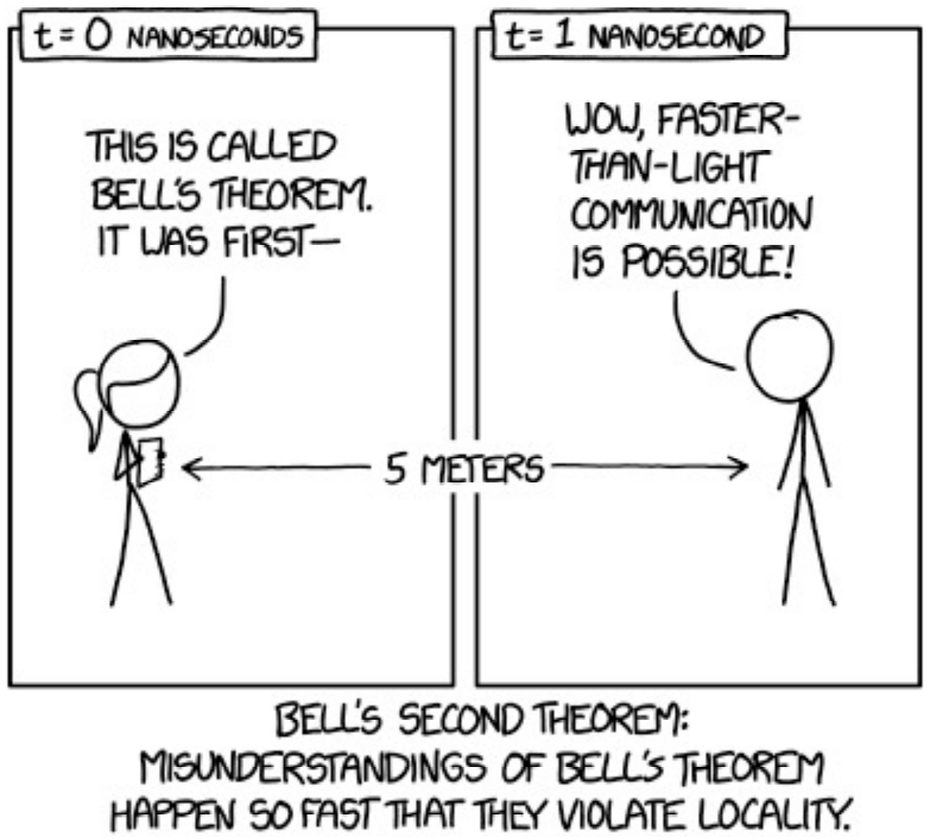
What comes next ?

Congratulations
You Survived Boot Camp

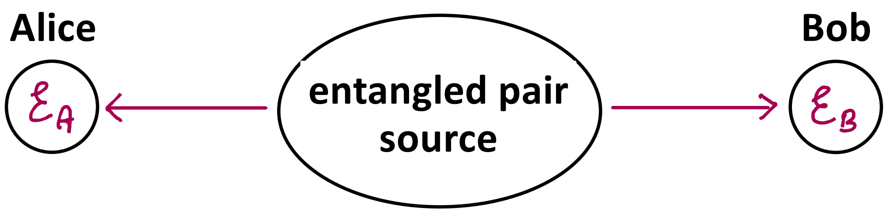
2 Spins, EPR States (Preskill ch. 2.5)



2 Spins, EPR States (Preskill ch. 2.5)

Basic Paradigm:

Shared pair of spin-1/2 particles



2 – spin state space: $\mathcal{E} = \mathcal{E}_A \otimes \mathcal{E}_B$

Product State Basis: $|\uparrow_{\hat n} \uparrow_{\hat n'}\rangle, |\uparrow_{\hat n} \downarrow_{\hat n'}\rangle, |\downarrow_{\hat n} \uparrow_{\hat n'}\rangle, |\downarrow_{\hat n} \downarrow_{\hat n'}\rangle$

Example of entangled state : $|\Phi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z \uparrow_z\rangle + |\downarrow_z \downarrow_z\rangle\right)$

