Quantum Computation with Electrons Trapped on Liquid Helium

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Electron trapped on liquid He forms a unique two dimension system and is actively explored for qubit architecture. This paper discusses the fundamentals of trapped electron in liquid He and reviews the latest developments and challenges in this technology.

I. INTRODUCTION

Useful quantum computing and quantum information applications require robust, high-fidelity and massively scalable qubit technologies. To this end, qubits based on trapped ions [1] , quantum dots [2] , superconducting Josephson junction [3] , linear optical quantum systems [4] , nuclear spin of atoms doped in silicon [5], and topological states [6] are actively researched. Technological maturity, and figures of merit of each qubit technology is varied and clear winner has not emerged, as yet. Qubit count, gate fidelity, and decoherence time of qubit-state are figures of merits often used in comparing different qubit architectures. Fidelity is the signal to noise ratio of a quantum gate and higher fidelity desired. Decoherence is the time period where a unique qubit-state is held persistently in a qubit without the qubit-state thermalizing to the noise. Longest possible decoherence time is desired. Higher fidelity and long decoherence increase depth and breath of quantum algorithms for useful quantum computing application. Liquid-He-4 (LH4), where He^4 becomes a superfluid below $T_\lambda = 2.2$ K, is another emerging platform for qubits. Unlike the naturally abundant He^3, He^4 has zero nuclear spin and is an excellent system for electron spins qubits with long decoherence times.

Qubits are formed in LH4 when single electrons are trapped on the surface of LH4 by a columbic attraction to a positive image potential induced in the LH4. Such an electron cannot fall into the LH4 due to a 1-eV potential barrier between the He atoms and an electron. Within such a potential barrier the trapped electron forms hydrogen like eigenstates. The energy or spin of the trapped electron is tuned between two lower laying eigenstates forming a qubit. The spin or vibrational state of each trapped electron is interrogated by spectroscopy or by microwave photons within a cavity electrodynamic (QED) architecture similar to more mature superconducting Josephson junction qubits or trap-ion quantum systems. Robust and scalable methods have been developed to set and measure the quantum state of an electron trapped on LH4 during the past several decades. Even though trapped electrons in LH4 have been considered for long time, the use of these trapped electrons as qubits states is taking this technology to a new Renaissance period. There are currently technology startups which are exploring electrons-trapped in LH4 for building a commercial quantum computer. One reason for this excitement is that qubits formed on trapped electrons on the surface of LH4 has the smallest possible decoherence where qubit states can live up to into millisecond time scale which is far above other qubit technologies. However, unlike the mature qubit technologies, there no quantum gates have been thus far been fabricated using trapped electrons on LH4. This paper discusses the fundamentals of trapped electron in LH4 and reviews the latest developments and challenges in this technology.

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II. THEORY

LH4 is testbed for fundamental investigations in many-body physics, Quantum Electrodynamics, mesoscopic physics, charge and polaron transport, relativistic physics, topology, symmetry, and even quantum-gravity research.[7] An electron when bought close to the LH4 interface is trapped nanometers above LH4 in a potential well create by an image-potential and Coulomb-potential with He atoms on the interface. Due to the image potential, the electron acquires a Hydrogen atom type electronic structure normal to the LH4 plane. Interestingly, in the in-plane direction the trapped-electrons have no potential barriers and could move at exceptionally high mobility. The first qubit architectures were conceived with trapped electrons within eigenstates normal to the He. The image-potential is $V = -\Lambda q2/z$, where $\Lambda = (\epsilon - 1)/4(\epsilon + 1)$, e is the charge of an electron and z is the coordinate of the electron perpendicular to the interface, and ϵ the dielectric constant of the LH4. The ϵ is 1.057 for LH4 giving $\Lambda = 0.01$. The 1D time independent Schrodinger equation for a bound electron above the interface ($z > 0$) in this image potential is given by equation 1:

$$\frac{-\hbar^2}{2m}\nabla^2\Psi - \frac{q^2}{z}\Psi = E\Psi \tag{1}$$

Below the surface of ($z < 0$) the $\Psi = 0$. The solution is $\Psi = \Phi(x)exp(iK.R)$ where $\Phi(x)$ is the 1D wavefunction describing the electron motion perpendicular to the LH4 and the complex exponential is the free motion across the lateral surface of the electron. $\Phi(x)$ is expressed as a hydrogenic potential as equation 2:

