Wigner-Weisskopf Theory of Spontaneous Decay

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Time Dep. Perturbation Theory:

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$$c_{1,1_{\vec{k},\lambda}}(t) = -i\, g^{*}_{\vec{k},\lambda}\int_0^t e^{-i(\omega_{eg}-\omega_{\vec{k}})t'}\, c_{2,0}(t')\,dt'$$

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda}|g_{\vec{k},\lambda}|^2\int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')}\, c_{2,0}(t')\,dt'}$$

Time Dep. Perturbation Theory:

Study short-time limit, $c_{2,0}(t)\sim 1$

Wigner-Weisskopf Theory of Spontaneous Decay

Expand

$$\boxed{|\psi(t)\rangle = c_{2,0}(t)|2,0\rangle + \sum_{\vec{k},\lambda} c_{1,1_{\vec{k},\lambda}}(t)|1,1_{\vec{k},\lambda}\rangle}$$

$$\dot{c}_{2,0}(t) = -i\sum_{\vec{k},\lambda} g_{\vec{k},\lambda}\, e^{i(\omega_{eg}-\omega_{\vec{k}})t} c_{1,1_{\vec{k},\lambda}}(t)$$

$$\dot{c}_{1,1_{\vec{k},\lambda}}(t) = -i\, g^*_{\vec{k},\lambda}\, e^{-i(\omega_{eg}-\omega_{\vec{k}})t} c_{2,0}(t)$$

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$$c_{1,1_{\vec{k},\lambda}}(t) = -i\, g^*_{\vec{k},\lambda} \int_0^t e^{-i(\omega_{eg}-\omega_{\vec{k}})t'}\, c_{2,0}(t')\, dt'$$

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Study short-time limit, $c_{2,0}(t) \sim 1$

→ **Fermi's Golden Rule**

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Expand

$$|\psi(t)\rangle = c_{2,0}(t)|2,0\rangle + \sum_{\vec{k},\lambda} c_{1,1_{\vec{k},\lambda}}(t)|1,1_{\vec{k},\lambda}\rangle$$

$$\dot{c}_{2,0}(t) = -i\sum_{\vec{k},\lambda} g_{\vec{k},\lambda}\, e^{i(\omega_{eg}-\omega_{\vec{k}})t} c_{1,1_{\vec{k},\lambda}}(t)$$

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Study short-time limit, $c_{2,0}(t) \sim 1$

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No good for this problem!

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Wigner-Weisskopf Theory of Spontaneous Decay

$$\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'$$

Quantization in a box w/periodic B. C.

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{C}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} C_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$\Rightarrow$ $k_i = n_i \frac{2\pi}{L}$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

➡ $k_i = n_i \frac{2\pi}{L}$, n **integer**

Wigner-Weisskopf Theory of Spontaneous Decay

$$\dot{C}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} C_{2,0}(t')\,dt'$$

Quantization in a box w/periodic B. C.

➡ $k_i = n_i \frac{2\pi}{L}$, n **integer**

In $\vec{k}$ space the modes form a grid:

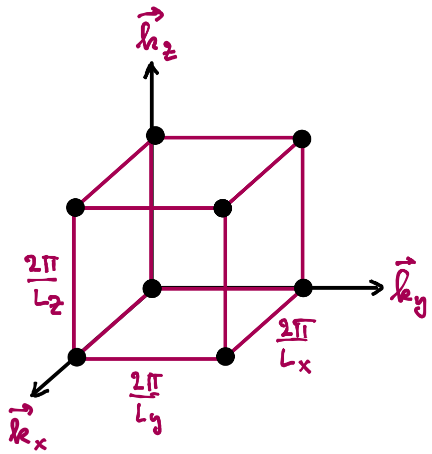
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$\Rightarrow$ $k_i = n_i \frac{2\pi}{L}$, n **integer**

In $\vec{k}$ **space the modes form a grid:**



Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

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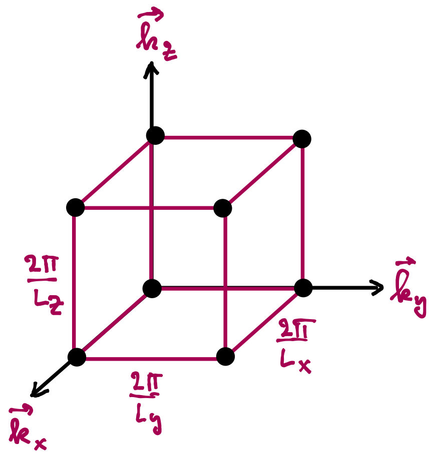


$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)}$$

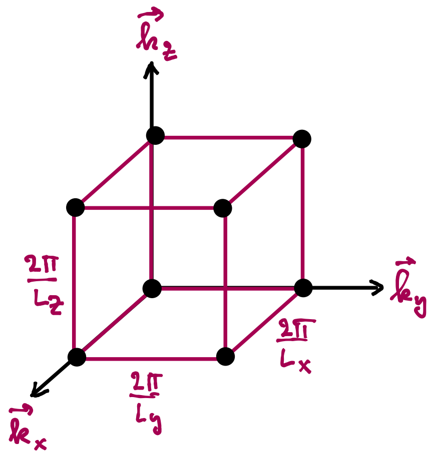
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{C}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} C_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

⇨ $k_i = n_i \frac{2\pi}{L}$, n **integer**

In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{V}{(2\pi)^3}$$

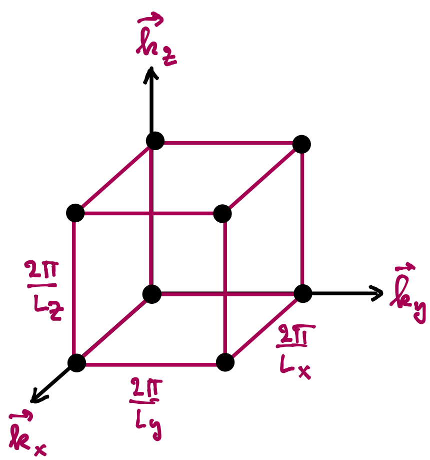
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$\Rightarrow$ $k_i = n_i \frac{2\pi}{L}$, n **integer**

In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)}$$

$$= \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\, dt'}$$

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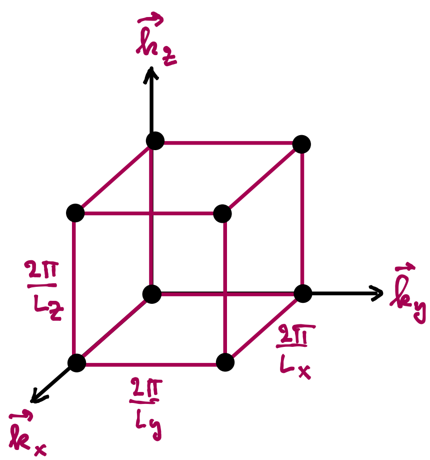


