Superconducting Quantum Computing

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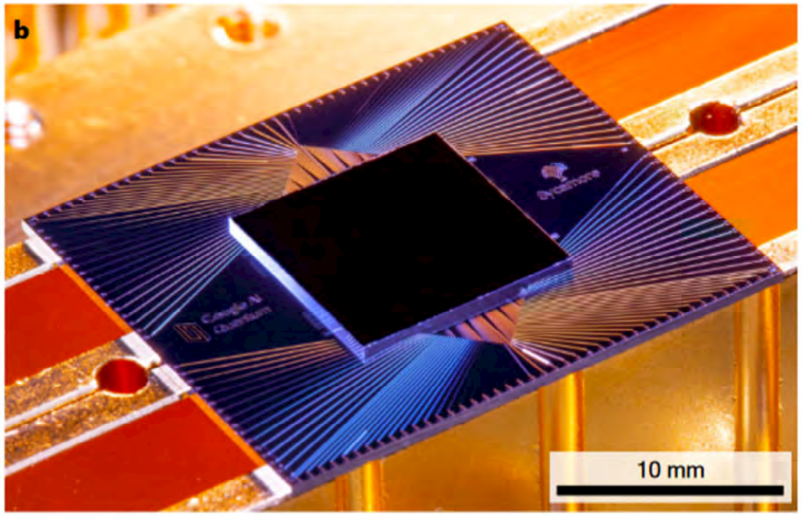
OPTI 646 Final Lecture

Popular implementation in industry

- Google Sycamore of quantum supremacy experiment
- IBM’s Condor, Heron, etc.
- Amazon braket’s OQC and Rigetti

Similarity to classical computer chips

- Circuit form factor is similar to existing computer chips
- Can be made using established techniques
- Straightforward to couple qubits and connect to instruments for readout and control



Arute 2019

Superconductivity

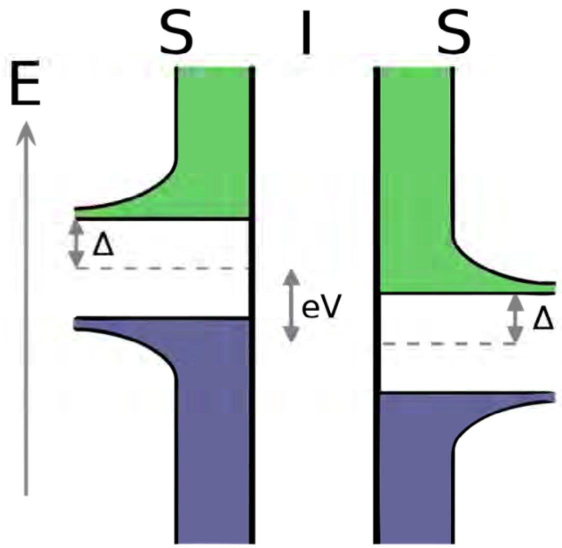
- Resistance discontinuously drops to zero at finite low temperature
- Band gap of $\pm\Delta$ around Fermi energy
- Macroscopic quantum effect, allowing current and voltage to act as quantum objects
- Low temperature requirements for quantum computing are much lower than T_c, ~10mK

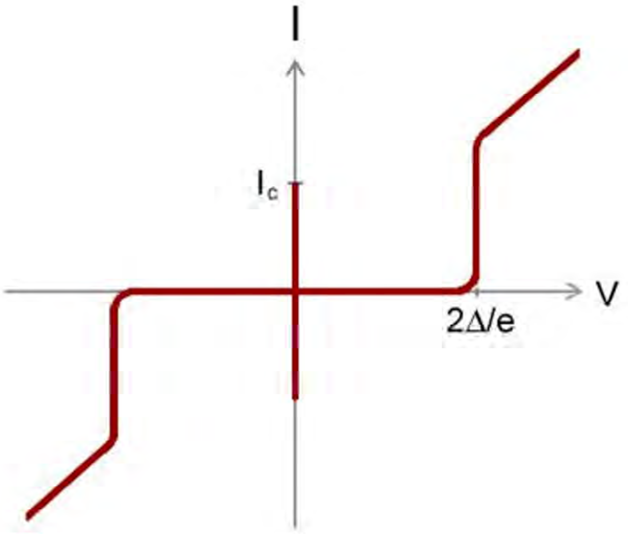
BCS theory

- Electrons have attractive interactions due to interactions with the lattice
- Bind into pairs called Cooper pairs which travel freely in material
- Weak binding explains low critical temperature

