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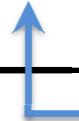
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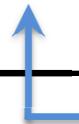
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Formal Solution:

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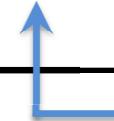
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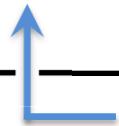
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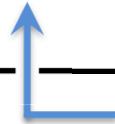
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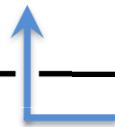
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# Wigner-Weisskopf Theory of Spontaneous Decay

Expand

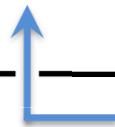
$$|\Psi(t)\rangle = C_{2,0}(t) |2,0\rangle + \sum_{\vec{k}, \lambda} C_{1,1_{\vec{k}, \lambda}}(t) |1,1_{\vec{k}, \lambda}\rangle$$

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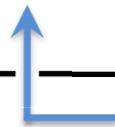
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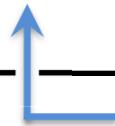
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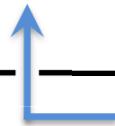
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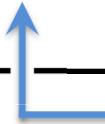
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Quantization in a box w/periodic B. C.

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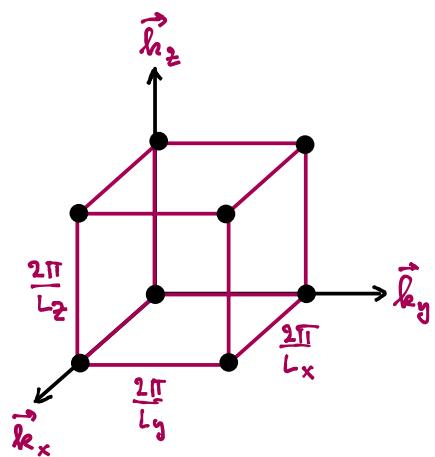
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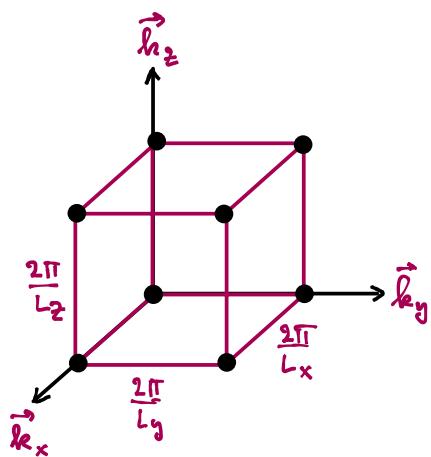
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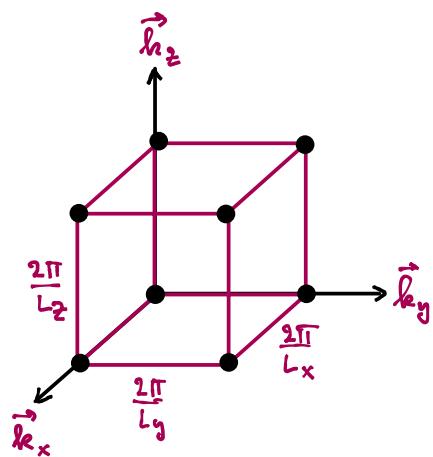
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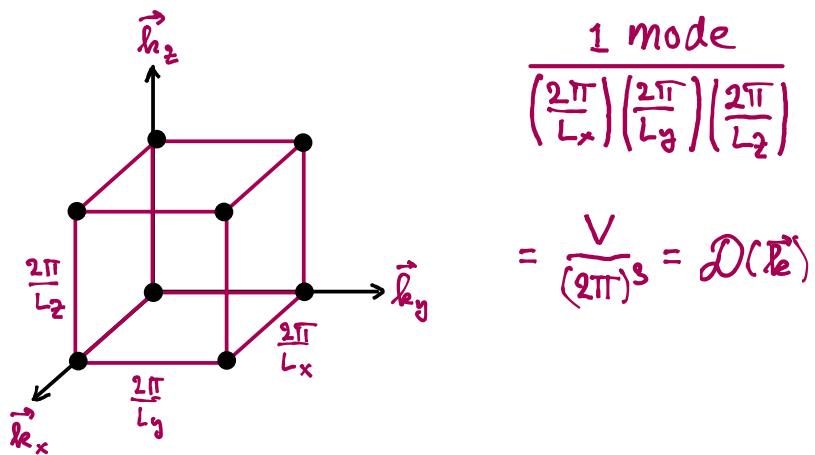
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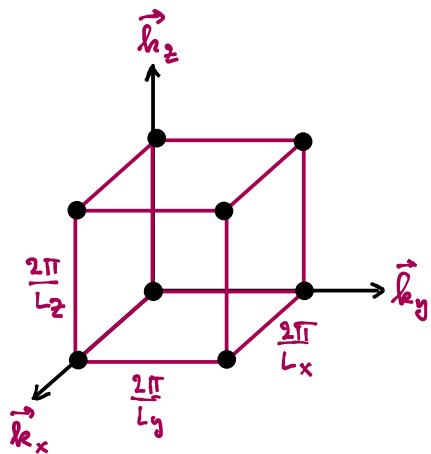
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Density of Modes