$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

Density of Modes

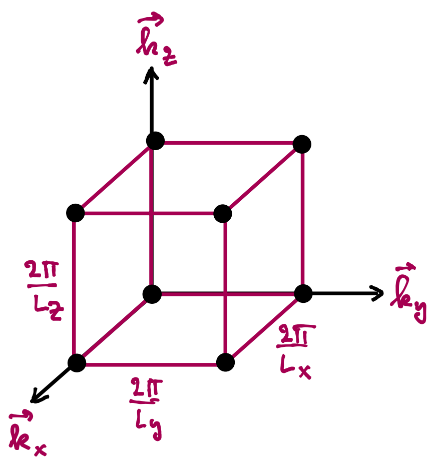
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{C}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} C_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$\Rightarrow$ $k_i = n_i \frac{2\pi}{L}$, n **integer**

In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

Density of Modes

Convert sum to integral over modes:

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

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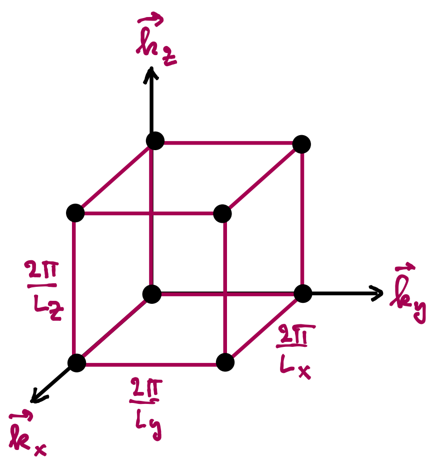

$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

Density of Modes

Convert sum to integral over modes:

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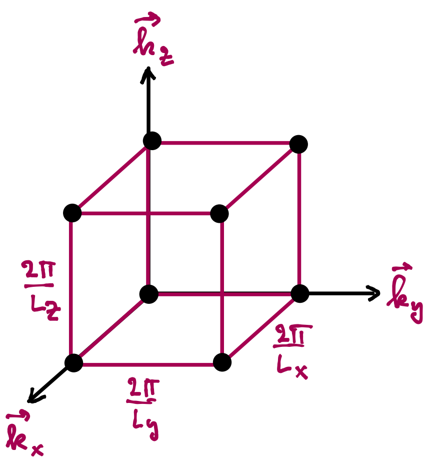

$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)}$$

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Density of Modes

Convert sum to integral over modes:

$$\sum_{\vec{k}} \rightarrow \int d^3\vec{k}\, \mathcal{D}(\vec{k}) = \int k^2 d(\hat{k})\, dk\, \mathcal{D}(\vec{k})$$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

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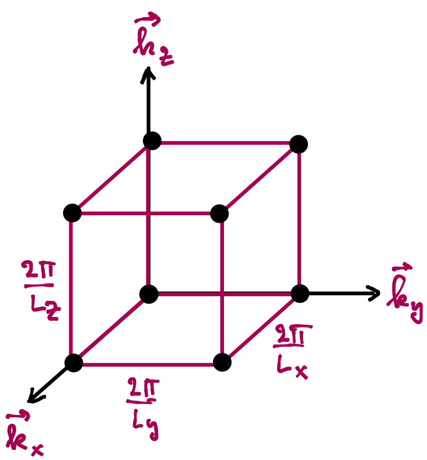

$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)}$$

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Convert sum to integral over modes:

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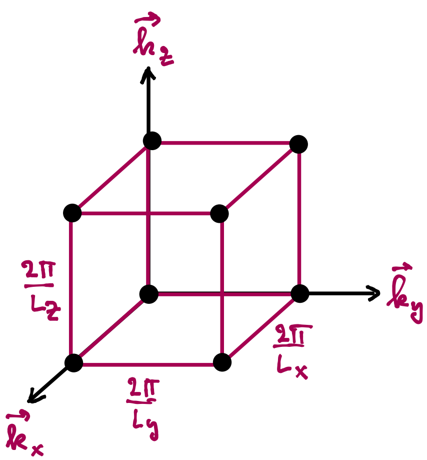

$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)}$$

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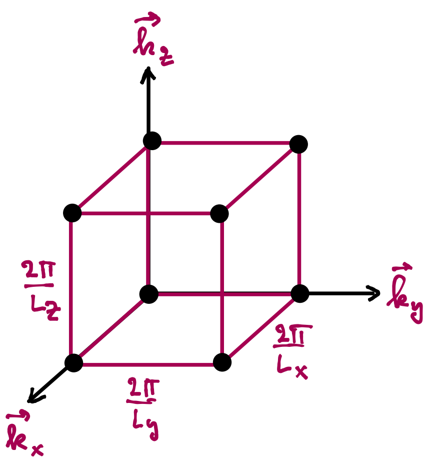
In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)}$$

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Convert sum to integral over modes:

$$\sum_{\vec{k}} \to \int d^3\vec{k}\, \mathcal{D}(\vec{k}) = \int k^2 d(\hat{k})\, dk\, \mathcal{D}(\vec{k})$$

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where $\hat{k} = \vec{k}/k$ **is a unit vector along** $\vec{k}$

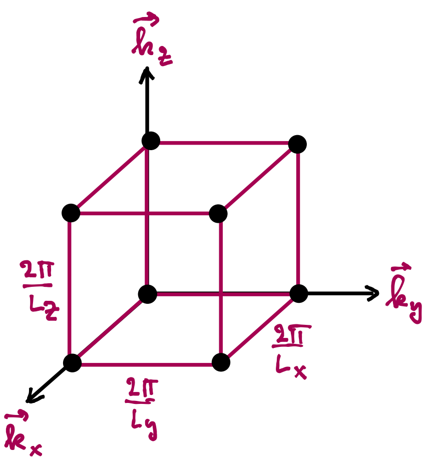
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')}\, c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$\Rightarrow$ $k_i = n_i \frac{2\pi}{L}$, n **integer**

In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

Density of Modes

Convert sum to integral over modes:

$$\sum_{\vec{k}} \to \int d^3\vec{k}\, \mathcal{D}(\vec{k}) = \int k^2 d(\hat{k})\, dk\, \mathcal{D}(\vec{k})$$

$$= \int d(\hat{k})\, d\omega_{\vec{k}} \frac{\omega_{\vec{k}}^2}{c^3} \mathcal{D}(\vec{k}) = \int d(\hat{k})\, d\omega_{\vec{k}}\, \mathcal{D}(\omega_{\vec{k}})$$

where $\hat{k} = \vec{k}/k$ **is a unit vector along** $\vec{k}$ **and**

$$\mathcal{D}(\omega_{\vec{k}}) = \frac{V}{(2\pi)^3} \frac{\omega_{\vec{k}}^2}{c^3}$$

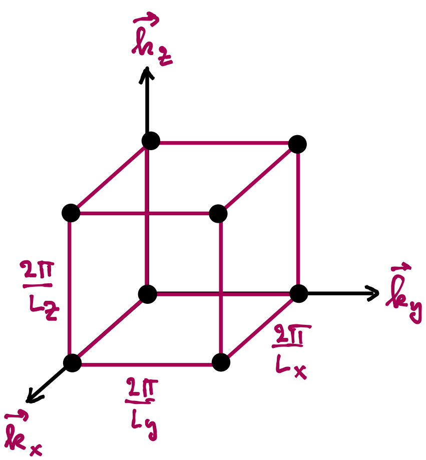
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')}\, c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$k_i = n_i \frac{2\pi}{L}$, n **integer**