Josephson effect

- Two superconductors separated by thin layer of insulator
- Allows tunneling current up to maximum I_c at 0V, then none until $2\Delta/e$
- Linear I-V curve at higher voltage





https://en.wikipedia.org/wiki/Superconducting_tunnel_junction

Theory of Josephson effect

- A/B for each side of the junction

$$\psi_{\mathrm{A}} = \sqrt{\mathrm{n_A}} e^{i\phi_A} \quad (1)$$

$$\hat{\mathrm{H}} = \begin{pmatrix} \mathrm{eV} & \mathrm{K} \\ \mathrm{K} & \mathrm{-eV} \end{pmatrix} \quad (2)$$

$$i\hbar \frac{\partial}{\partial t} \sqrt{n_A} e^{i\phi_A} = eV\sqrt{n_A} e^{i\phi_A} + K\sqrt{n_B} e^{i\phi_B} \quad (3)$$

$$i\hbar \frac{\partial}{\partial t} \sqrt{n_B} e^{i\phi_B} = K\sqrt{n_A} e^{i\phi_A} - eV\sqrt{n_B} e^{i\phi_B} \quad (4)$$

$$\varphi \equiv \phi_B - \phi_A$$

$$\dot{n}_A = \dot{n}_B = \frac{2K\sqrt{n_A n_B}}{\hbar} sin\varphi \quad (5)$$

$$\dot{\phi}_A = -\frac{1}{\hbar}\left(eV + K\sqrt{\frac{n_B}{n_A}} cos\varphi\right) \quad (6)$$

$$\dot{\phi}_B = \frac{1}{\hbar}\left(eV - K\sqrt{\frac{n_A}{n_B}} cos\varphi\right) \quad (7)$$

Theory of Josephson effect

- Current between ±I_c depending on φ, which evolves based on applied voltage

$$I(t) \sim \dot{n}_A = I_c sin\varphi(t) \quad (8)$$

$$\dot{\varphi} = \frac{2eV(t)}{\hbar} \quad (9)$$

Theory of Josephson effect

- Can act like a non-linear inductor

$$\frac{\partial I}{\partial t} = I_c cos\varphi \frac{2eV}{\hbar} \quad (10)$$

$$V = \frac{\hbar}{2eI_c cos\varphi} \frac{\partial I}{\partial t} \quad (11)$$

- Charging energy

$$W = \int_{t_1}^{t_2} IV\,dt = \int_{\varphi_1}^{\varphi_2} I_c sin\varphi \frac{\hbar}{2e}\,d\varphi = -\frac{I_c \hbar}{2e}\Delta(cos\varphi) \quad (12)$$

