Quantum non-demolition measurements in quantum optomechanics

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Outline

- What are quantum non-demolition (QND) measurements?
  - Measuring the position of a free particle
  - Interest in quantum optomechanics: the standard quantum limit (SQL)
- General formulation for QND measurements
  - Difference from the most general measurement in QM
- Examples of quantum non-demolition measurements in quantum optomechanics
  - Stroboscopic measurement of position
  - Energy level jumps of a quantum harmonic oscillator

Measuring the position of a free particle

What happens when we try to make a precise measurement of $x(t)$ of a free particle?

Position and momentum are observables which do not commute:

$$[\hat{x}, \hat{p}] = i\hbar$$

$$\Delta x \Delta p \geq \frac{\hbar}{2} \rightarrow \Delta p \geq \frac{\hbar}{2\Delta x}$$

$$x(t) = x(0) + v(0)t$$

$$= x(0) + \frac{p(0)t}{m}$$

Braginsky and Khalili, *Quantum Measurement*.

Measuring the position of a free particle

$$x(t) = x(0) + \frac{p(0)t}{m}$$

$$[\Delta x(t)]^2 = [\Delta \mathrm{x}(0)]^2 + \left[\frac{\Delta p(0)}{m}\right]^2 t^2$$

With $\Delta p \geq \frac{\hbar}{2\Delta x}$:

$$[\Delta x(t)]^2 \geq [\Delta \mathrm{x}(0)]^2 + \left[\frac{\hbar}{2m\Delta x(0)}\right]^2 t^2$$

$$= \Delta x_{measure}^2 + \Delta x_{added}^2$$

- Measurement increases variance/noise!
    - Altering the state of the particle (determined by x and p)

Braginsky and Khalili, *Quantum Measurement*.

SQL for a free particle

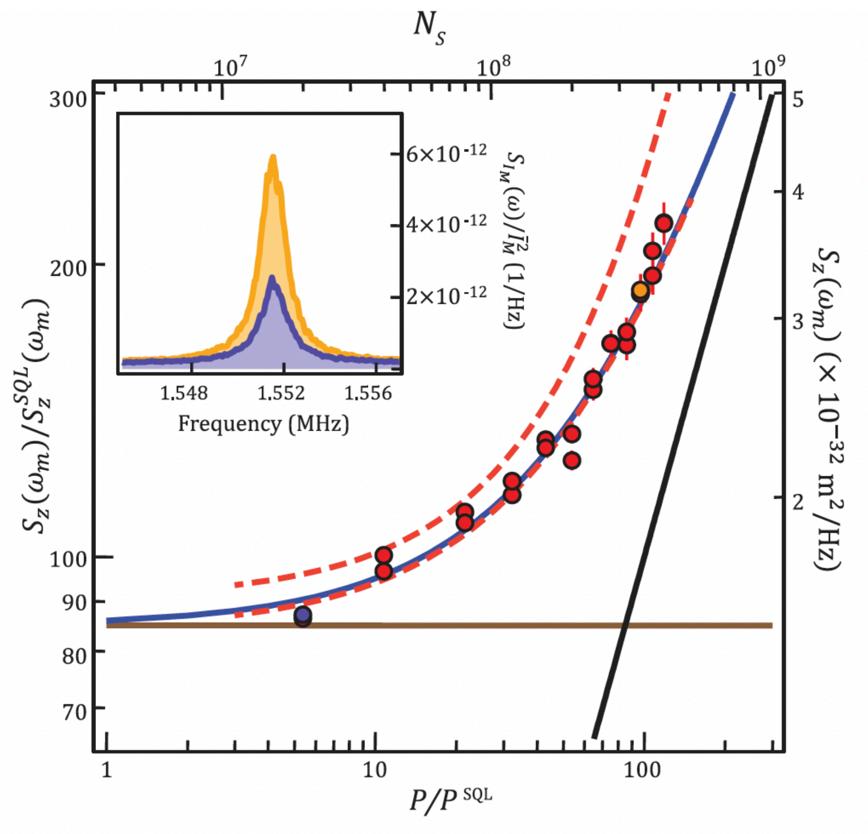
$$[\Delta x(t)]^2 \geq [\Delta \mathrm{x}(0)]^2 + \left[\frac{\hbar}{2m\Delta x(0)}\right]^2 t^2$$