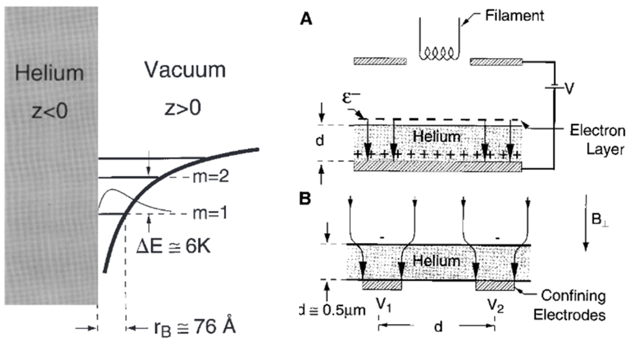

FIG. 1. Geometry of an electron trapped at a helium vacuum interface. Lateral control of the 2D film of electrons on the LH4 interface.

$$\frac{-\hbar^2}{2m}\frac{d^2\Phi}{dz^2} - \frac{q^2}{z}\Phi = E_n \tag{2}$$

Where E_n is the eigenvalue given as $E_n = -R/n^2(n = 1, 2, 3 \dots)$ where R is the Rygberg energy $R = mq^4/(2\hbar^2)$. The eigenfunction for the eigenstates E_1 is $\phi_1(x) = 2a^{-3/2}z.exp(-z/a)$ where $a = \hbar^2/(mq^2)$. As shown the Bohr radius is 76-ansgtroms and the ground state binding energy is 7.6 K. Control of the trapped-electron qubits is done by changing the electric field normal to the helium surface via electrodes submerged in LH4. The normal field changed the energy levels of the qubit and was used for tuning the resonance to couple to other qubit states or to an external microwave radiation. The major relaxation pathways were coupling to ripplons which propagate opposite directions or bulk phonons which travel lateral to the substrate. In plane electrons states to LH4 are also considered for qubit states. As the LH4 substate is free from magnetic defects, the trapped electrons have long-lived spin states, and these spin-states are also can form the eigenstates for a spin qubit.

The surface of the LH4 has height variation due to thermal excitation. Height variation on the LH4 surface leads to propagating capillary waves on the LH4 surface known as ripplons. Coupling to ripplon states is a non-radiative relaxation from E_2 to E_1 eigenstates. It is seen that the relaxation rate is a factor of one million smaller than the transition frequency between the lowest two hydrogen energy levels. This means that E_2 and E_1 states can be used as qubit. The first work which demonstrated this effect was Platzman et al. which showed that microwaves with a field of 1 Vcm^{-1} created Rabi oscillation between the two states.[8] Moreover, in the above paper, a 2D electron layer is formed on the LH4 surface by capacitive coupling. The electrons form a Wiger lattice by freezing in to a closed packed orientation due to the coulomb repulsion between the electrons. Using a split conductors potential different between the lateral electrons were created. This allowed the control of addressing different qubit systems each with a single trapped electron. This is similar to how quantum-dots function in semiconductors systems. However, lateral field creates an energy separation between the in-plane energies, which suppress decay rates to the ripplon modes, Electron trapped in LHF4 has a decoherence million times slower than GaAs type 2DEG quantum dot systems. Moreover, a perpendicular magnetic field can further increase the quantization between the in-plane states for further isolation from the ripplons. As shown in panel 2 , an electron on top of the V_1 electrode can be put in an excited state and then V_2 can be swept such that the two levels pass though resonance and then out of resonance to create linear superposition between the qubits. The qubit states can be read by a reverse potential to the capacitor and collecting the electrons via an imaging channel plate which detects electrons. The key is that electrons which are strongly bound (i.e. at E_1 state) will not get collected on the imaging channel. One of the main challenges in this first work is the need for 120-GHz or mm wavelength microwaves for the interrogating the qubit.

III. QED MANIPULATION OF TRAPPED ELECTRONS IN LIQUID HE

Schuster et al. proposed a more efficient method of coupling to the electrons in the qubits using cavity QED. [9] The advantage is that the quantized in-plane motion has transition frequencies of a few GHz and can be coupled to an on-chip cavity for nondestructive readouts similar to a superconducting qubit. The main idea is that by operating sub-50-mK temperature the electrons are suppressed in the lower state normal to the LH4. A split-ring potential barrier on the in-pane surface creates a trapping potential giving the Hamiltonian given in 3:

$$H_e = \frac{\hat{p}^2}{2m_e} + \frac{1}{2}m_e\omega_x^2\hat{x}^2 + \hbar\alpha\frac{\hat{x}^4}{3a_x^4} \tag{3}$$