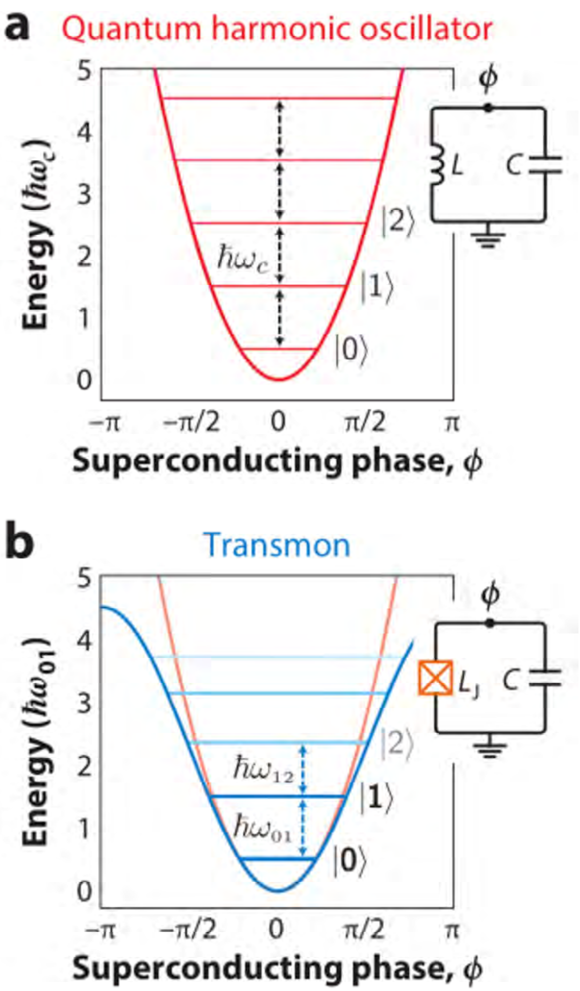
$$E(\varphi) = -E_J cos\varphi \quad (13)$$

LC circuit

- Low temp LC circuit forms quantum harmonic oscillator
- However the uniform energy spacing makes qubit control impossible
- Need to stay in |0〉 and |1〉 subspace

Modified LC circuit

- Capacitor energy goes as square of the time derivative of φ which is proportional to voltage
- The Josephson junction energy is approximately quadratic at low energy levels but drops off like a pendulum
- Creates necessary anharmonicity



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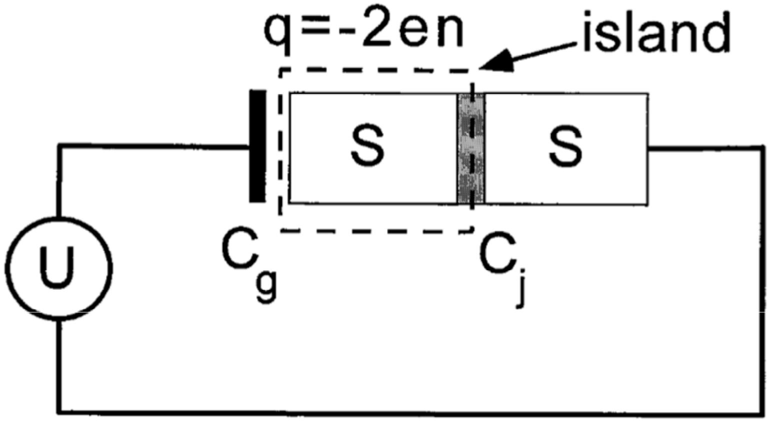
Charge qubits