Local description of spin A ➡ **Need reduced Density Operator**

$$\rho_A = Tr_B[\rho_{AB}] = \sum_{i=\uparrow,\downarrow} {}_B\langle i|\tfrac{1}{2}\left(|\uparrow_z\uparrow_z\rangle + |\downarrow_z\downarrow_z\rangle\right)\left(\langle\uparrow_z\uparrow_z| + \langle\downarrow_z\downarrow_z|\right)|i\rangle_B$$

$$= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$ ⬅ **maximally mixed**

Note: $\boxed{\rho_A \text{ contains } \underline{\text{no information}}\,!}$

Explicitly we have

$$P(a) = Tr[P_a \rho_A] = Tr\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}}_{\text{basis } |a\rangle, |a'\rangle} = Tr\begin{pmatrix} 1/2 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}$$

for any observable, any outcome

observable A
outcomes a, a'
eigenbasis $|a\rangle, |a'\rangle$

Local Measurements, Correlations ?

2 Spins, EPR States (Preskill ch. 2.5)

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Local Measurements

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➡ **Alice has** $\left\{ \begin{matrix} |\uparrow_z\rangle_A \\ |\downarrow_z\rangle_A \end{matrix} \right.$ **w/** $P = 1/2$

➡ $\rho_A = \frac{1}{2}\left(|\uparrow_z\rangle_{AA}\langle\uparrow_z| + |\downarrow_z\rangle_{AA}\langle\downarrow_z|\right) = \frac{1}{2}\mathbb{1}$

2. **Bob measures** S_x ➡ **outcomes** $\left\{ \begin{matrix} |\uparrow_x\rangle_B \\ |\downarrow_x\rangle_B \end{matrix} \right.$ **w/** $P = 1/2$

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<u>**Note**</u>: **This holds for any max entangled state and any measurement Bob can make.**

Same ρ_A ➡ **No "faster than light" communications**

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1. Bob measures S_z on many pairs ➡ $\uparrow\downarrow\uparrow\downarrow\downarrow\ldots$

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Pure State Distillation:

1. Bob tells Alice he measured S_x , keeps measurement record $\uparrow\downarrow\uparrow\downarrow\downarrow\ldots$ to himself

   Alice keeps spins w/out measuring

   ➡ $\rho_A = \frac{1}{\sqrt{2}}\left(|\uparrow_x\rangle\langle\uparrow_x| + |\downarrow_x\rangle\langle\downarrow_x|\right)$

2. Bob shares measurement record with Alice, who then knows which spins are up and which are down. She flips the latter.

   ➡ Alice can "distill" a pure state from the ensemble

Conclusion:

$\boxed{\rho_A \neq \rho_A + \text{information}}$

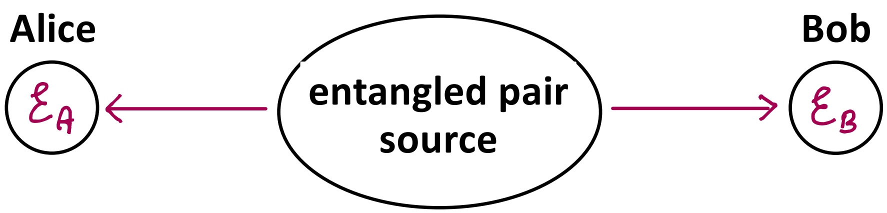
- Information is physical -

The above scenarios and variants thereof are central to Quantum Communication !

Begin 10-04-2023

EPR and Bell Inequalities (Preskill ch. 4.1)

We return to our basic scenario:



Einstein: If A & B are separated in space then measurements on A & B can be spacelike separated events ⇒ Alice and Bob cannot exchange light speed signals so one of them will know the result of the others measurement before performing their own

⇓

In a complete description of physical reality a measurement performed on A must not modify the description of B.

Seems reasonable, given what we know about **Special Relativity** and **Causality**

How to think about this in a rigorous, testable way?

