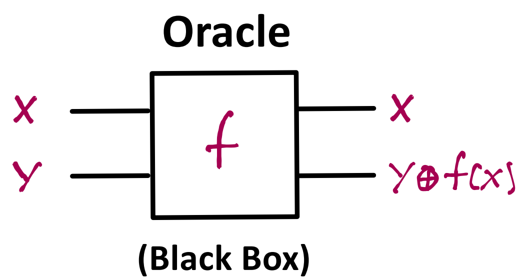
Begin
08-30-2023

Introduction and Overview (Preskills Notes)

Quantum Advantage

David Deutsch: Toy problem that shows Quantum Advantage



Is the function

constant $f(0) = f(1)$

balanced $f(0) \neq f(1)$

Classical Box: Need 2 queries $f(0)$ & $f(1)$

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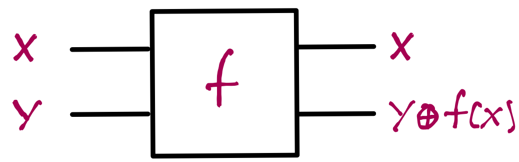

(Black Box)

Is the function

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Quantum Computation:

Input $|x\rangle|y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$U_f: \frac{1}{2}\left((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right)(|0\rangle - |1\rangle)$

Measure 1st qubit in basis $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$\rightarrow$ $|+\rangle$ if constant, $|-\rangle$ if balanced

Quantum Speedup: can solve w/1 query

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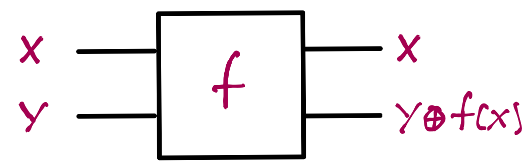

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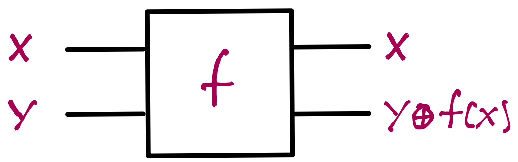

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Quantum Speedup: can solve w/1 query

Key aspect of Deutsch's algorithm:

We are looking for a global property of the function f

N bit binary number

Generally: $U_f: |x\rangle|0\rangle \rightarrow |x\rangle|f(x)\rangle$

Input $|\psi_{in}\rangle = \left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]^{\otimes N}|0\rangle$

$$= \frac{1}{2^{N/2}}\sum_{x=0}^{2^N-1}|x\rangle|0\rangle$$

compute once

Output $|\psi_{out}\rangle = \frac{1}{2^{N/2}}\sum_{x=0}^{2^N-1}|x\rangle|f(x)\rangle$

Global properties encoded in state, trick is to extract desired information

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Quantum Advantage

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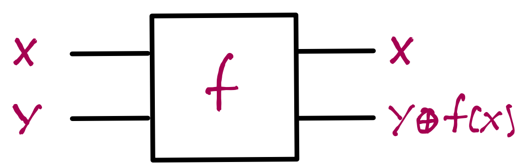

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Peter Shor: Period finding, QFT, Factoring

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Next: Will this work with real-world Quantum Hardware ?

Faulty gates, decoherence !

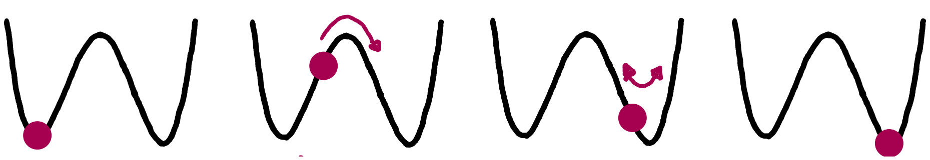
Introduction and Overview (Preskills Notes)

Quantum Error Correction

Fundamental Problem

Quantum States are fragile, especially when entangled

Classical Computation ? **Dissipation helps**



bit | logic operation | error | damping

No dissipation ⇨ **Errors build up**

Quantum Computation ⇨

* Cannot tolerate dissipation
* Destroys superposition and entanglement

What to do ? **Error Correction !**

<u>Classical</u> Error Correction:

Simple example: **<u>Redundancy</u> protects against bit flips**

<u>Encode:</u>
$0 \rightarrow (000)$
$1 \rightarrow (111)$

<u>Errors:</u>
$(000) \rightarrow (100)$
$(111) \rightarrow (011)$

correct by majority vote

Introduction and Overview (Preskills Notes)