Bouchiat 1998

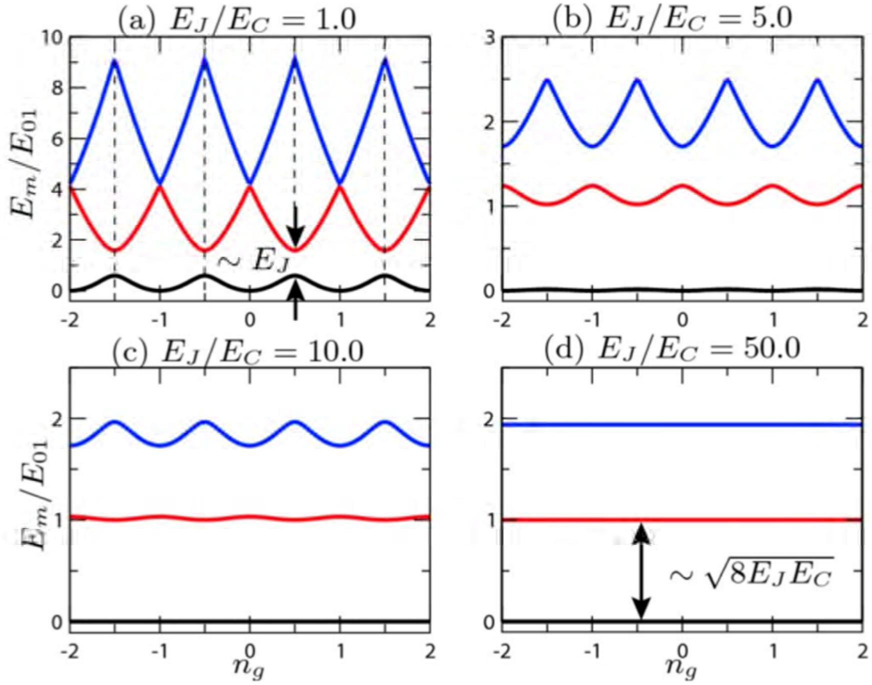
- Hamiltonian can be written as

$$\hat{H} = 4E_C\left(\hat{N} - n_g\right)^2 - E_J cos\hat{\varphi} \quad (14)$$

- Where the N operator corresponds to the number of Cooper pairs on the capacitor and n_g is an applied bias voltage
- For large E_C, N is a good quantum number
- This is the charge qubit or Cooper pair box
- First experimentally realized superconducting qubit (1999)
- Sensitive to charge/gate voltage noise

Transmon qubits

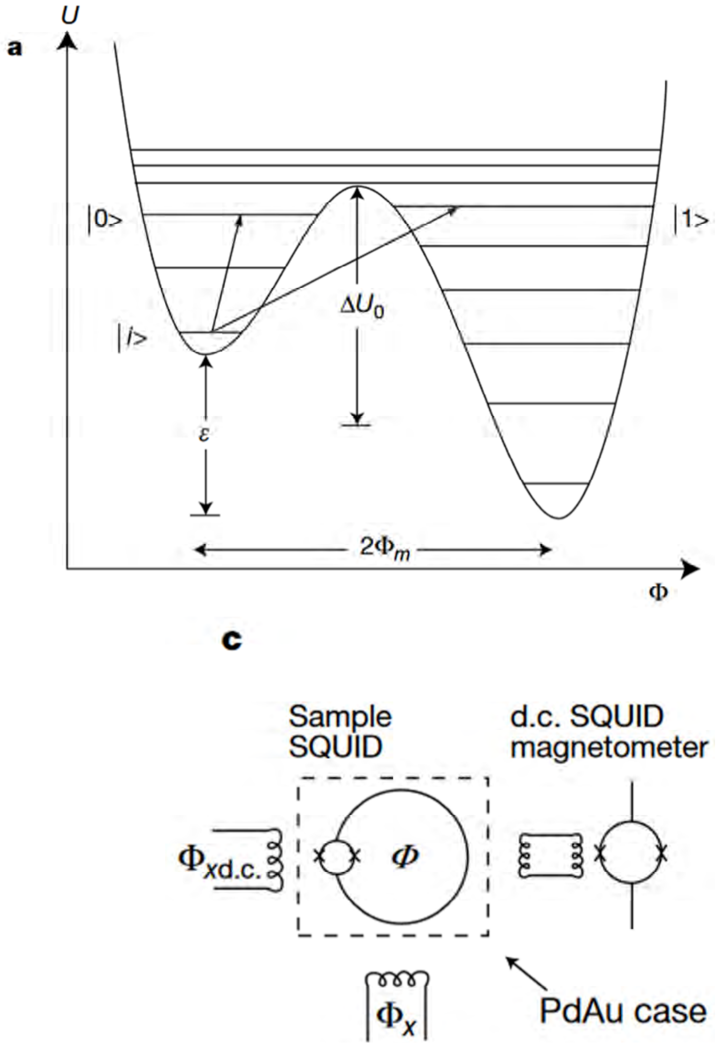
- Based on charge qubits but with greater capacitive shunting, $E_J > E_C$
- Leads to reduced sensitivity to charge noise
- φ becomes the quantum number
- Anharmonicity is also reduced, needs to be balanced with reduction in noise