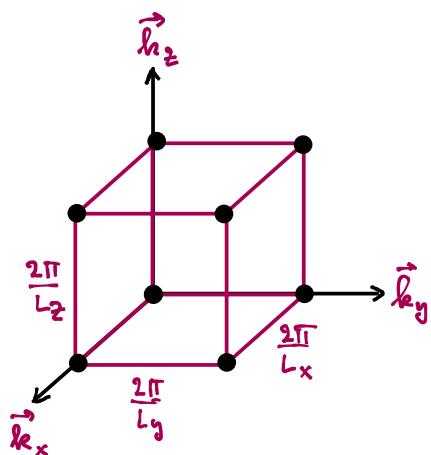
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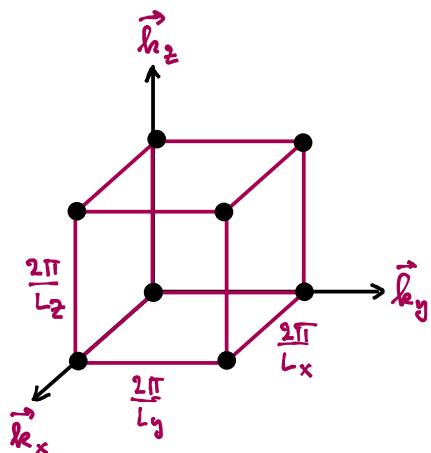
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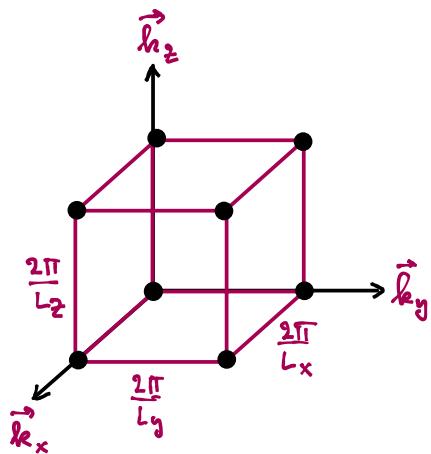
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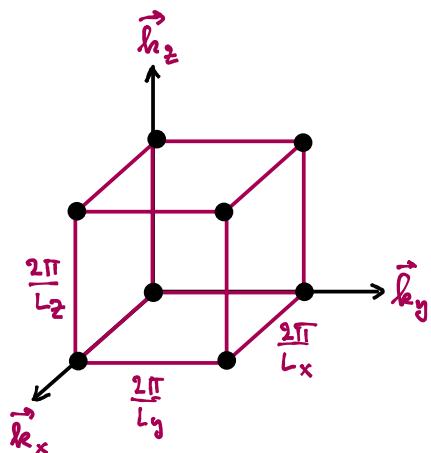
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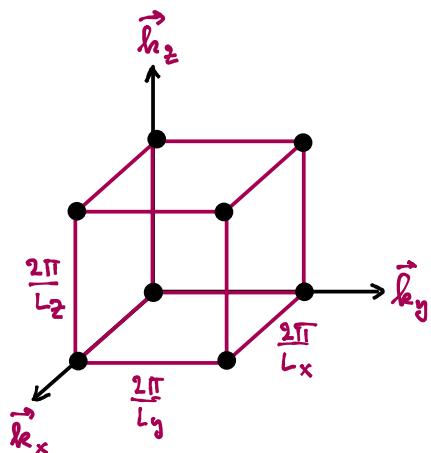
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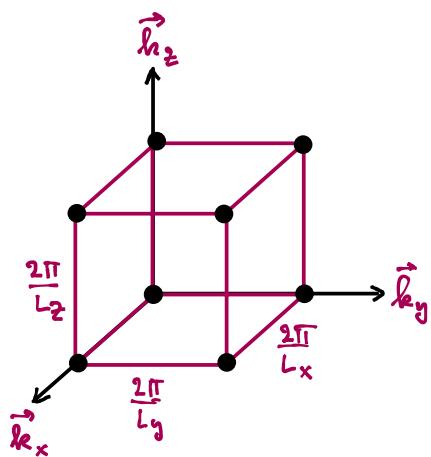
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where  $\hat{k} = \vec{k}/k$  is a unit vector along  $\vec{k}$