Thought experiment: Scheduling a date

1. Alice and Bob share qubits in the entangled state

$$\boxed{|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z \uparrow_z\rangle + |\downarrow_z \downarrow_z\rangle\right)}$$

They agree that if Bob measures $\uparrow_z$ at a specified later date then he will visit Alice.

2. Alice travels to a galaxy far, far away

3. At the agreed-upon time Alice and Bob measure their qubits

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Alice instantly knows about Bob's travel plans. Moreover, this information did not exist prior to their measurements

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Let us be clear:

(1) Alice and Bob have no control over the outcome of their measurements – it is equally likely that they both get $\uparrow_z$ or they both get $\downarrow_z$. Thus they cannot signal each other to say "I am bored, come visit"

(2) Alice and Bob would be in the same situation if they shared a mixed state,

$$\rho = \frac{1}{2}\left(|\uparrow_z\uparrow_z\rangle\langle\uparrow_z\uparrow_z| + |\downarrow_z\downarrow_z\rangle\langle\downarrow_z\downarrow_z|\right)$$

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Classical balls ➡ Info was there all along

Entangled spins ➡ Info came into existence when the measurement was made

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Nevertheless , the quantum and classical physics appears fundamentally different

Bells inequalities:

Guidelines to set up experiments with entangled spins so quantum mechanics and "reasonable classical models" (Hidden Variable Theories) make different and testable predictions

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Local Hidden Variable (LHV) Theories

Measurement is fundamentally deterministic. It appears probabilistic only because the state of a system is described by the quantum state plus a set of hidden variables whose values are not known and cannot be controlled

QM: Preparation ➡ spin in state $|\psi_S\rangle$

LHV: Preparation ➡ spin state $(|\psi_S\rangle, \{\lambda\})$

LHVs

Example: $|\psi_S\rangle = |\uparrow_z\rangle$, one HV $0 \le \lambda \le 1$, uniformly distributed

Measure: $\sigma_\theta = \begin{cases} |\uparrow_\theta\rangle & \text{for } 0 \le \lambda \le \cos^2 \theta/2 \\ |\downarrow_\theta\rangle & \text{for } \cos^2 \theta/2 \le \lambda \le 1 \end{cases}$

Deterministic if we know λ, probabilistic otherwise

Take this seriously ? Definitely !

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Einstein: There exist a LHV description of a spin-1/2. Thus, once prepared, the outcome of measuring $\sigma_{\hat{n}} = \vec{\sigma}\cdot\hat{n}$ is completely determined by the LHV state $(|\psi_s\rangle, \{\lambda\})$, and we could predict the outcome deterministically if only we knew $|\psi_s\rangle$ and the values of all the HV's in the set $\{\lambda\}$.

QM says: Measure $\sigma_{\vec{n}}$ ➡ wipe out info predicting outcomes of later measurements $\sigma_{\vec{m}}$, where $\vec{n}\cdot\vec{m} = 0$.

HV Theory: This happens because measuring $\sigma_{\vec{n}}$ disturbs the values of the HV's in ways we cannot control and cannot know.

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Physical Reality of HV's ?

If QM is always correct and HV theories make no measurably different predictions, then we conclude the $\{\lambda\}$ do not represent any element of physical reality (Occams Razor)

EPR experiment with spins:

(1) Prepare 2 spins in state $|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Note: Total spin $\vec{J} = \vec{S}_1 + \vec{S}_2$, $|\psi_-\rangle = |j=0, m=0\rangle$

(Singlet state, rotationally invariant)

(2) Separate and measure $\sigma_{\vec{n}}(A)$ and $\sigma_{\vec{m}}(B)$ as spacelike separated events ➡

Local descriptions $\begin{cases} \rho_A = Tr_B\left(|\psi_-\rangle\langle\psi_-|\right) \\ \rho_B = Tr_A\left(|\psi_-\rangle\langle\psi_-|\right) \end{cases}$

ρ_A, ρ_B ➡ $\begin{cases}\text{no info about correlations, in QM} \\ \text{no local description is possible}\end{cases}$

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To test, assign a LHV state $(\rho_i, \{\lambda\})$, $i = A, B$ where complete knowledge of the HVs allows deterministic predictions regarding measurements of $\sigma_{\vec{n}}(A), \sigma_{\vec{m}}(B)$ and their correlations.