Koch 2007

Flux qubits

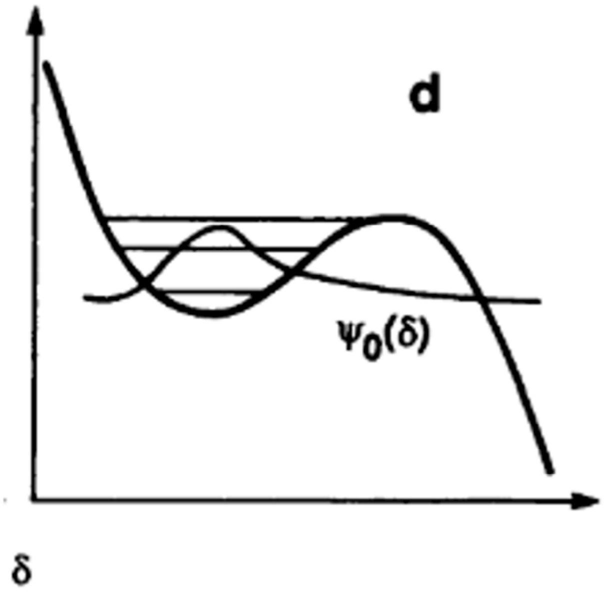
- Operate in large E_J regime, larger than transmons
- Apply magnetic flux to superconducting loop with Josephson junction
- Flux quantization condition generates persistent current cw or ccw
- Can replace junction with SQUID for further tunability of potential landscape
- Introduces sensitivity to flux noise



Friedman 2000

Phase qubits

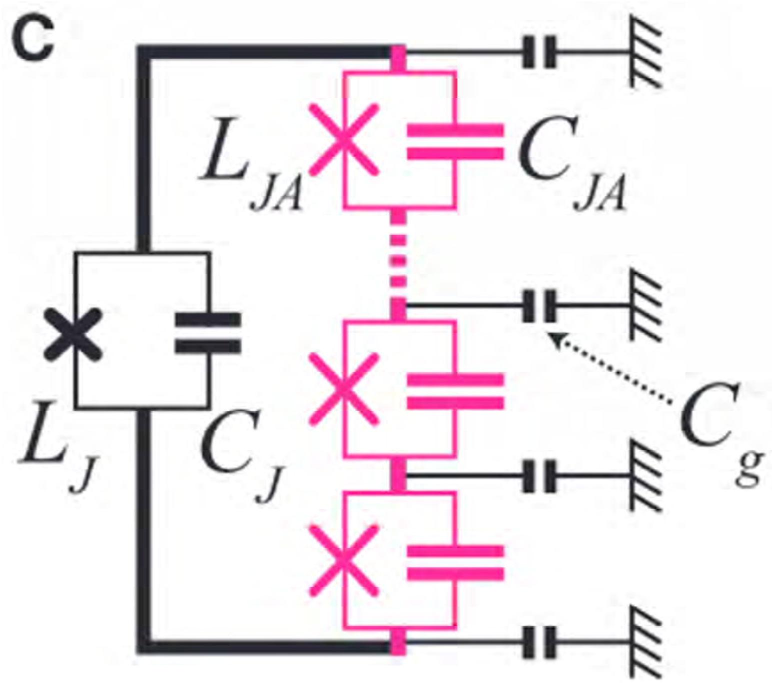
- Apply a bias current to a Josephson junction shunted by a capacitor
- Results in a “washboard” potential
- Bound states of phase possible for small enough bias current



Clarke 1988

Fluxonium qubits

- Series of large area junctions shunts a smaller junction
- Islands between large junctions are grounded
- Very low charge and flux noise sensitivity but low energy transitions necessitate very low operating temperatures
- Also more complicated to fabricate