In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

Density of Modes

Convert sum to integral over modes:

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where $\hat{k} = \vec{k}/k$ **is a unit vector along** $\vec{k}$ **and**

$$\mathcal{D}(\omega_{\vec{k}}) = \frac{V}{(2\pi)^3} \frac{\omega_{\vec{k}}^2}{c^3}$$

mode density in shell of $\vec{k}$ - space of radius $\omega_{\vec{k}}/c$

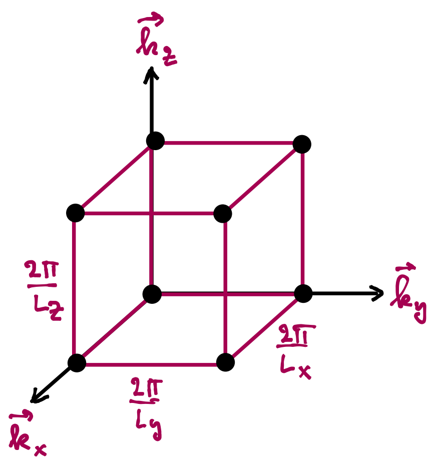
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$\Rightarrow$ $k_i = n_i \frac{2\pi}{L}$, n integer

In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

Density of Modes

Convert sum to integral over modes:

$$\sum_{\vec{k}} \to \int d^3\vec{k}\, \mathcal{D}(\vec{k}) = \int k^2 d(\hat{k})\, dk\, \mathcal{D}(\vec{k})$$

$$= \int d(\hat{k})\, d\omega_{\vec{k}} \frac{\omega_{\vec{k}}^2}{c^3} \mathcal{D}(\vec{k}) = \int d(\hat{k})\, d\omega_{\vec{k}}\, \mathcal{D}(\omega_{\vec{k}})$$

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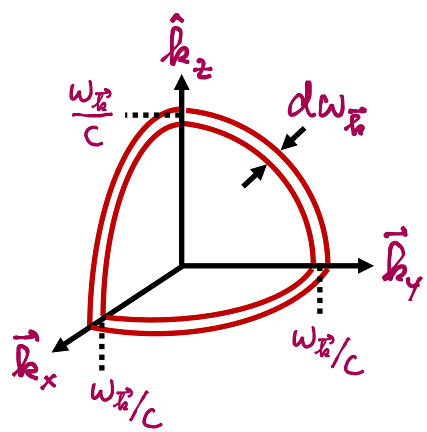


$$\mathcal{D}(\omega_{\vec{k}}) = \frac{V}{(2\pi)^3} \frac{\omega_{\vec{k}}^2}{c^3}$$

mode density in shell of $\vec{k}$ - space of radius $\omega_{\vec{k}}/c$

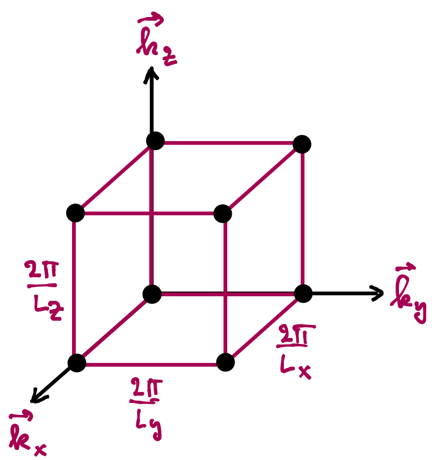
Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

Quantization in a box w/periodic B. C.

$\Rightarrow$ $k_i = n_i \frac{2\pi}{L}$, n integer

In $\vec{k}$ space the modes form a grid:



$$\frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)}$$

$$= \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k})$$

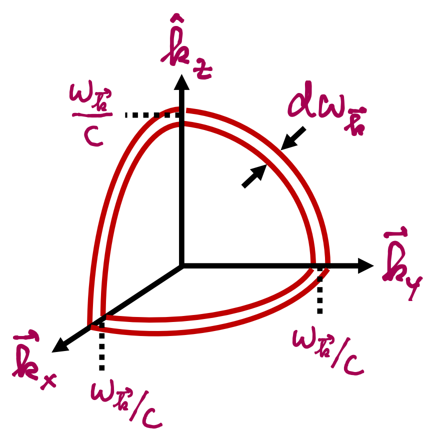
Density of Modes

Convert sum to integral over modes:

$$\sum_{\vec{k}} \to \int d^3\vec{k}\,\mathcal{D}(\vec{k}) = \int k^2 d(\hat{k})\,dk\,\mathcal{D}(\vec{k})$$

$$= \int d(\hat{k})\,d\omega_{\vec{k}} \frac{\omega_{\vec{k}}^2}{c^3} \mathcal{D}(\vec{k}) = \int d(\hat{k})\,d\omega_{\vec{k}}\,\mathcal{D}(\omega_{\vec{k}})$$

where $\hat{k} = \vec{k}/k$ is a unit vector along $\vec{k}$ and



$$\boxed{\mathcal{D}(\omega_{\vec{k}}) = \frac{V}{(2\pi)^3} \frac{\omega_{\vec{k}}^2}{c^3}}$$

mode density in shell of $\vec{k}$ - space of radius $\omega_{\vec{k}}/c$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')}\, c_{2,0}(t')\,dt'}$$

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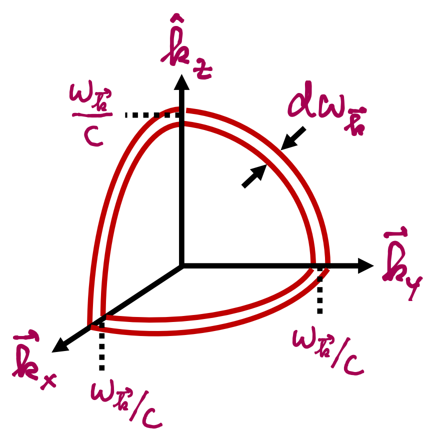

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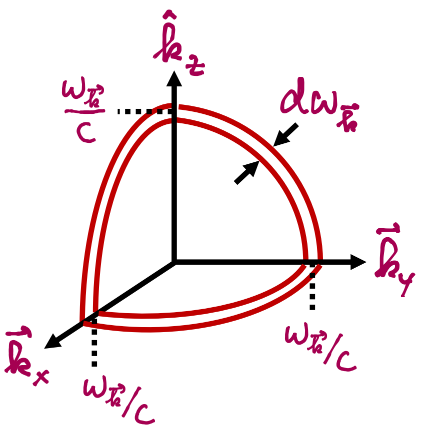

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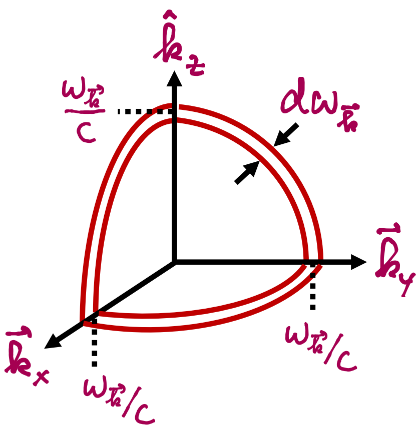