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* Experiments ⇨ we know measurements of $\sigma_{\vec{n}}(A), \sigma_{-\vec{n}}(B)$ are always perfectly correlated

  ⇨ source must build in correlations between $\{\lambda\}_A, \{\lambda\}_B$ to make this happen

* Even so, we still cannot predict if outcomes will be $\uparrow_{\vec{n}}, \downarrow_{-\vec{n}}$ or $\downarrow_{\vec{n}} \uparrow_{-\vec{n}}$

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(4) Spacelike interval ⇨ Bobs measurement cannot alter the LHV state $(\rho_A, \{\lambda\}_A)$

* Bob measures $\sigma_{\vec{m}}(B)$ ⇨ we know result if Alice were to measure $\sigma_{\vec{m}}(A)$

* Instead Alice measures $\sigma_{\vec{n}}(A)$ ⇨ we have effectively measured complementary observables $\sigma_{\vec{n}}(A)$, $\sigma_{\vec{m}}(A)$, $\vec{n} \neq \vec{m}$

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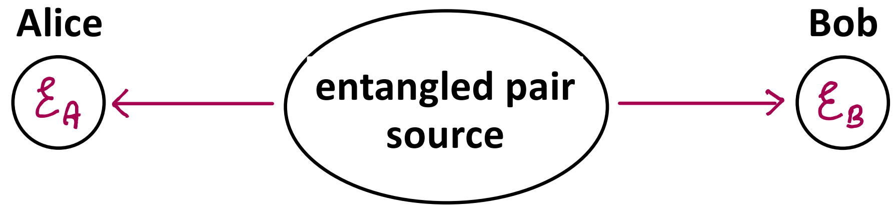

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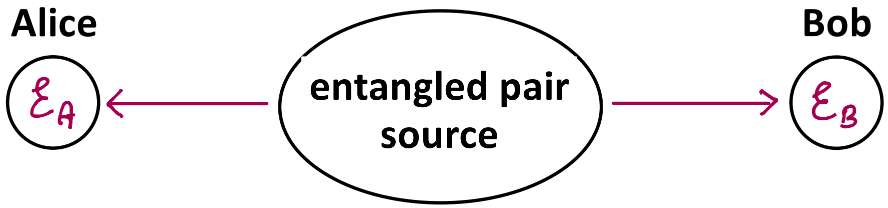

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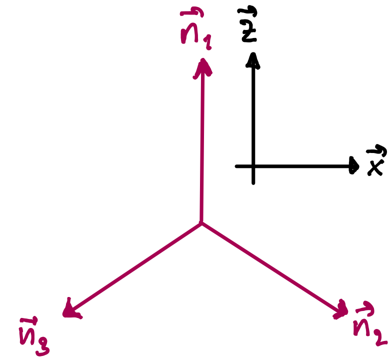
Setup: Alice and Bob choose at random between measurement axes

Alice chooses among

$\vec{n}_1 = (0,0,1)$

$\vec{n}_2 = \left(\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}\right)$

$\vec{n}_3 = \left(-\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}\right)$



Bob chooses among

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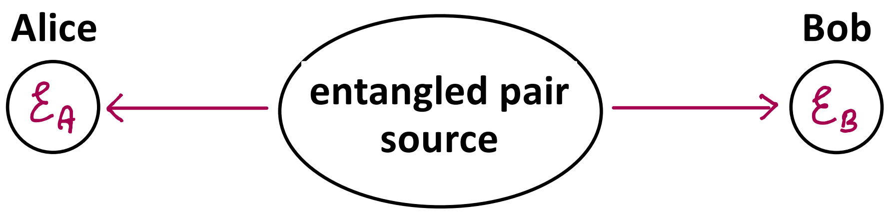
EPR and Bell Inequalities (Preskill ch. 4.1)