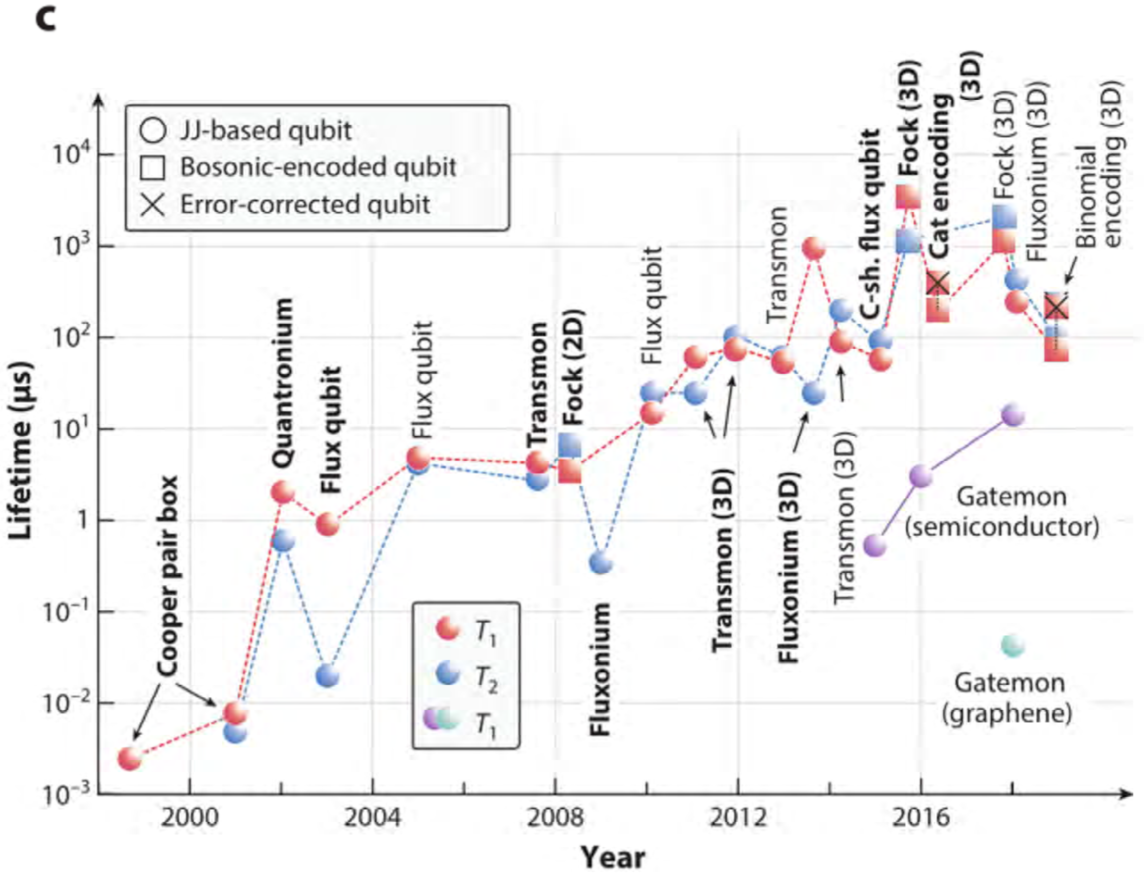
Manucharyan 2009

Other noise sources

- Photon number fluctuations and quasiparticles
- Photon number fluctuations are primarily driven by retained radiation in the resonator from before cooling
- Quasiparticles are unpaired electrons