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Von Neumann:

* A classical computer w/faulty components can work, given enough redundancy
* Classical error correction is well developed and highly sophisticated…

* Quantum Errors

1) Bit Flip $|0\rangle \to |1\rangle$, $|1\rangle \to |0\rangle$, phase flip $|0\rangle \to |0\rangle$, $|1\rangle \to -|1\rangle$

2) Small errors $a|0\rangle + b|1\rangle$ a, b can change by ε errors accumulate

3) Measurement disturbs ➡ collapse of quantum states

4) No cloning ➡ Cannot protect by making copies

Introduction and Overview (Preskills Notes)

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Example: Peter Shor's code for bit flip error when $P(\text{error}) \ll 1$

Encode: $|0\rangle \to |\bar{0}\rangle \equiv |000\rangle$, $|1\rangle \to |\bar{1}\rangle \equiv |111\rangle$ (3 bit code)

$$a|0\rangle + b|1\rangle \to a|000\rangle + b|111\rangle$$

Single-qubit measurement ⇨ collapse of state, destroys info, no majority voting!

Collective 2-qubit measurement:

- for $|x, y, z\rangle$ measure $y \oplus z$, $x \oplus z$ (never measure individual bits)
- if $|000\rangle$, $|111\rangle$ these observables $= 0$
- if one bit-flip, at least one observable $= 1$
- easy to check that $(y \oplus z, x \oplus z) =$ binary address of qubit flip

$|000\rangle \to |010\rangle$ $(1, 0) =$ 2nd bit

Introduction and Overview (Preskills Notes)

Example: Peter Shor's code for bit flip error when P(error) << 1

Encode:

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$$|1\rangle \to |\bar{1}\rangle \equiv |111\rangle$$

(3 bit code)

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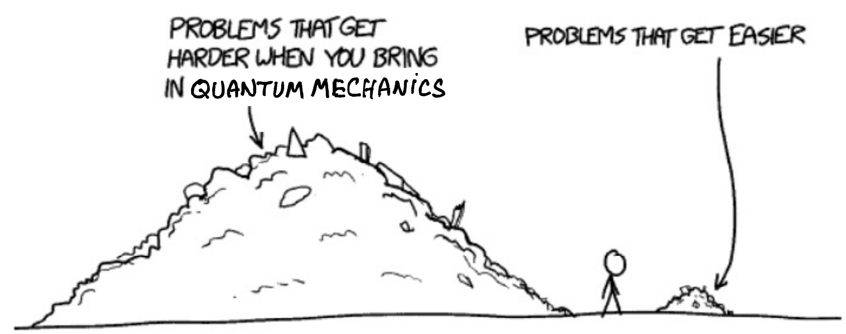
$|000\rangle \to |010\rangle$ $\quad (1,0)$ = 2nd bit

Small errors:

$$|000\rangle \to |000\rangle + \varepsilon|001\rangle$$
$$|111\rangle \to |111\rangle + \varepsilon|110\rangle$$

Quantum mechanics to the rescue !

- mostly no error detected
  ➡ collapse into $|000\rangle$ resp. $|111\rangle$
- sometime error detected
  ➡ collapse into $|001\rangle$ resp. $|110\rangle$
  ➡ full bit flip, correct as such



Source: xkcd.com

Introduction and Overview (Preskills Notes)

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How to implement ?

Quantum circuit + single qubit measurement

Quantum Gates – work on superpositions, and entangled states

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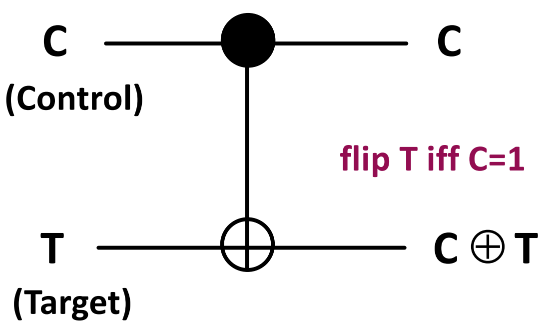
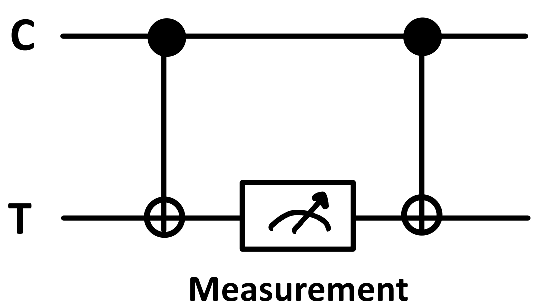
Controlled-NOT (CNOT)