Here $a_x = (\hbar/m_e\omega_x)^{1/2}$ is the standard deviation of the ground state and the α is the anharmonicity of the potential well. The transition frequency between the n and $n+1$ is $\omega_{x,n} = \omega_{x,0}+(n+1)\alpha$ and this lateral electron motion is given as a qubit then $|\alpha|$ is larger than the decoherence rate. ω_x and α is given by the width W, depth d and applied potential V_e as $\omega_x = 2\pi(eV_t/m_eW^2)^{1/2}$, $\alpha = (2\pi/W)2\hbar/8m_e$ and $V_t = V_e e^{-2\pi.d/W}$. Therefore, for $d = W = 500 - nm$, $V_e = 10 - mV$ a trap depth of $eV_t/h = 20 - GHz$ is formed (for QED applications defining energy is given in units of frequency). The transition frequency is $\omega_x = 5GHz$. The trapped electron, electromagnetic field in the cavity and the interaction can be presented in the Jaynes-Cummings Hamiltonian as in 4,

$$H = H_e + H_r + \hbar.g(a^\dagger c + ac^\dagger) \tag{4}$$

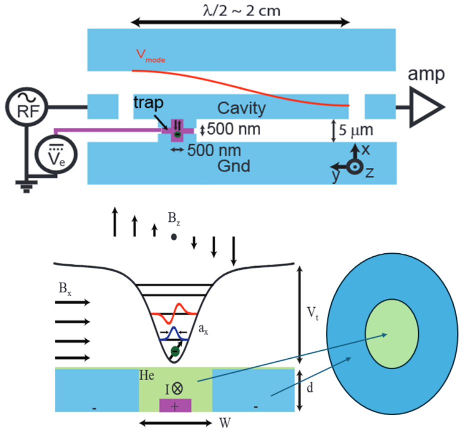

FIG. 2. Electrostatic electron trap where the electron state can be manipulated by RF inputs. Middle panel is view of the trap electrode energy levels and right panel is a view of the spin ring potential barrier architecture.

Where H_e is the Hamiltonian of the electron, H_r is the Hamiltonian of the cavity given as $H_r = \hbar.\omega_r(a^\dagger a+1/2)$, g is the vacuum Rabi frequency $2g = 2^{1/2}ea_xE_o/\hbar$ with E_o is the zero point electric field in the cavity given as $Eo\ 2V/m$. a and c are the photon and mortional quanta annihilation operator respectively. The main upshot of this is the large vacuum Rabi coupling strength of 20 MHz due to the larger electron dipole-moment of ea_x 2000 Debye which enables fast manipulation of the electrons in the qubit. In addition to detecting the occupation of motional states of the electron in the cavity, above QED procedure can be used to detect spins via the spin-orbit interaction by creating a magnetic field in the z direction by passing a current through the center electron into the page. This allows the manipulation and readout of individual spins.

A practical implementation of QED based LH4 trapped electron qubit system was reported by Koolsta et al.[10] A superconducting microwave resonator with a center frequency of fo = 6.339 GHz with a line width of 0.4-MHz, shown in red, is fabricated integrated to the LH4-trapped electron quantum dot. Once the structure is filled with LH4, electrons are deposited into the reservoir via a thermal emission or by a negative DC bias of the resonator and positive on the filament. All measurement is done with the V_{res} = 0.6-V for constant electron density at T = 25-mK under low incident microwave power to keep the system in linear mode. A potential barrier set in the trap is used as a gate to move the electron in the reservoir into the quantum-dot. The dispersive resonance frequency shifts in the of the reservoir as the electrons leak out to the dot. Transferring the charge to the dot by opening the barrier. The resonator frequency shift gives clear signals when electrons are loaded to the dot and unload from it.

The coupling strength $g/2\pi$ = 4.8 MHz and total electron line $\gamma/2\pi$ = 77 MHz. This linewidth is extraordinarily high compared to the fundamental limitations governed by electron-phonon coupling via the ripplons (¡ 0.1 MHz). The authors of the above work point to classical noise in the He due to lack of vibration isolation in the dilution refrigerator as the cause of the noise. Nevertheless, this highlights technical challenges in obtaining the ideal condition of keeping a superfluid immune to experimental conditions. Any type of fluctuation of the He/vacuum interface will drive the system of the equilibrium conditions which impacts the electronic structure of the electrons bound on the He surface.