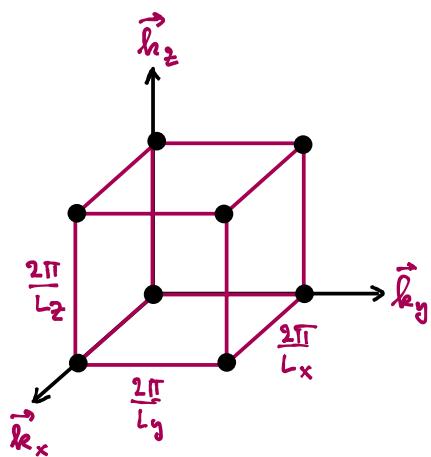
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$$\begin{aligned} \sum_{\vec{k}} &\rightarrow \int d^3 \vec{k} D(\vec{k}) = \int \vec{k}^2 d(\hat{k}) d\vec{k} D(\vec{k}) \\ &= \int d(\hat{k}) d\omega_{\vec{k}} \frac{\omega_{\vec{k}}^2}{C^3} D(\vec{k}) = \int d(\hat{k}) d\omega_{\vec{k}} D(\omega_{\vec{k}}) \end{aligned}$$

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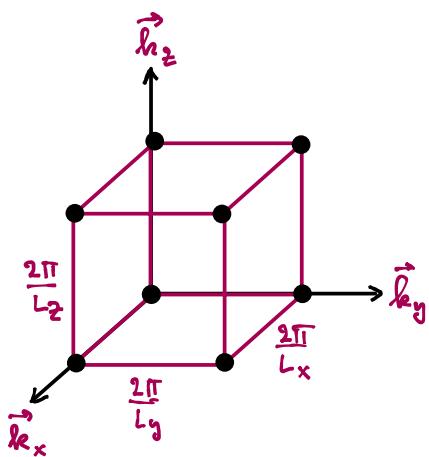
# Wigner-Weisskopf Theory of Spontaneous Decay

$$\dot{C}_{2,0}(t) = -\sum_{\vec{k}, \lambda} |g_{\vec{k}, \lambda}|^2 \int_0^t e^{i(\omega_{\text{eg}} - \omega_{\vec{k}})(t-t')} C_{2,0}(t') dt'$$

Quantization in a box w/periodic B. C.

$$\rightarrow \vec{k}_i = n_i \frac{2\pi}{L}, \quad n \text{ integer}$$

In  $\vec{k}$  space the modes form a grid:



$$\begin{aligned} & \frac{1 \text{ mode}}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} \\ &= \frac{V}{(2\pi)^3} = \mathcal{D}(\vec{k}) \end{aligned}$$