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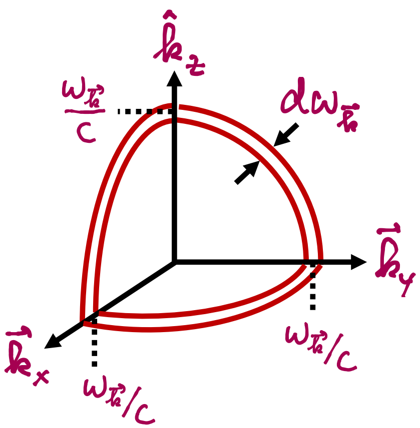

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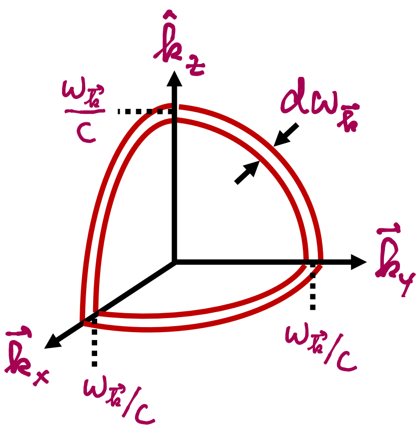

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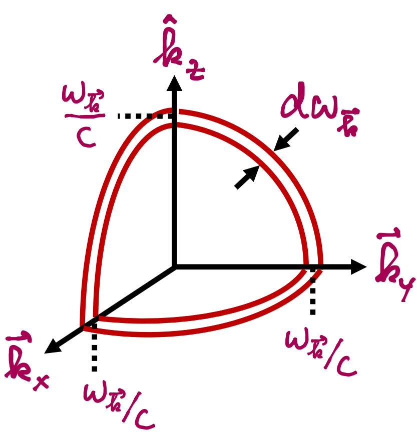

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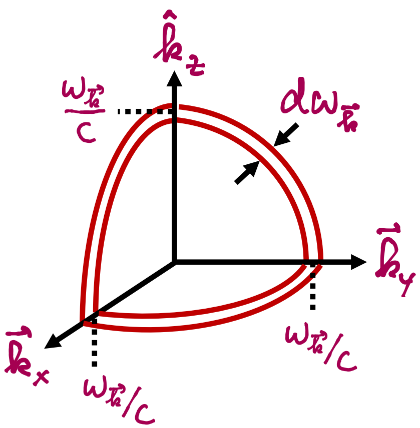

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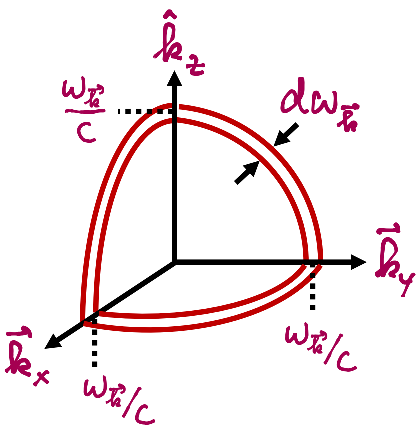

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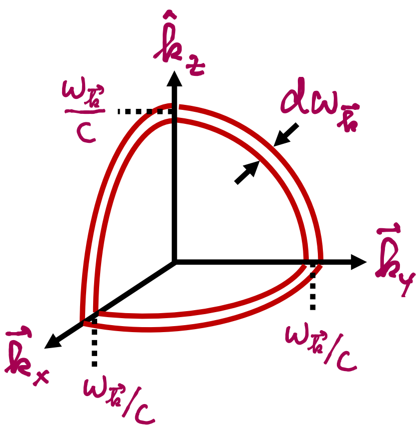

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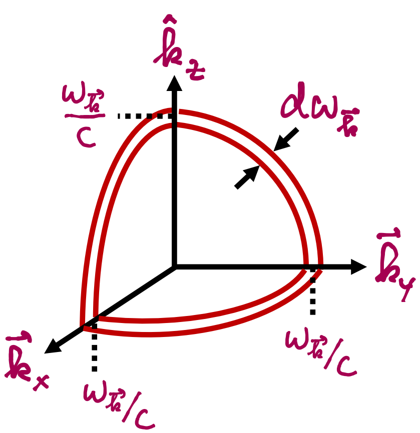

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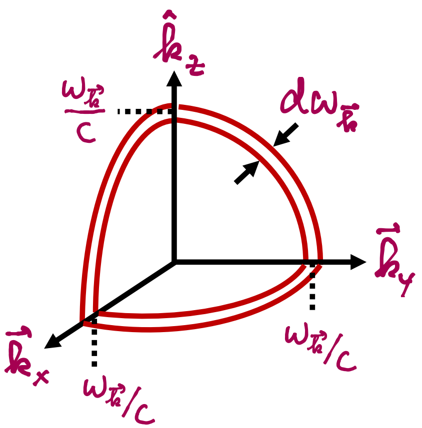

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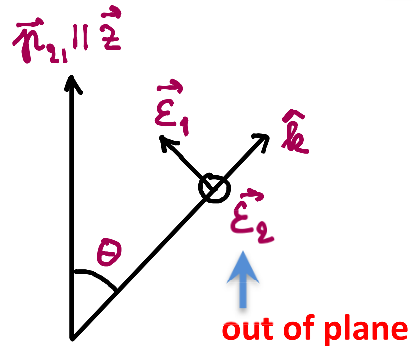

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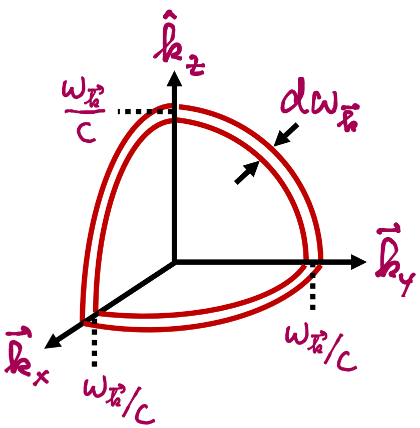

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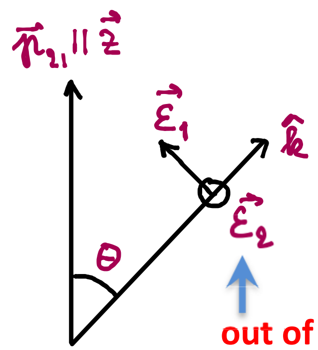

in polar coordinates

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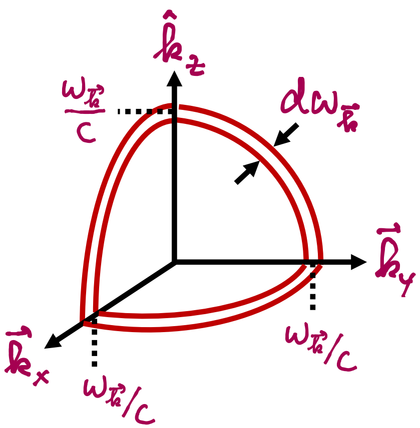