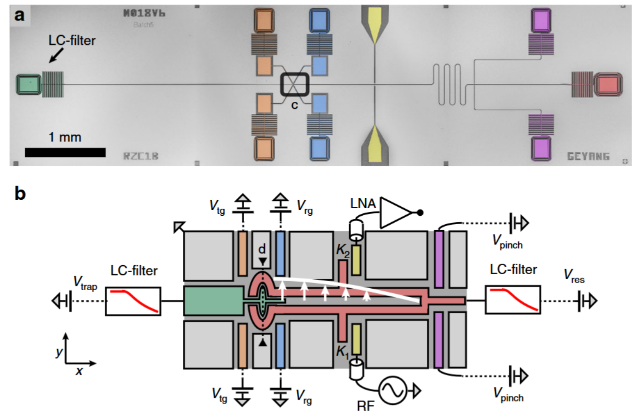

FIG. 3. Practical implementation of a electron-on-LH4 quantum dot.

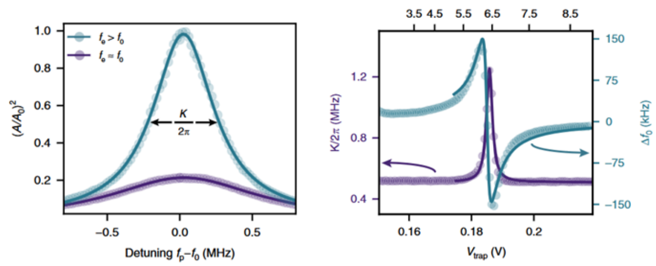

FIG. 4. Single electron spectroscopy using the microwave resonance coupling. (a) Transmission amplitude of the of the resonator at microwave probe detuning. Blue color is the far-off resonance and purple is where the electron is resonant with the cavity. (b) Cavity resonance frequency shift as a function of trap voltage by fitting Lorentzian to the spectra at a taken at different bias points.

IV. SPIN QUBITS ON TRAPPED-ELECTRONS IN He^4

Apart from using hydrogen-like states normal to the LH4 or confined quantum-dot type states lateral to to the LH4 surface as qubit states, its also possible to use spins states of electrons trapped in the LH4 as spin qubits. Lack of nuclear spin in He^4 gives rise to long-lived electron spin states of the trapped electrons. Dykman et al. proposed a scheme to create quantized spin states of trapped electron on the LH4 and methods to control the interaction of spin states in adjacent qubits. The electrons are confined in an electrostatic potential using a spit-ring electrode and a strong magnetic field parallel to the surface creates the quantized spin states in the electrons. The spin states of the localized are controlled by magnetic field Bx applied along the x axis which impose a Larmor frequency $\omega L = 2\mu_B B_x/\hbar$. here μ_B is the Bohr magneton and we have approximated the g factor by 2. As example for $B_x = 0.2$ T the Larmor frequency is $\omega_L = 2\pi$ x 5.6 GHz. The eigenvalues of the spin operator S_x is given as equation 5

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} |\downarrow\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} \tag{5}$$

The Hamiltonian for the spin-qubit is given by considering both vibrational and spin energy in equation 6.

$$H_0 = \sum_{i=x,y} \hbar\omega_i a_i^\dagger a_i - \hbar\omega_L S_x \tag{6}$$

Where a_i and $a_i^\dagger$ are the annihilator and creator operators of x and y vibrational modes. ω_x and ω_y are the eigenfrequencies. Spin orbit coupling can be manipulated by sending a current through a superconducting nanowire going through separate quantum-dots or both together. The spin-orbit coupling is used to control coupling of each spin state in each quantum dot to the driving field.

V. QUANTUM GATES USING TRAPPED-ELECTRONS IN He^4

State of art in trapped electrons in LH4 qubit systems has been unable to create even a single qubit gate operation. A recent theoretical work has shed some light on the possibility of creating single and double qubit gates as well as the use of centimeter waves instead of the millimeter waves are used in the past to probe the vibrational energy states normal to the LH4 surface.[11] In contrast to the vibration eigenstates of the electron normal to the LH4 surface, lateral anharmonic vibrations of each electron each trap is used to encode the qubit. Since this energy is smaller than the vibrations in the normal direction to the LH4, centimeter-waves can be used to manipulate the qubits. The qubits are set and read using a coplanar waveguide transmission line resonator. Figure 5 shows single gate and double gate operation on a linear array of trapped electrons. The coplanar waveguide resonator,acts as a data bus to couple different qubit states.

$$H_p = \hbar\omega_r \hat{a}_r^\dagger \hat{a}_r + \frac{1}{2}\hbar\omega_p \hat{\sigma}_p^z + \hbar g_p(\hat{a}_r^\dagger \hat{\sigma}_p^- + \hat{a}_r \hat{\sigma}_p^+) + \hbar \hat{a}_r \xi^* e^{i\omega_d t} + \hbar \hat{a}_r^\dagger \xi e^{-i\omega_d t} \tag{7}$$

For a single qubit (p) the Hamiltonian for the resonator coupled to a trapped-electron is given by equation 7.

