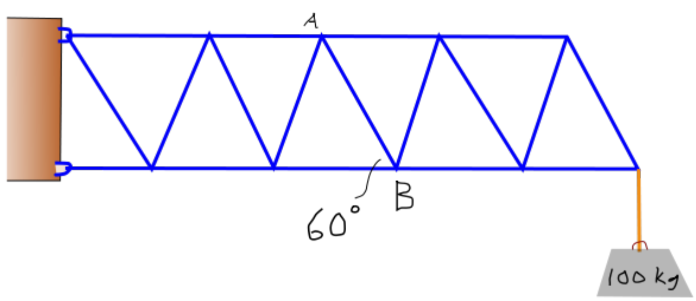
Figure (c) – Dismembered crane showing member forces. From the point of view of the structure as a whole, these forces are considered to be internal forces.

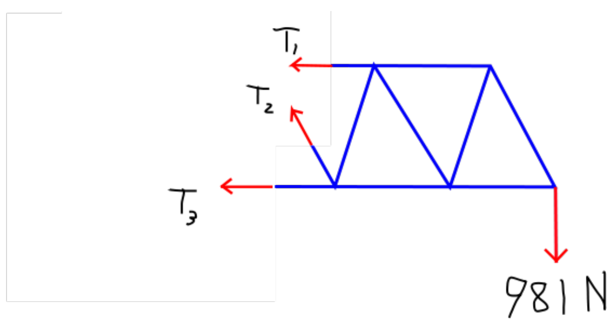
The internal forces conform to Newton’s third law – the forces of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense.

When structures, like the one shown above, contain members other than two force members, they are considered to be frames or machines. Typically, frames are rigid structures and machines are not.

Analysis of structures – Method of sections

Divide structure along sections, rather than joints.
Solve for equilibrium.