- Lower bound on $\Delta x(t)$ → "Standard quantum limit" (SQL)
- SQL for $\Delta x(t)$: $\sqrt{\hbar t/2m}$
- Other SQL's (not just for the position of a free particle)
- In general, the act of measuring an observable alters the system
  - "Quantum backaction" or "measurement backaction"
- For certain experiments, repeated/continuous measurement is desirable
  - But you don't want your measurement to disturb what you are trying to measure!

Braginsky and Khalili, *Quantum Measurement*.

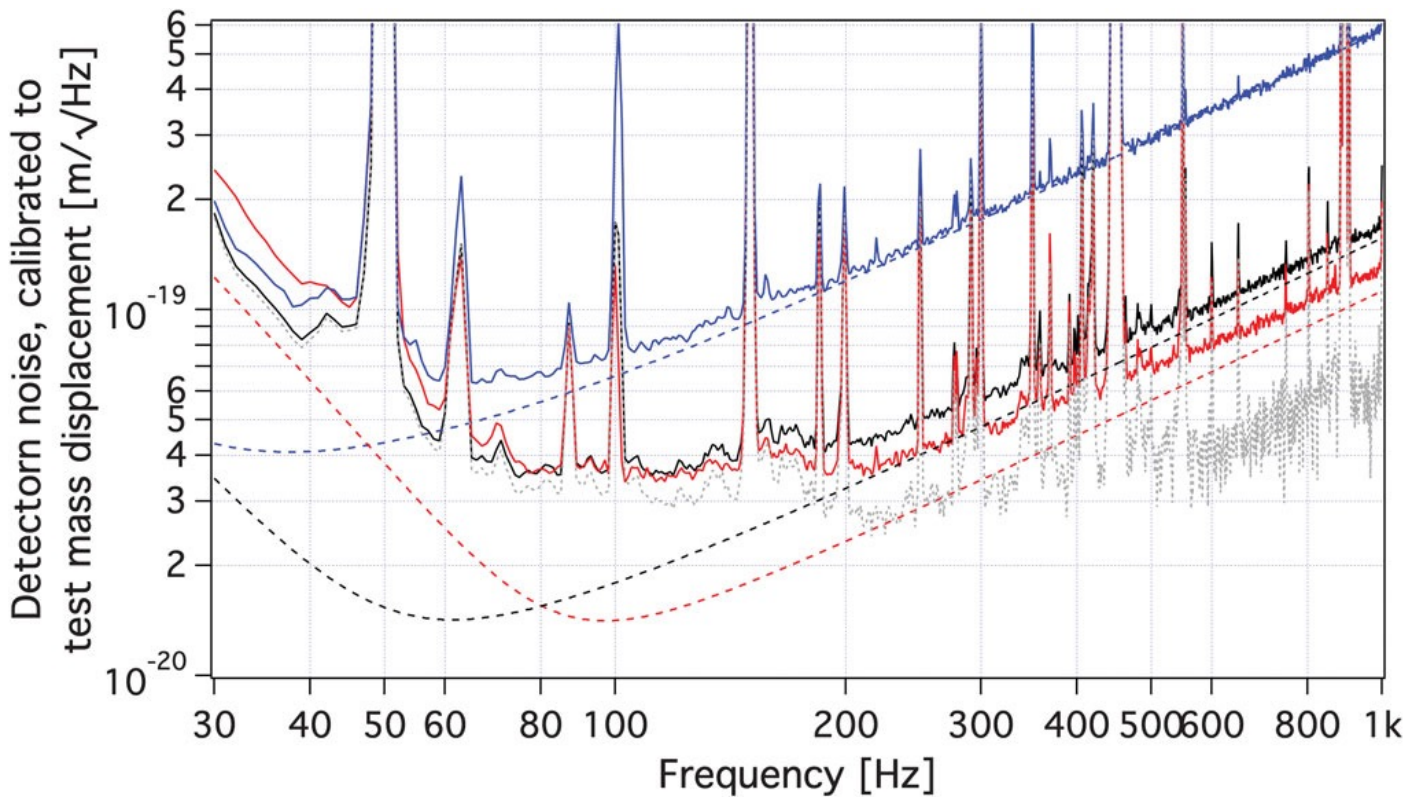
Quantum backaction on a mechanical oscillator