Coherence lifetime

- ~100-1000 us for the best Josephson junction based qubit designs



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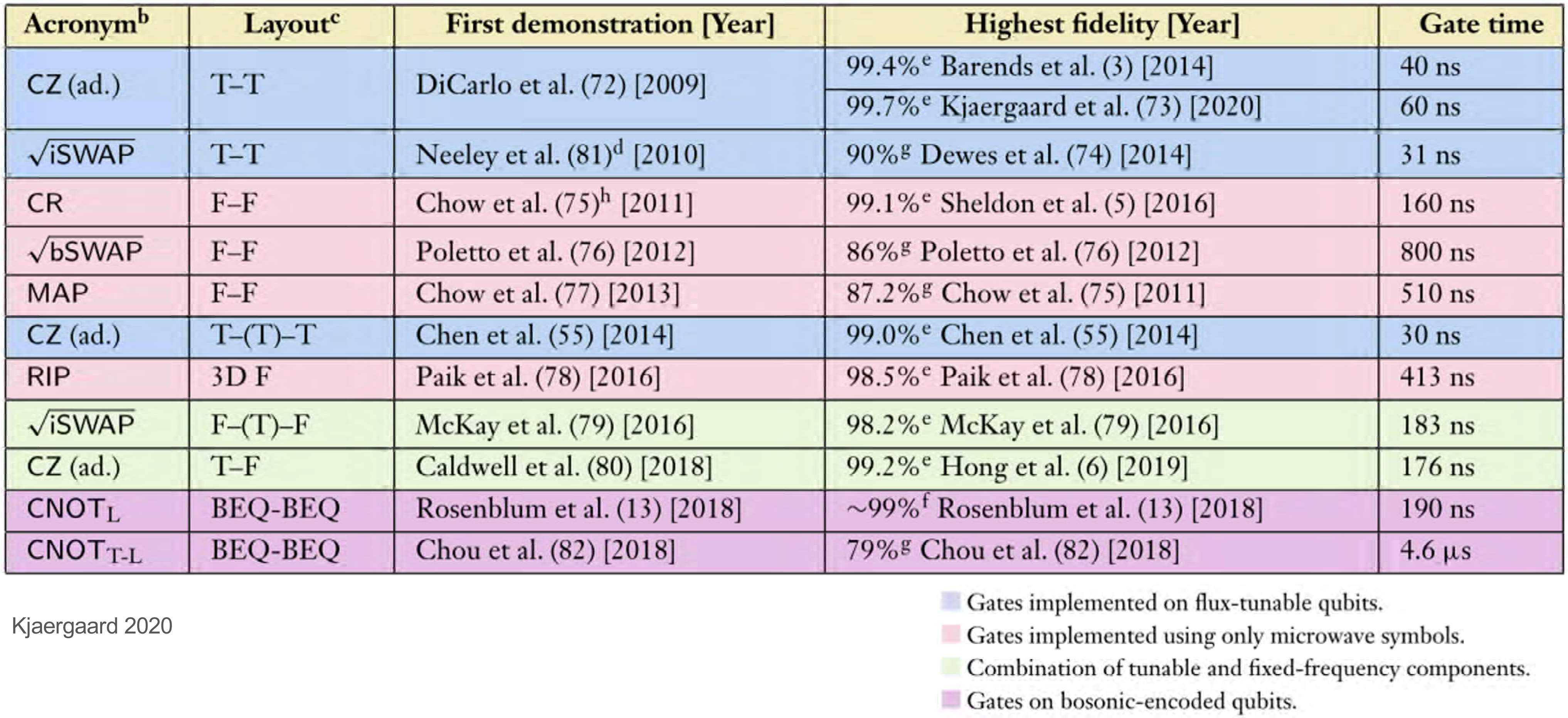
Gates