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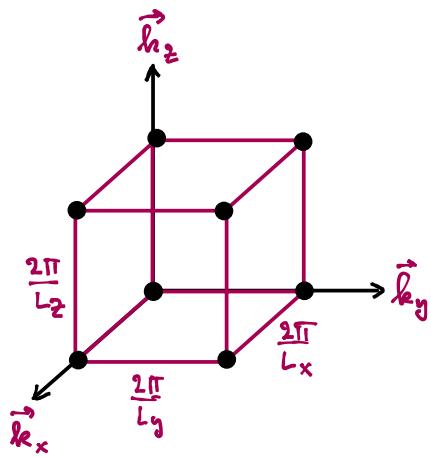
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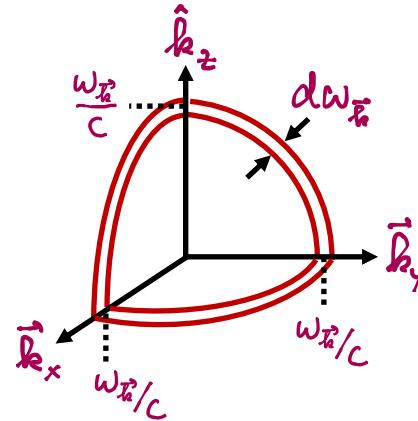
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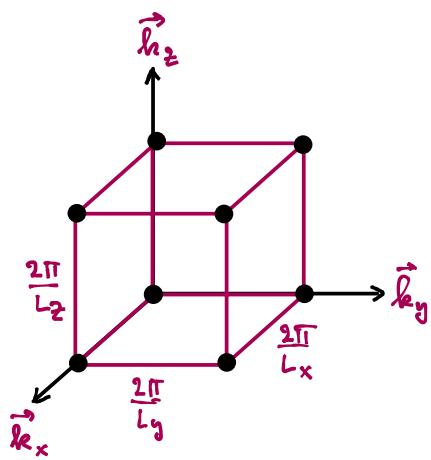
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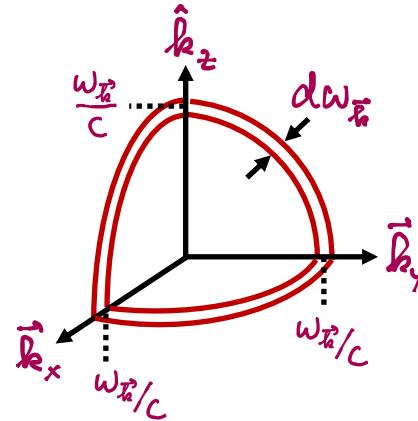
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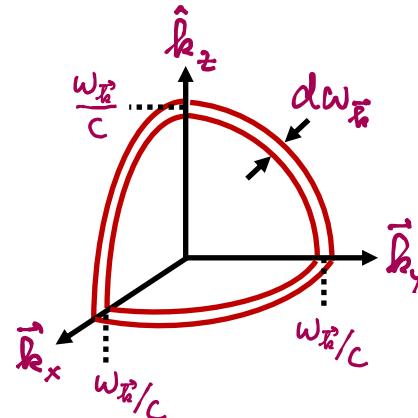