Truth Table

C	T	C ⊕ T
0	0	0
0	1	1
1	0	1
1	1	0

Quantum Circuit for joint measurement



Measurement in $\{|0\rangle, |1\rangle\}$ basis

yields $C \oplus T$

Circuit maps logical basis states as

C	T
0	0
1	1
0	1
1	0

CNOT ⇒

C	T
0	0
1	0
0	1
1	1

meas. T ⇒

C	T
0	0
1	0
0	1
1	1

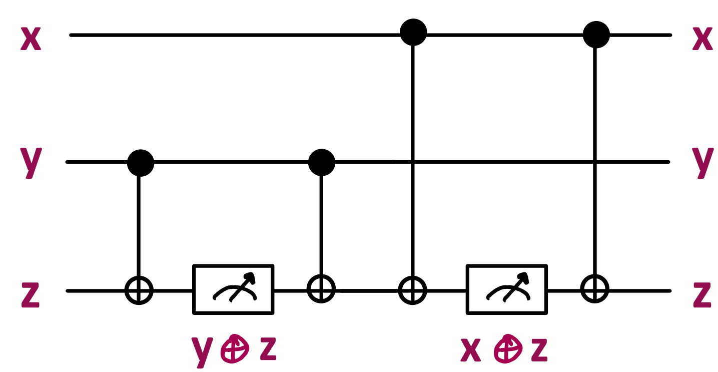
CNOT ⇒

C	T
0	0
1	1
0	1
1	0

$T=0$ iff same

$T=1$ iff different

Full circuit to obtain Error Syndrome



* iff qubit flip, binary address $= (y \oplus z, x \oplus z)$

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Circuit maps logical basis states as

C	T
0	0
1	1
0	1
1	0

CNOT ⇨

C	T
0	0
1	0
0	1
1	1

meas. T ⇨

C	T
0	0
1	0
0	1
1	1

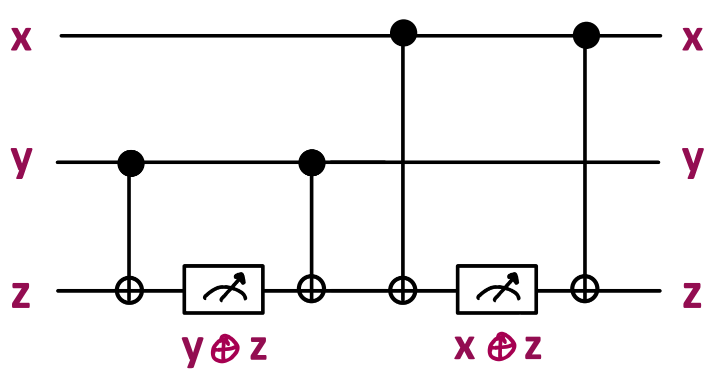
CNOT ⇨

C	T
0	0
1	1
0	1
1	0

$T=0$ iff same

$T=1$ iff different

Full circuit to obtain Error Syndrome



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Circuit maps logical basis states as

C	T
0	0
1	1
0	1
1	0

CNOT

C	T
0	0
1	0
0	1
1	1

meas. T

C	T
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1	0
0	1
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CNOT

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$T=0$ iff same

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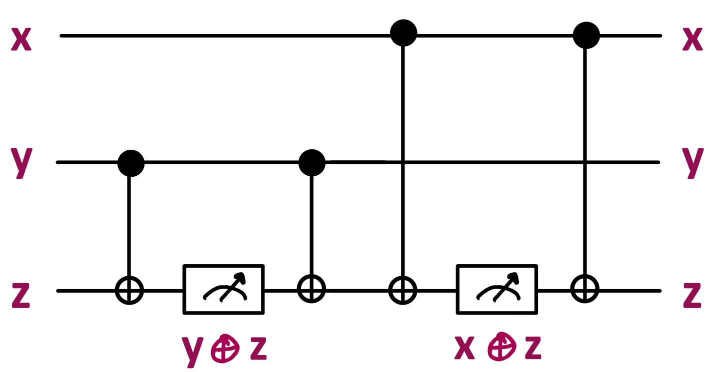
Full circuit to obtain Error Syndrome



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Quantum Phase Error