Where the σ_p^z, σ_p^+ and σ_p^- are the Pauli operators for the p-th qubit. The first 2 terms are the free Hamiltonian for the cavity and vibrations states of the trapped electron. The third term is the interaction between the qubit and the waveguide resonator. The ω_d and ξ are the driving frequency and amplitude. By ignoring the higher frequency components by rotating wave approximation, the Hamiltonian in equation 7 is simplified leaving only the interaction terms between the resonator and the quantum dots as given by $H_I = \hbar g_p(\hat{a}_r^\dagger \hat{\sigma}_p^- + \hat{a}_r \hat{\sigma}_p^+)$. The $\hat{\sigma}_p^+ = |1\rangle\langle 0|$ and $\hat{\sigma}_p^- = |0\rangle\langle 1|$ are the pseudo-spin Pauli operators of the qubit. As shown by Yufen et al the g_p is was estimated to be 2π x 14.92 MHz is higher than the Coulomb interaction (2π x 0.93 kHz) between the electrons in adjacent quantum-dots. [11]. There are two rules of thumb to implement quantum gates using the trapped-electrons in liquid He^4. (a) Maintain low coupling strength (i.e. g_p) to satisfy the rotating wave condition, and (b) qubit-qubit interactions can be generated by using the a resonator to couple different qubits together.

Multi-qubit operations are described with the interaction Hamiltonian as $\hat{H}_{qp} = \frac{2\hbar g^2}{\Delta}(\hat{\sigma}_p^- \hat{\sigma}_q^+ + \hat{\sigma}_p^+ \hat{\sigma}_q^-)$. The unitary evolution operator $exp(-\frac{i}{\hbar}H_{qp}(t)$ can be written as:

$$exp(-\frac{i}{\hbar}H_{qp}(t) = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & cos(\kappa t) & -isin(\kappa t) & 0\\ 0 & -isin(\kappa t) & cos(\kappa t) & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \tag{8}$$

The $\kappa = 2g^2/\Delta$ where Δ is the detuning between the lateral vibrational frequency of the qubit and the driving field. This indicates that *by controlling the evolution time* $t = \pi/(2\kappa)$ a i-SWAP gate operation is realized between p and q qubits.

VI. CONCLUSION

Trapped electron in liquid He^4 is a highly scalable and low-decoherence qubit technology. Vibrational states normal and lateral to the liquid-He are explored as qubits. Lateral qubit states have approximately an order of magnitude less energy difference which can be manipulated via centimeter waves. Both single and multi-qubit gates can be implement with by using a co-planner wave guide resonator to dirve one or more qubit states. External vibrations and classical affect the fidelity of the trapped-electron qubits.

VII. REFERENCES

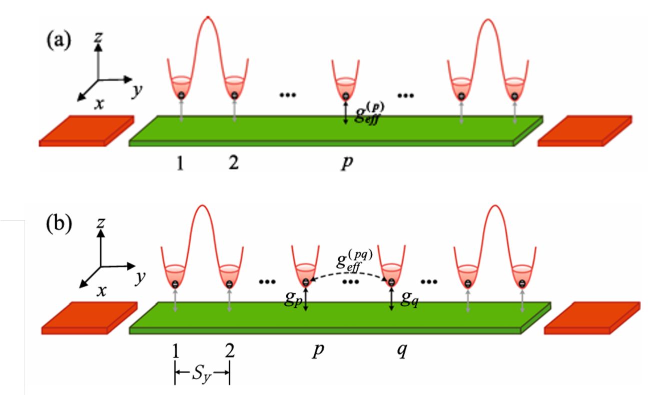

FIG. 5. trapped electron array to implement quantum gate operations. The qubits are encoded by the ground and first excited states of the vibration of the trapped electron in the y-direction. The co-planar waveguide transmission line resonator is the data bus to implement the operations and duties.

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