- Continuous measurement of position (7 ng mechanical oscillator)
- Meter (light field) disrupts the measurement of the position of the oscillator, increases total noise



Purdy, Peterson, and Regal, “Observation of Radiation Pressure Shot Noise on a Macroscopic Object.”

Quantum backaction in VIRGO



The Virgo Collaboration et al., “Quantum Backaction on Kg-Scale Mirrors.”

Can we avoid backaction?

- Trying to beat the SQL
- What happens if you try to monitor $\hat{p}(t)$ for a free particle?
- Momentum measurement will necessarily perturb position
- With no external force, $p(t) = p(0)$
  - No coupling position and momentum– no backaction!
  - As a result, no SQL
  - $\hat{p}$ is a “QND variable” or “QND observable”
  - Useful for monitoring a weak classical force

Braginsky and Khalili, *Quantum Measurement*.

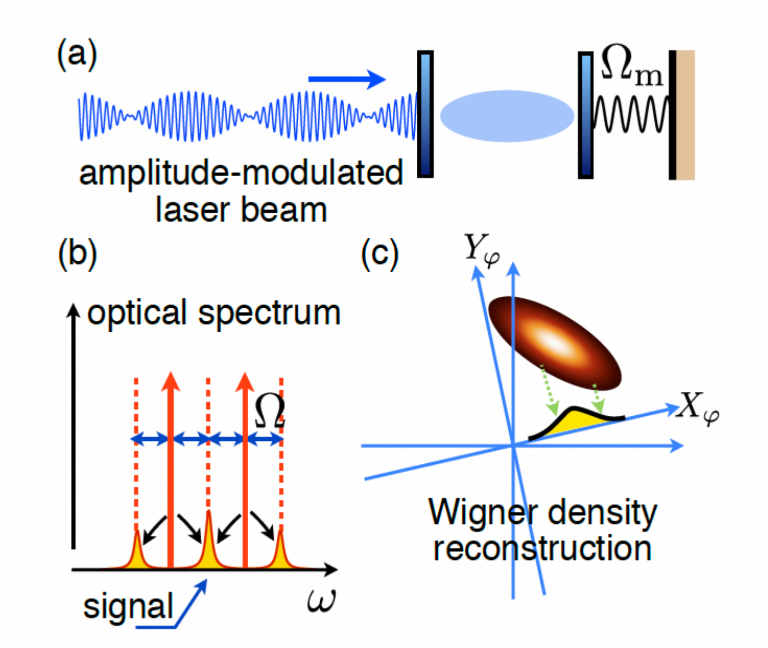
Quantum non-demolition measurements

- "Sequence of precise measurements such that the result of the measurement is completely predictable from the result of the first measurement"
  - "back-action evading measurement"
- Most general approach: entangle your system of interest with a "meter" system, then read out the meter
  - $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_M + \mathcal{H}_I$
    - $\mathcal{H}_I = K\hat{A}\hat{M}$
- Measurement observable must commute with itself at different times: $\left[\hat{A}(t_1), \hat{A}(t_2)\right] = 0$
  - Stroboscopic and continuous QND observables
- To avoid back-action on the measured obserbable: $\left[\hat{A}(t), \hat{\mathcal{H}}_I(t)\right] = 0$
  - Still might contaminate other observables of the system

Braginsky and Khalili, *Quantum Measurement*.