Acronym[b]	Layout[c]	First demonstration [Year]	Highest fidelity [Year]	Gate time
CZ (ad.)	T–T	DiCarlo et al. (72) [2009]	99.4%[e] Barends et al. (3) [2014]	40 ns
			99.7%[e] Kjaergaard et al. (73) [2020]	60 ns
$\sqrt{\text{iSWAP}}$	T–T	Neeley et al. (81)[d] [2010]	90%[g] Dewes et al. (74) [2014]	31 ns
CR	F–F	Chow et al. (75)[h] [2011]	99.1%[e] Sheldon et al. (5) [2016]	160 ns
$\sqrt{\text{bSWAP}}$	F–F	Poletto et al. (76) [2012]	86%[g] Poletto et al. (76) [2012]	800 ns
MAP	F–F	Chow et al. (77) [2013]	87.2%[g] Chow et al. (75) [2011]	510 ns
CZ (ad.)	T–(T)–T	Chen et al. (55) [2014]	99.0%[e] Chen et al. (55) [2014]	30 ns
RIP	3D F	Paik et al. (78) [2016]	98.5%[e] Paik et al. (78) [2016]	413 ns
$\sqrt{\text{iSWAP}}$	F–(T)–F	McKay et al. (79) [2016]	98.2%[e] McKay et al. (79) [2016]	183 ns
CZ (ad.)	T–F	Caldwell et al. (80) [2018]	99.2%[e] Hong et al. (6) [2019]	176 ns
CNOT_L	BEQ-BEQ	Rosenblum et al. (13) [2018]	~99%[f] Rosenblum et al. (13) [2018]	190 ns
$\text{CNOT}_\text{T-L}$	BEQ-BEQ	Chou et al. (82) [2018]	79%[g] Chou et al. (82) [2018]	4.6 μs

🟦 Gates implemented on flux-tunable qubits.
🟥 Gates implemented using only microwave symbols.
🟩 Combination of tunable and fixed-frequency components.
🟪 Gates on bosonic-encoded qubits.

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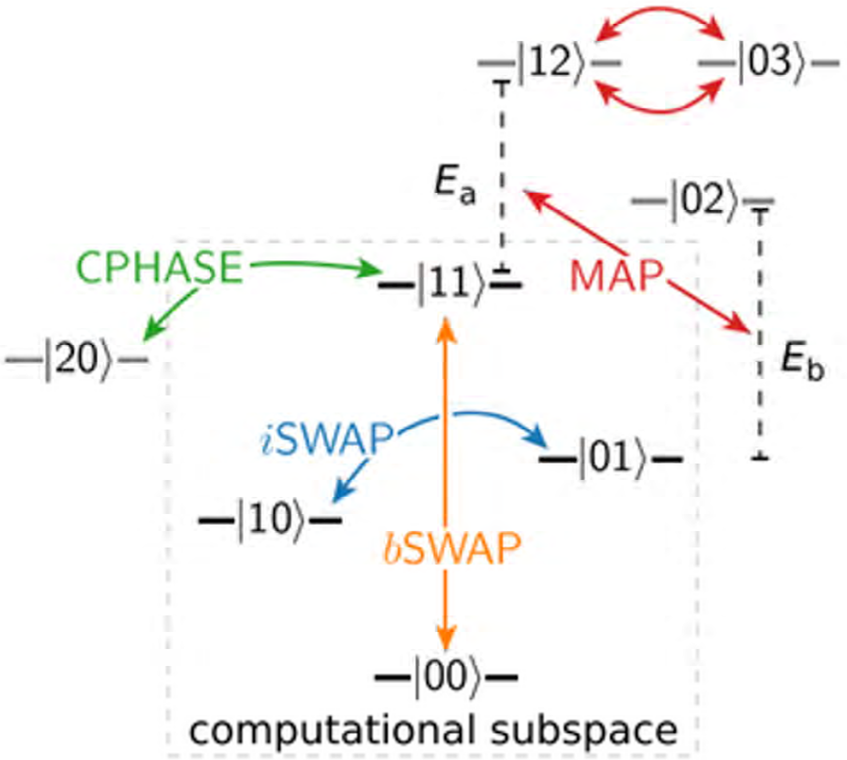
Gates

- CZ puts phase on $|11\rangle$
- CR rotates between $|00\rangle$ and $|01\rangle$ and between $|10\rangle$ and $|11\rangle$
- MAP and RIP both implement $\sigma_z \otimes \sigma_z$ type interactions



Krantz 2019

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