OPTI 646 **Homework Set #4** **Jessen**

Posted Monday 11/14, 2022, Due Monday 12/5, 2022

I
Some simple quantum circuits

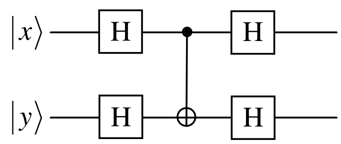
The Hadamard (**H**) and phase (**P**) gates

$$\mathbf{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad \mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

are two important single-qubit gates. The Hadamard and controlled-**P** gate (**P** is applied to the 2nd (target) qubit iff the 1st (control) qubit = 1) together make up a universal set. Two consecutive applications of the controlled-**P** gate makes up a controlled-**Z** (or controlled-σ_z).

(a) The controlled-NOT gate applies $\sigma_x = \mathbf{X}$ to the 2nd (target) qubit iff the 1st (control) qubit equals 1. Find a quantum circuit that implements it using Hadamard and controlled-**Z** gates.

(b) The controlled-NOT gate appears to act on control and target bits in fundamentally different ways. Find the action of the circuit

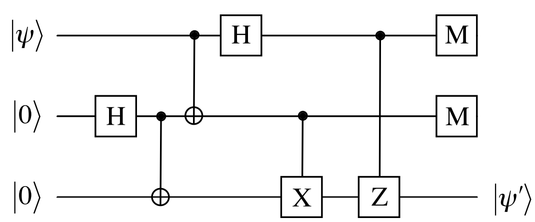


on the four two-qubit logical basis states, and show that with an appropriate change of qubit basis states the role of control and target qubits in a controlled-NOT gate are reversed.

(c) Find a quantum circuit based on single- and two-qubit gates which maps the (unentangled) 2-qubit logical basis states onto the Bell states.

II
Teleportation with a quantum circuit

Consider the following quantum circuit



where $\mathbf{X}=\sigma_x$, $\mathbf{Z}=\sigma_z$ and **M** indicates an orthogonal measurement in the logical basis.

(a) Show that the circuit achieves "teleportation" in the sense that, after the first two qubits are measured, the state of the third is $|\psi'\rangle = |\psi\rangle$.

Of course this is not strictly teleportation, as qubit one and three must undergo a direct quantum mechanical interaction. (We can easily achieve real teleportation by modifying the circuit so that the **X** and **Z** gates are controlled by the outcomes of the two measurements.) The circuit does however succeed in moving the quantum state $|\psi\rangle$ from one qubit to another.

(b) Find a much simpler circuit that interchanges the quantum states of two qubits without the need for a 3rd ancilla qubit.