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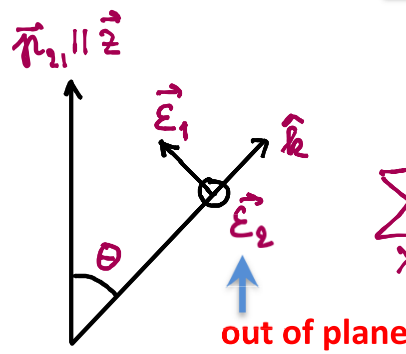

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$$\sum_\lambda |\vec{p}_{21} \cdot \vec{\varepsilon}_{\vec{k},\lambda}|^2 = \sin^2\theta\, |\vec{p}_{12}|^2$$

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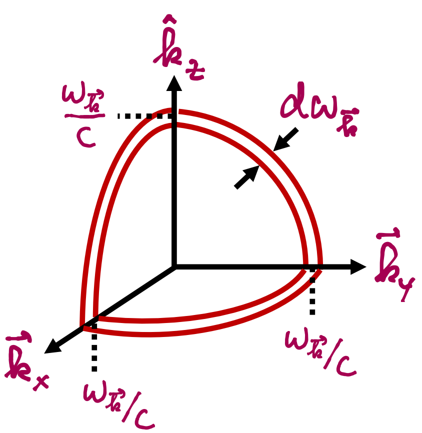

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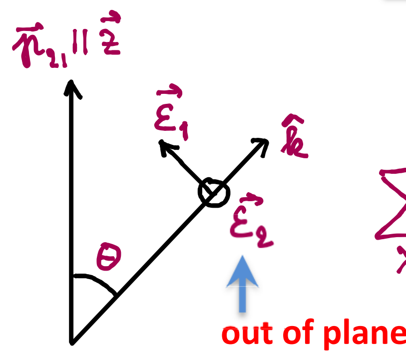

in polar coordinates

$$\sum_\lambda |\vec{p}_{21}\cdot\vec{\varepsilon}_{\vec{k},\lambda}|^2 = \sin^2\theta\, |\vec{p}_{12}|^2$$

no φ dependence

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{\dot{c}_{2,0}(t) = -\sum_{\vec{k},\lambda} |g_{\vec{k},\lambda}|^2 \int_0^t e^{i(\omega_{eg}-\omega_{\vec{k}})(t-t')} c_{2,0}(t')\,dt'}$$

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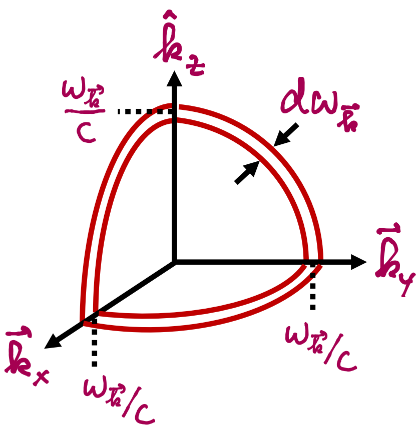

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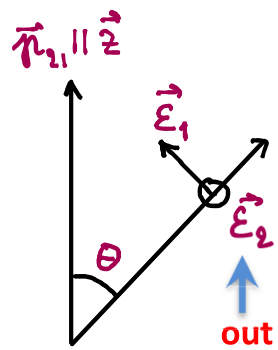
$\mathcal{E}_{\vec{k}}^2$



out of plane

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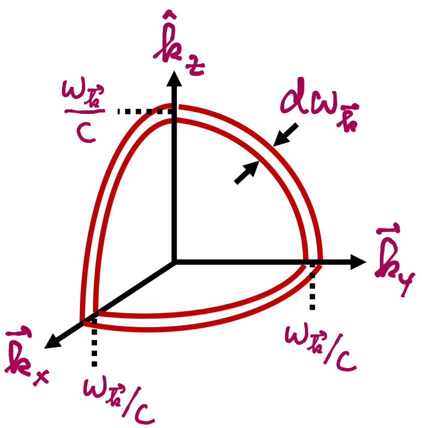

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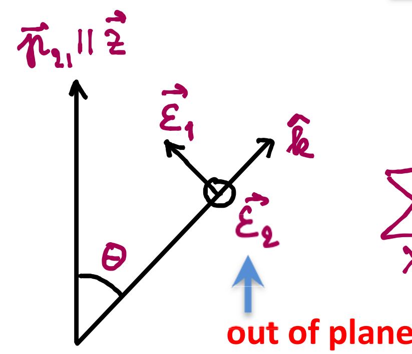

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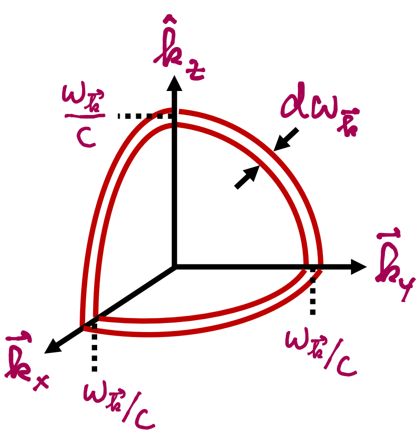

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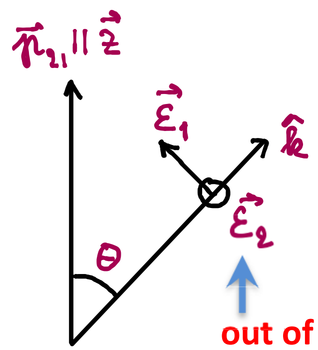

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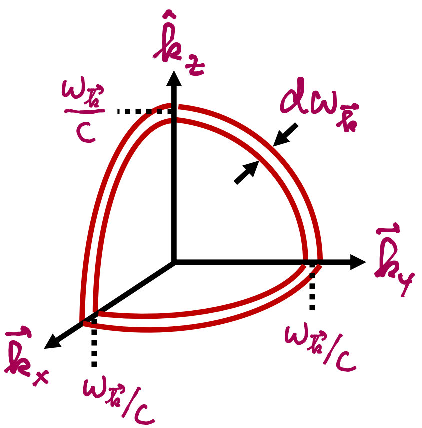

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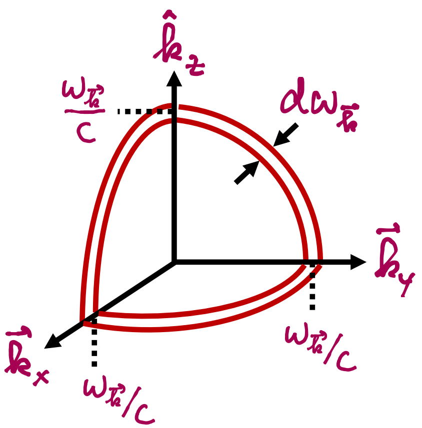