Surpassing the SQL: single quadrature measurements of a harmonic oscillator

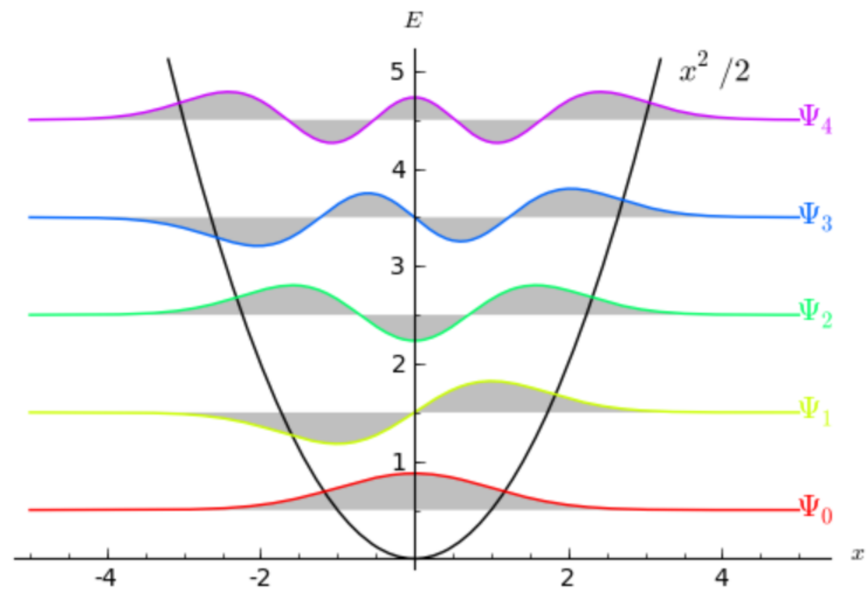
- Measure a single quadrature $\left(\hat{X} \text{ or } \hat{Y}\right)$ of motion for a harmonic oscillator:
$$\hat{x}(t) = \hat{X}\cos(\Omega_m t + \phi) + \hat{Y}\sin(\Omega_m t + \phi)$$
- Choose to measure $\hat{X}$: will perturb $\hat{Y}$
$$\left[\hat{X}, \hat{Y}\right] = i\hbar/m\Omega_m$$
- Measure just $\hat{X}$, not both $\hat{X}$ and $\hat{Y}$ simultaneously
- Alternatively, $x(t)$ for only certain times:
$$[\hat{x}(t_1), \hat{x}(t_2)] = \left(\frac{i\hbar}{m\Omega_m}\right)\sin(\Omega_m \delta t)$$
- Amplitude modulate or pulse laser to measure every $t_{measure} = n\pi/\Omega_m$
  - With the correct phase, $\rightarrow \sin(\Omega_m t + \phi) = 0$
  - Only measuring $\hat{X}$, no SQL (but limited measurement frequency)



Aspelmeyer, Kippenberg, and Marquardt, "Cavity Optomechanics."

Mechanical number state (phonon number) detection

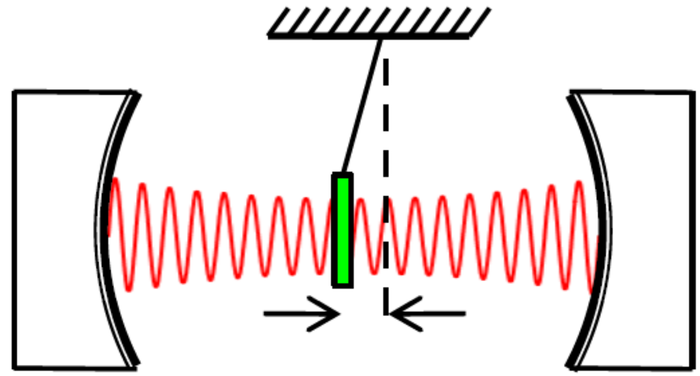
- Goal: Observe individual "quantum jumps" between QHO energy levels
- Need to make QND measurement not to exchange excitations between the meter and QHO
  - Absent an external force, $\mathcal{H}_S$ commutes with itself (energy is conserved)
- $\mathcal{H}_S = \frac{p^2}{2m} + \frac{1}{2} m \Omega_m x^2$
  - Make oscillator heavy and find probe that couples to x^2