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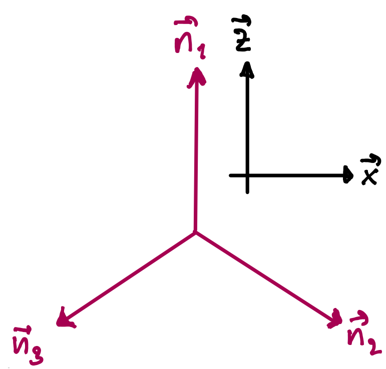

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(3) Accept LHV description ➡ must have info about 3 combinations $(i,j) = (1,2), (1,3), (2,3)$ simultaneously, all encoded in Alice's LHV state

Equivalent Scenario:

Flip 3 coins repeatedly, and each time pick 2 at random and look at those only. The coin flip process builds in correlations between the coins (HV values) that we can observe in measurements. This allows us to estimate $P_{same}(i,j)\ \forall (i,j)$

Note: If we flip 3 coins (heads or tails) then at least 2 of them must have the same value. This gives us the following Bell's inequality:

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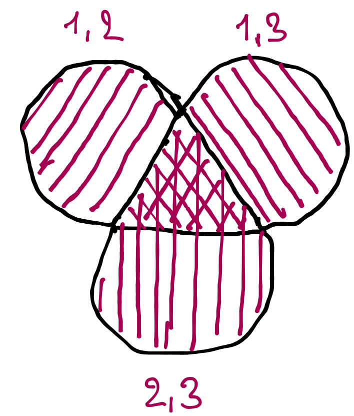

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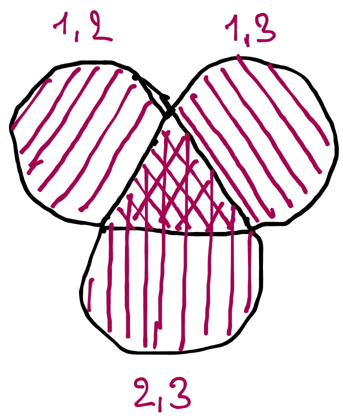

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<u>Quantum Mechanics</u>: (From Preskill)

The prob. of an outcome is the expectation value of the corresponding projector. We have

$$E(\vec{n},+) = |\uparrow_{\vec{n}}\rangle\langle\uparrow_{\vec{n}}| = \frac{1}{2}(\mathbb{1} + \vec{n}\cdot\vec{\sigma}) = \frac{1}{2}(1+\sigma_{\vec{n}})$$

$$E(\vec{n},-) = |\downarrow_{\vec{n}}\rangle\langle\downarrow_{\vec{n}}| = \frac{1}{2}(\mathbb{1} - \vec{n}\cdot\vec{\sigma}) = \frac{1}{2}(1-\sigma_{\vec{n}})$$

projectors for outcomes up/down along $\vec{n}$

Probability of identical outcomes

$$\mathcal{P}(\pm,\pm) = \langle\Psi_-|E^{(A)}(\vec{n},\pm)\,E^{(B)}(m,\pm)|\Psi_-\rangle \text{ 1)}$$

$$= \langle\Psi_-|\frac{1}{2}(\mathbb{1} + \sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(B)} \pm \sigma_{\vec{n}}^{(A)} \pm \sigma_{\vec{m}}^{(B)}|\Psi_-\rangle$$

$$= \frac{1}{4}(1 + \langle\Psi_-|\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(B)}|\Psi_-\rangle \text{ 2)}$$