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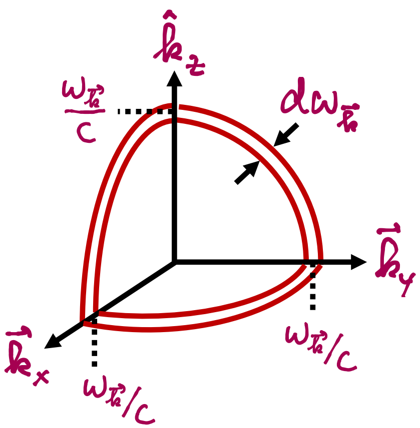

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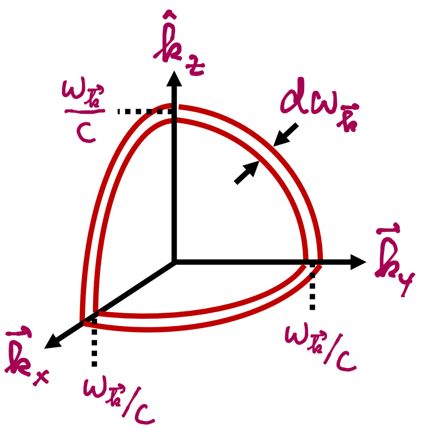

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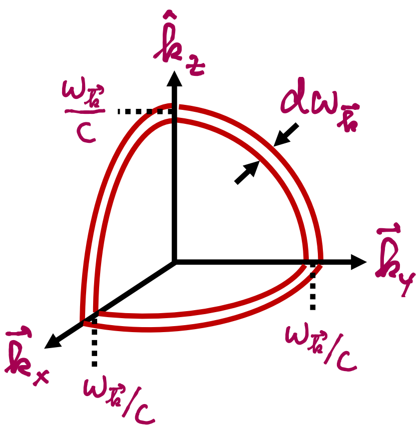

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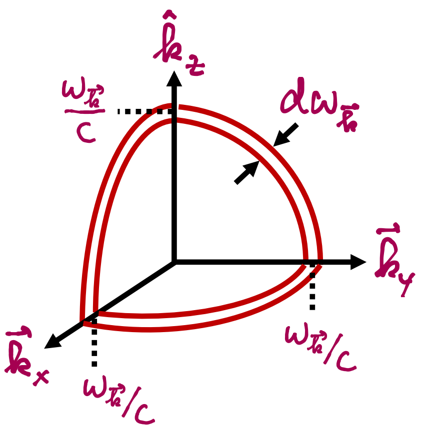

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Wigner-Weisskopf Theory of Spontaneous Decay

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Cohen-Tannoudji, Vol, 2, Ch. XIII, Compl D_{III} - 4

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Wigner-Weisskopf Theory of Spontaneous Decay

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We define the "polarization average"

$$\overline{|g(\omega_{\vec{k}})|^2} = \sum_{\lambda} \int d(\hat{k})\, |g_{\vec{k},\lambda}|^2 =$$

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First: This eq. is of the form $\dot{c}_{2,0}(t) = \beta c_{2,0}(t)$

$\Rightarrow \begin{cases} \text{solutions oscillate at freq. } & Im[\beta] \\ \text{and grow or decay at rate } & Re[\beta] \end{cases}$

Next: Atoms couple weakly to the vacuum

$c_{2,0}(t)$ **changes slowly on timescale** ω_{21}^{-1}**,**
evolving at a rate Γ **to be determined.**

That means we can let $t \to \infty$ **on timescale** ω_{21}^{-1}
while still keeping $t \ll \Gamma^{-1}$

Defining $-i\zeta(\omega_{\vec{k}} - \omega_{21}) = \int_0^\infty dt' \, e^{i(\omega_{21} - \omega_{\vec{k}})(t-t')}$

We can then rewrite

$$\dot{c}_{2,0}(t) = -\int_0^\infty d\omega_{\vec{k}} \, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left[-i\zeta(\omega_{\vec{k}} - \omega_{21})\right] c_{2,0}(t)$$

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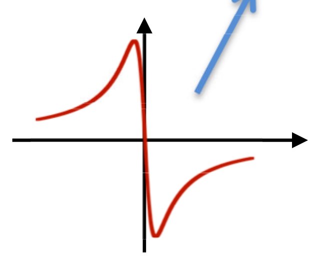

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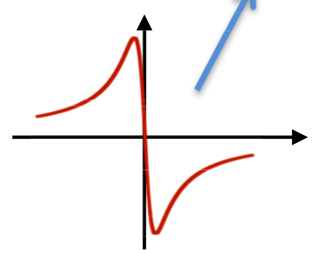

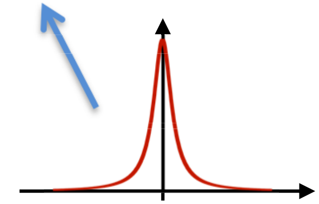

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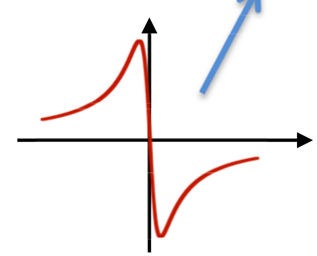

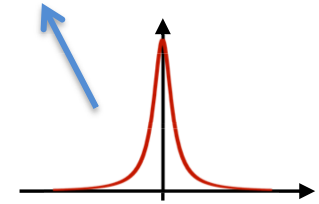

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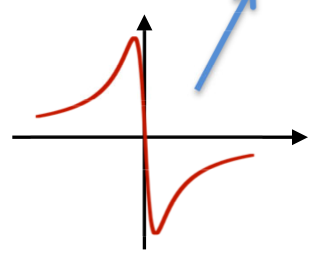

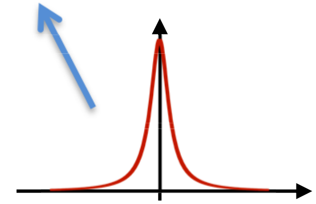

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Cauchy's Principal Part

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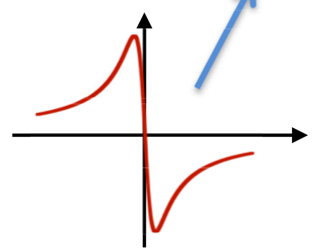

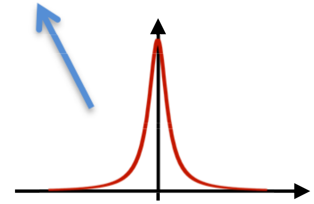

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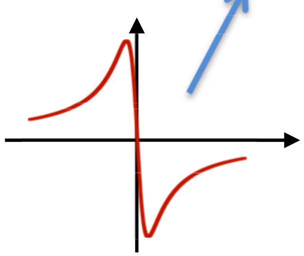

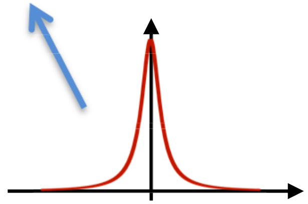

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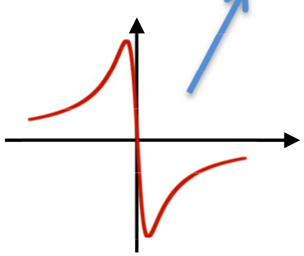