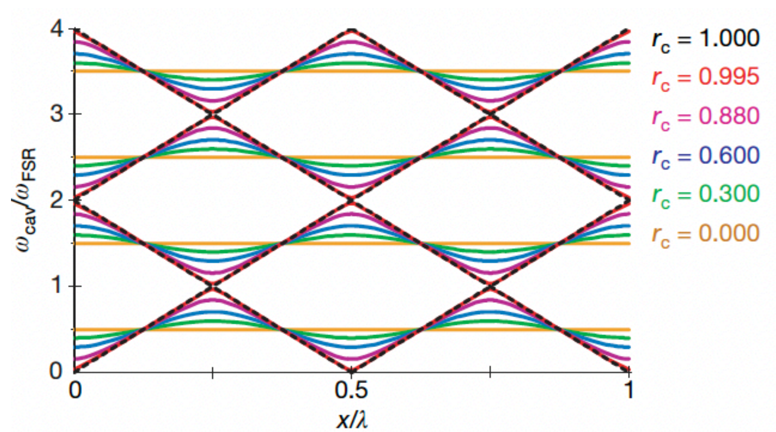
Aspelmeyer, Kippenberg, and Marquardt, "Cavity Optomechanics."

Mechanical number state (phonon number) detection

$$\mathcal{H}_S = \frac{p^2}{2m} + \frac{1}{2} m \Omega_m x^2$$



- Make light field responds to changes in x^2, not x
  - Phase shift is proportional to energy
- Place QHO at a node in the intracavity standing wave
  - Cavity frequency changes with respect to position of the QHO: $\frac{\partial \omega_c}{\partial x} = 0, \frac{\partial^2 \omega_c}{\partial x^2} \neq 0$



Aspelmeyer, Kippenberg, and Marquardt, "Cavity Optomechanics."
Thompson et al., "Strong Dispersive Coupling of a High-Finesse Cavity to a Micromechanical Membrane."

Bibliography/Further reading

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Scully, Marlan O., and M. Suhail Zubairy. *Quantum Optics*. Cambridge: Cambridge University Press, 1997. https://doi.org/10.1017/CBO9780511813993.

Measuring the spin of an electron

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z + |\downarrow\rangle_z)$$

Make a measurement along the z-axis: either get up or down along z

Repeated measurements yield the same result (no longer have a superposition of the two states)

Measurement changes the state of the system

Rigourous requirements for QND measurements

- System interacts with probe: $\frac{\partial \widehat{\mathcal{H}_I}}{\partial \widehat{A_S}} \neq 0$
- Meter tracks the motion of the system: $[A_M, \mathcal{H}_I] \neq 0$
- System should not be affected by its coupling to the probe: $[A_M, \mathcal{H}_I]$ = 0
- System Hamiltonian is not a function of the conjugate observable of the system: $\frac{\partial \mathcal{H}_S}{\partial A_S^c} = 0$
- For free particle momentum linearly coupled to a meter:
  - $\mathcal{H} = \frac{\hat{p}^2}{2m} + \lambda \hat{p} \widehat{M}$
  - $[\mathcal{H}, p] = 0$

Generally, $\left[\hat{A}(t), \hat{A}(t')\right] \neq 0$
Equivalent condition: $\left[\hat{A}, \widehat{H}\right] = 0$

Braginsky and Khalili, *Quantum Measurement*.
Scully and Zubairy, *Quantum Optics*.