$$\sum F_x = 0 = -T_1 - T_2 cos(60) - T_3$$

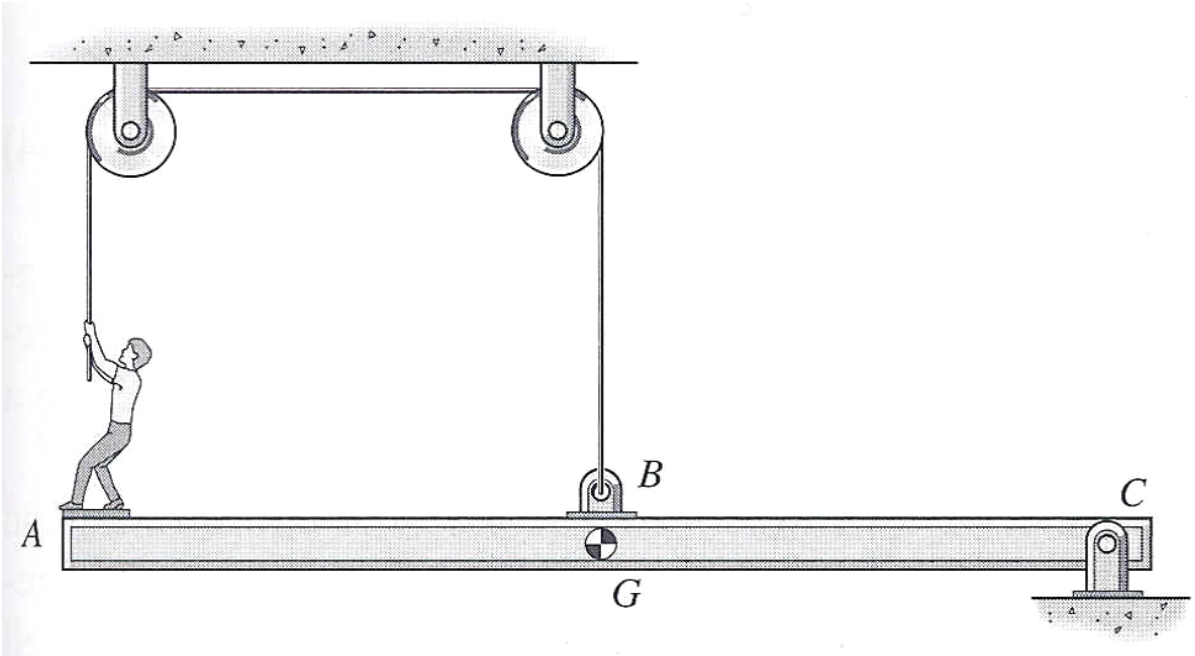
$$\sum F_y = 0 = T_2 sin(60) - 981N$$

The equation summing forces in the Y direction only has one unknown because all cut members except A-B are horizontal.

$$981N = T_2 sin(60)$$

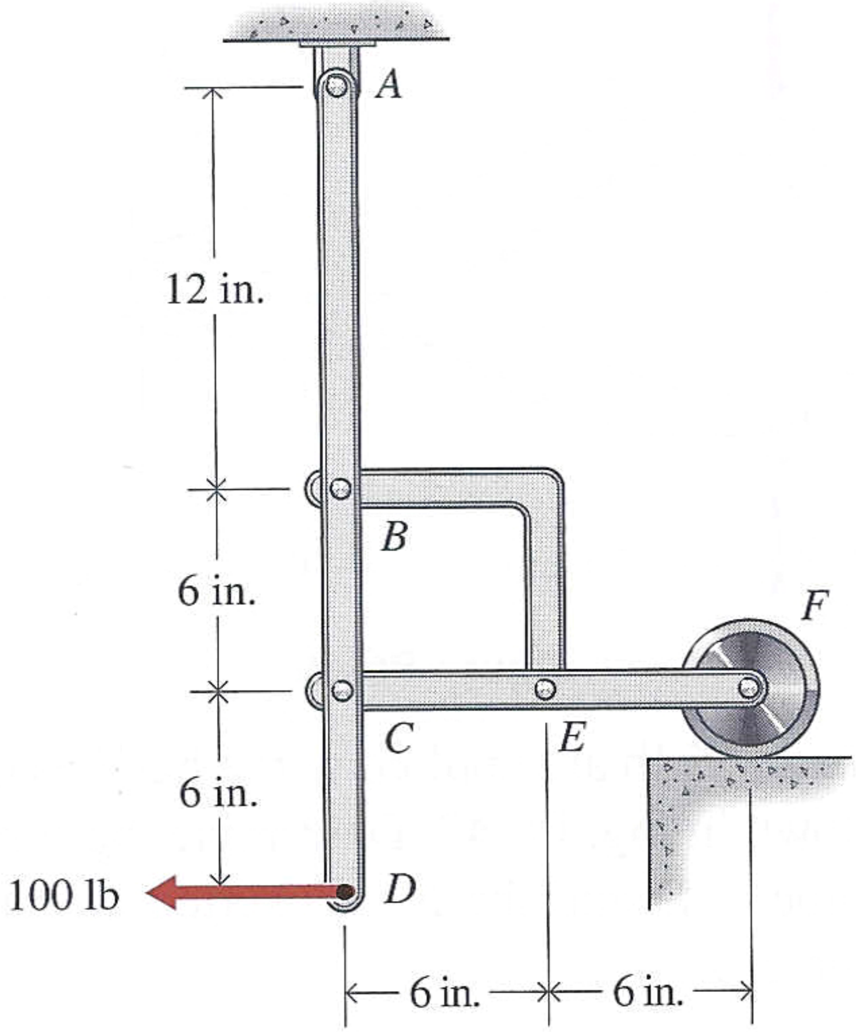
$$T_2 = 1132.761N$$

Because T_2 is positive, member A-B is in 1133N of tension



Assuming the beam does not fall, what is the direction of the force applied to the beam at C?

Example



Determine the forces acting on member ABCD.