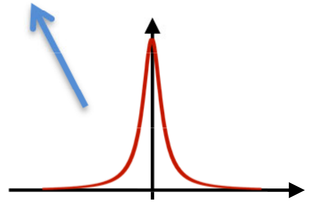

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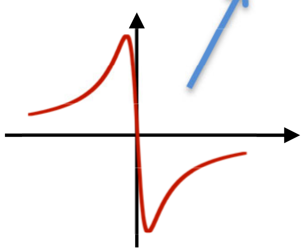

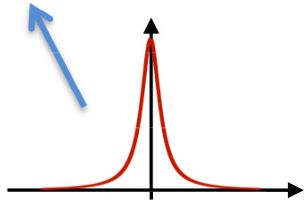

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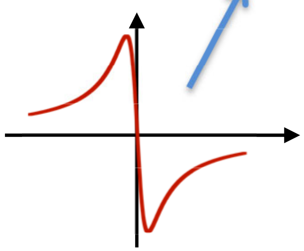

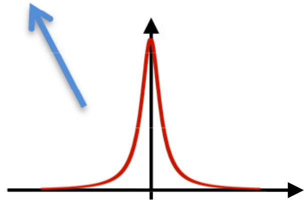

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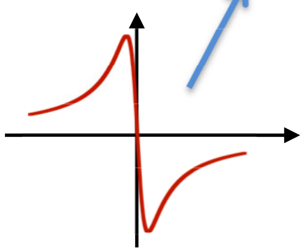

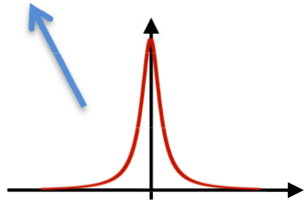

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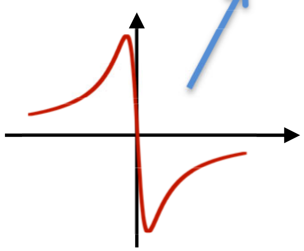

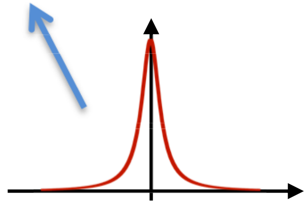

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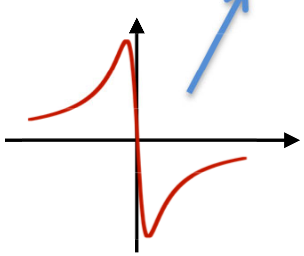

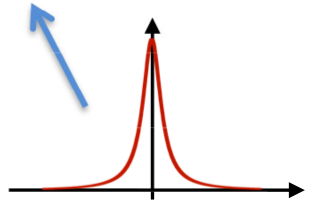

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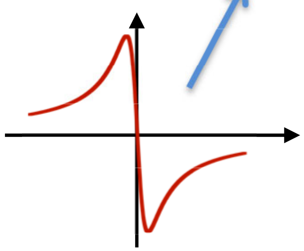

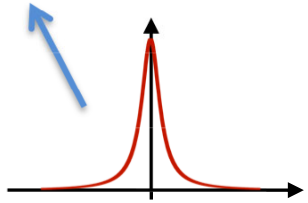

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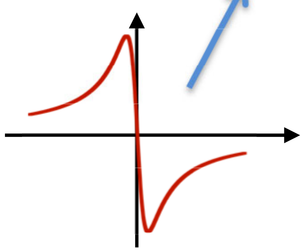

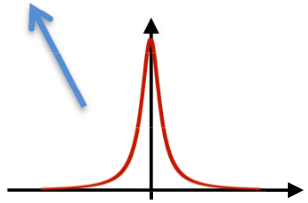

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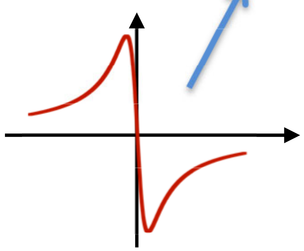

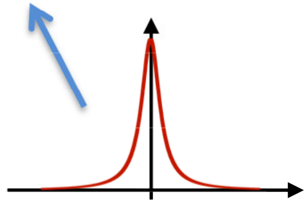

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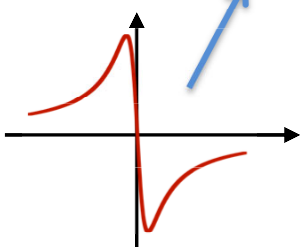

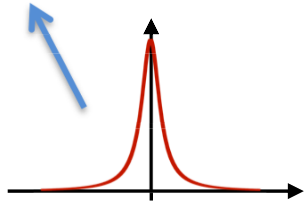

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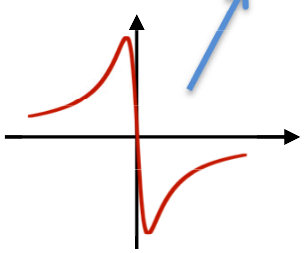

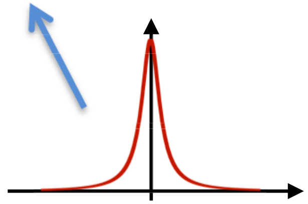

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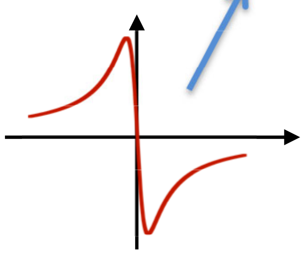

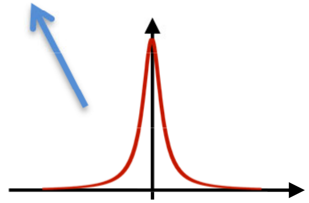

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$$\boxed{\begin{aligned} &\dot{C}_{2,0} = -\left(\frac{A_{21}}{2} - i\delta\right) C_{2,0}(t) \Rightarrow \\ &C_{2,0}(t) = e^{-A_{21}/2\, t}\, e^{i\delta(t)}\, C_{2,0}(0) \end{aligned}}$$

Decay Rate:

$$A_{21} = -\int_0^\infty d\omega_{\vec{k}}\, 2\pi\, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}})\, \delta(\omega_{21}-\omega_{\vec{k}})$$

$$= 2\pi |g(\omega_{21})|^2 \mathcal{D}(\omega_{21})$$

$$= 2\pi \times \frac{4\pi\omega_{21}}{3\hbar\varepsilon_0 V} |\vec{p}_{21}|^2 \times \frac{V}{(2\pi)^3} \frac{\omega_{21}^2}{c^3}$$

$$\Rightarrow$$

$$A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3}$$

Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{C}_{2,0}(t) \approx$$

$$-\left[\int_0^\infty d\omega_{\vec{k}}\, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi\delta(\omega_{21}-\omega_{\vec{k}}) - i\mathbb{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] C_{2,0}(t)$$

which is of the form

$$\dot{C}_{2,0} = -\left(\frac{A_{21}}{2} - i\delta\right) C_{2,0}(t)$$

$$C_{2,0}(t) = e^{-A_{21}/2\, t} e^{i\delta(t)} C_{2,0}(0)$$

Decay Rate:

$$A_{21} = -\int_0^\infty d\omega_{\vec{k}}\, 2\pi\, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}})\, \delta(\omega_{21}-\omega_{\vec{k}})$$