Next we use

$$\left(\vec{\sigma}^{(A)} + \vec{\sigma}^{(B)}\right)|\Psi_-\rangle = 0 \Rightarrow \vec{\sigma}^{(B)}|\Psi_-\rangle = -\vec{\sigma}^{(A)}|\Psi_-\rangle \Rightarrow$$

$$\langle\Psi_-|\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(B)}|\Psi_-\rangle = -\langle\Psi_-|\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(A)}|\Psi_-\rangle$$

$$= \mathrm{Tr}[\rho_A\sigma_{\vec{n}}^{(A)}\sigma_{\vec{m}}^{(A)}] = \frac{1}{2}\mathrm{Tr}[(\vec{n}\cdot\vec{\sigma})(\vec{m}\cdot\vec{\sigma})] \qquad \text{(using } \rho_A = \tfrac{1}{2}\mathbb{1}\text{)}$$

$$= -\frac{1}{2}\sum_{ij} n_i m_j \mathrm{Tr}[\sigma_i^{(A)}\sigma_j^{(A)}] = -\frac{1}{2}\sum_{ij} n_i m_j \delta_{ij} \qquad \text{(using } \mathrm{Tr}[\sigma_i\sigma_j] = \delta_{ij}\text{)}$$

$$= -\vec{n}\cdot\vec{m} = -\cos\theta, \quad \theta = \text{angle between } \vec{n}, \vec{m}$$

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1) $|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_{\vec{n}}^{(A)}\rangle|\downarrow_{\vec{n}}^{(B)}\rangle - |\downarrow_{\vec{n}}^{(A)}\rangle|\uparrow_{\vec{n}}^{(B)}\rangle\right)$, 2) $\langle\sigma_{\vec{n}}^{(A)}\rangle = \langle\sigma_{\vec{m}}^{(B)}\rangle = 0$

EPR and Bell Inequalities (Preskill ch. 4.1)

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Note: σ_i are the Pauli operators $i=1,2,3$, and $\sigma_{\vec{n}}$ is the component of the Pauli vector along $\vec{n}$

This gives us

$$\boxed{\begin{aligned}\mathcal{P}(\pm,\pm) &= \frac{1}{4}(1-\cos\theta)\\ \mathcal{P}(\pm,\mp) &= \frac{1}{4}(1+\cos\theta)\end{aligned}}$$

In John Bell's version of the EPR experiment the angles between the $\vec{n}_i$ and the $\vec{m}_j$ are all 60°, $\cos 60^\circ = 1/2$.

QM: $P_{same}(1,2)+P_{same}(1,3)+P_{same}(2,3) = 3/4$

Whereas

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This is in conflict with the LHV model of the experiment
Actual experiments agree with QM, rules out LHV's by many standard deviations.

David Mermin: Is reality really real ? (Physics Today)

Possible Resolutions

* **New physics** beyond QM – no sign so far
* **Complementarity** – Alice did not measure $\sigma^{(A)}_{\vec{n}_i}$, $\sigma^{(A)}_{\vec{n}_j}$ and it is meaningless to assign probs to measurements that were not done.
* **Nonlocality** – Bobs choice affects outcomes of Alice's measurements
* **Alternative**: Take QM at face value.

  Nature does not allow us to assign LHV descriptions to Alice and Bob's qubits if they are entangled. Only the Global State Has objective Physical Reality

  This is not hard to accept if we embrace the viewpoint of Quantum Information Science

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Quantum States are States of Knowledge. Thus, in the EPR experiment a global observer is permitted by nature to have only as much information as can be incorporated in the global state vector. This allows to predict correlations and nothing else. Nature does not allow local observers with access to only one spin to have any information about it.

Real EPR Experiments:

* Tend to use photons with entangled polarization states
* Earliest experiments used photons produced in atomic cascades (Aspect); modern experiments use photon pairs from spontaneous parametric downconversion.
* Photon experiments use polarization states $|x\rangle$, $|y\rangle$ corresponding to linear polarizations forming a 90° angle

Relevant formulae contains angles that are half of those for spins

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Loopholes – some examples

* Locality (space-like separated measurements)
* Fair Sampling (detection efficiency)
* Freedom of choice (truly random meas. settings)
* Coincidence – time (locally defined detection windows)
* Memory (trials not identical and independent)

See selection of papers on the EPR paradox under the "Reading" tab on the OPTI 646 website.

* First good experiments   Aspect et al. (3 papers)
* Loophole free experiments

  Hensen et al., Giustina et al., Shalm et al.