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$$A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}$$

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$$-\left[\int_0^\infty d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi\delta(\omega_{21}-\omega_{\vec{k}}) - i\mathbb{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] C_{2,0}(t)$$

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$$\boxed{\begin{aligned} \dot{C}_{2,0} &= -\left(\frac{A_{21}}{2} - i\delta\right) C_{2,0}(t) \\ C_{2,0}(t) &= e^{-A_{21}/2\, t}\, e^{i\delta(t)}\, C_{2,0}(0) \end{aligned}}$$

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Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{c}_{2,0}(t) \approx -\left[\int_0^\infty d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi\delta(\omega_{21}-\omega_{\vec{k}}) - i\mathcal{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = -\left(\frac{A_{21}}{2} - i\delta\right) c_{2,0}(t)$$

$$c_{2,0}(t) = e^{-A_{21}/2\, t}\, e^{i\delta(t)}\, c_{2,0}(0)$$

Decay Rate:

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$$= 2\pi |g(\omega_{21})|^2 \mathcal{D}(\omega_{21})$$

$$= 2\pi \times \frac{4\pi\omega_{21}}{3\hbar\varepsilon_o V} |\vec{p}_{21}|^2 \times \frac{V}{(2\pi)^3} \frac{\omega_{21}^2}{c^3}$$

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_o \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}}$$

This is the value used earlier!

Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{C}_{2,0}(t) \approx$$

$$-\left[\int_0^\infty d\omega_{\vec{k}}\, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi\delta(\omega_{21}-\omega_{\vec{k}}) - i\mathbb{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] C_{2,0}(t)$$

which is of the form

$$\dot{C}_{2,0} = -\left(\frac{A_{21}}{2} - i\delta\right) C_{2,0}(t)$$

$$C_{2,0}(t) = e^{-A_{21}/2\, t}\, e^{i\delta(t)}\, C_{2,0}(0)$$

Decay Rate:

$$A_{21} = -\int_0^\infty d\omega_{\vec{k}}\, 2\pi\, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}})\, \delta(\omega_{21}-\omega_{\vec{k}})$$

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This is the value used earlier!

Frequency shift:

Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{c}_{2,0}(t) \approx$$

$$-\left[\int_0^\infty d\omega_{\vec{k}}\, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi\delta(\omega_{21}-\omega_{\vec{k}}) - i\mathcal{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] c_{2,0}(t)$$

which is of the form

$$\boxed{\begin{aligned} \dot{c}_{2,0} &= -\left(\frac{A_{21}}{2} - i\delta\right) c_{2,0}(t) \\ c_{2,0}(t) &= e^{-A_{21}/2\, t}\, e^{i\delta(t)}\, c_{2,0}(0) \end{aligned}}$$

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Frequency shift:

$$\delta = \lim_{\varepsilon\to 0}\left(i\int_0^\infty d\omega_{\vec{k}}\, |\overline{g(\omega_{\vec{k}})}|^2 \mathcal{D}(\omega_{\vec{k}}) \frac{\omega_{21}-\omega_{\vec{k}}}{(\omega_{21}-\omega_{\vec{k}})^2+\varepsilon^2}\right)$$

Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

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$$-\left[\int_0^\infty d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi \delta(\omega_{21}-\omega_{\vec{k}}) - i\mathcal{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] c_{2,0}(t)$$

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Integral is not well behaved!

Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{c}_{2,0}(t) \approx$$

$$-\left[\int_0^\infty d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi\delta(\omega_{21}-\omega_{\vec{k}}) - i\mathcal{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] c_{2,0}(t)$$

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Integral is not well behaved!

Further development by Willis Lamb

Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{C}_{2,0}(t) \approx$$

$$-\left[\int_0^\infty d\omega_{\vec{k}}\, |g(\omega_{\vec{k}})|^2 \mathcal{D}(\omega_{\vec{k}}) \left(\pi\delta(\omega_{21}-\omega_{\vec{k}}) - i\mathbb{P}\left(\frac{1}{\omega_{21}-\omega_{\vec{k}}}\right)\right)\right] C_{2,0}(t)$$

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Integral is not well behaved!

Further development by Willis Lamb

Wigner-Weisskopf Theory of Spontaneous Decay

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Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Wigner-Weisskopf Theory of Spontaneous Decay

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Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi \varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi \hbar \lambda^3}}$$

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Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

- **accurate calculations for**

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}}$$

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Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

- **accurate calculations for**

$$\lambda = 121.57\,\text{nm}$$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}}$$

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Frequency shift:

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Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

- **accurate calculations for**

$$\lambda = 121.57\,\text{nm}$$

$$|\vec{p}_{21}| = 0.745\, a_0 e$$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}}$$

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Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

- accurate calculations for

$$\lambda = 121.57\,\text{nm}$$

$$|\vec{p}_{21}| = 0.745\, a_0 e = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \times 0.745 e$$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}}$$

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Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

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Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

- accurate calculations for

$$\lambda = 121.57\,nm$$

$$|\vec{p}_{21}| = 0.745\, a_0 e = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} \times 0.745 e$$

$$(A_{21})^{-1} = \tau_{2P} = 1.595\,ns$$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}}$$

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Integral is not well behaved!

Further development by Willis Lamb

⇓

Lamb shift

Hydrogen 2P lifetime:

- accurate calculations for

$$\lambda = 121.57\,\text{nm}$$

$$|\vec{p}_{21}| = 0.745\, a_0 e = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} \times 0.745 e$$

⇓

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Wigner-Weisskopf Theory of Spontaneous Decay

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Frequency shift:

$$\delta = \lim_{\varepsilon\to 0}\left(i\int_0^\infty d\omega_{\vec{k}} |\overline{g(\omega_{\vec{k}})}|^2 \mathcal{D}(\omega_{\vec{k}}) \frac{\omega_{21}-\omega_{\vec{k}}}{(\omega_{21}-\omega_{\vec{k}})^2+\varepsilon^2}\right)$$

Integral is not well behaved!

Further development by Willis Lamb

Lamb shift

Hydrogen 2P lifetime:

- accurate calculations for

$\lambda = 121.57\ \text{nm}$

$$|\vec{p}_{21}| = 0.745\, a_0 e = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \times 0.745 e$$

$$\boxed{(A_{21})^{-1} = \tau_{2p} = 1.595\ \text{ns}}$$

Measured: $\tau_{2p} = 1.60\ \text{ns}$

Wigner-Weisskopf Theory of Spontaneous Decay

$$\boxed{A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi\varepsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi\hbar\lambda^3}}$$

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Lamb shift

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- accurate calculations for

$$\lambda = 121.57\,\mathrm{nm}$$

$$|\vec{p}_{21}| = 0.745\, a_0 e = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} \times 0.745 e$$

$$\boxed{(A_{21})^{-1} = \tau_{2p} = 1.595\,\mathrm{ns}}$$

Measured: $\tau_{2p} = 1.60\,\mathrm{ns}$