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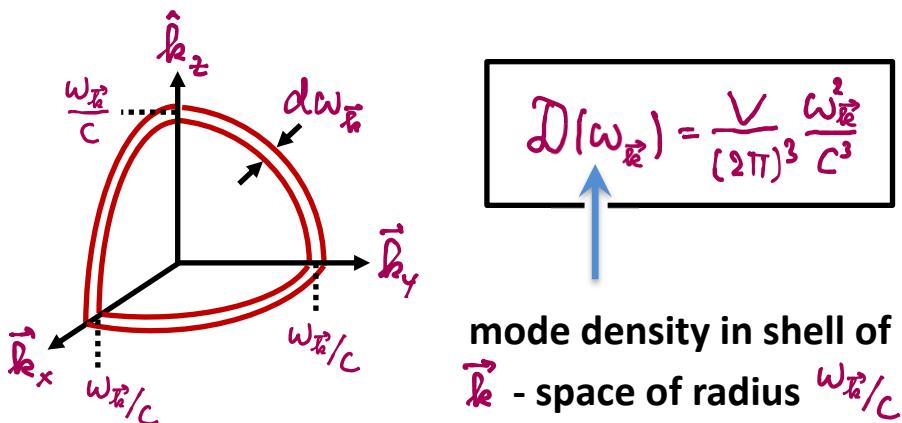
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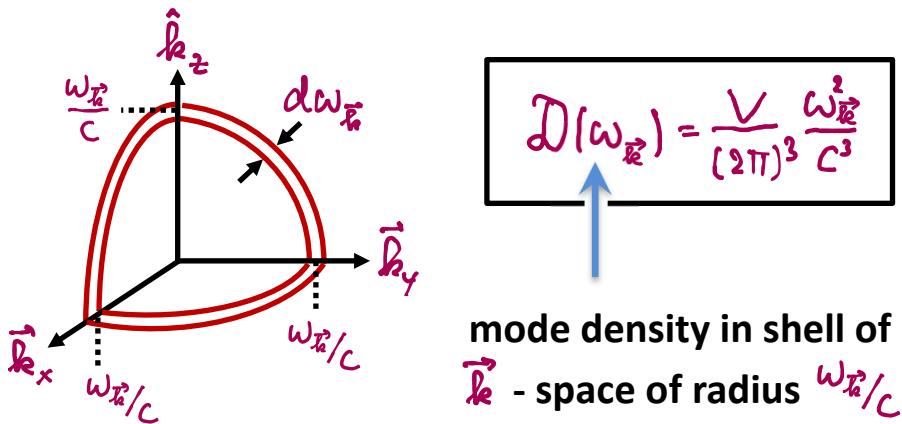
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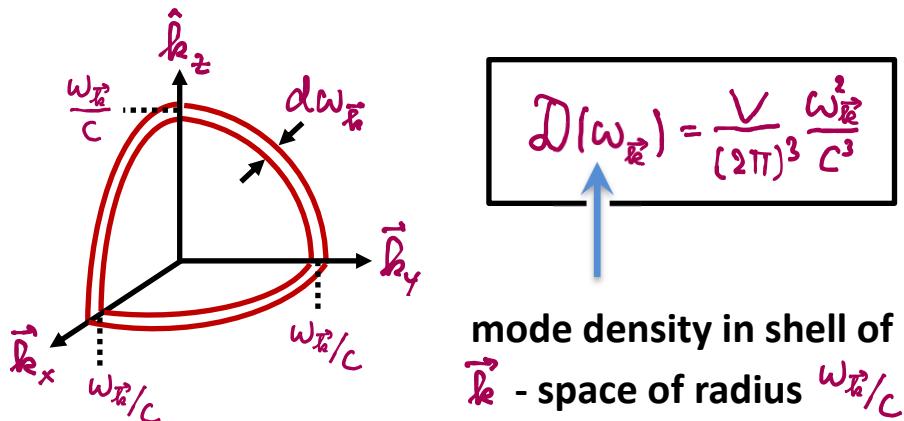
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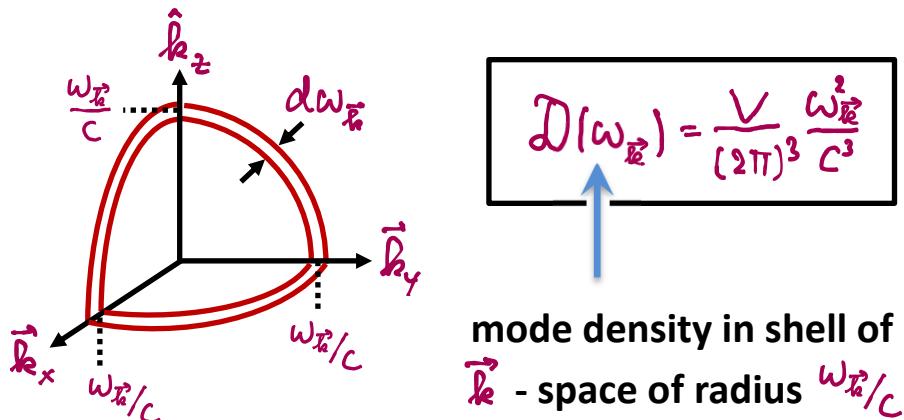
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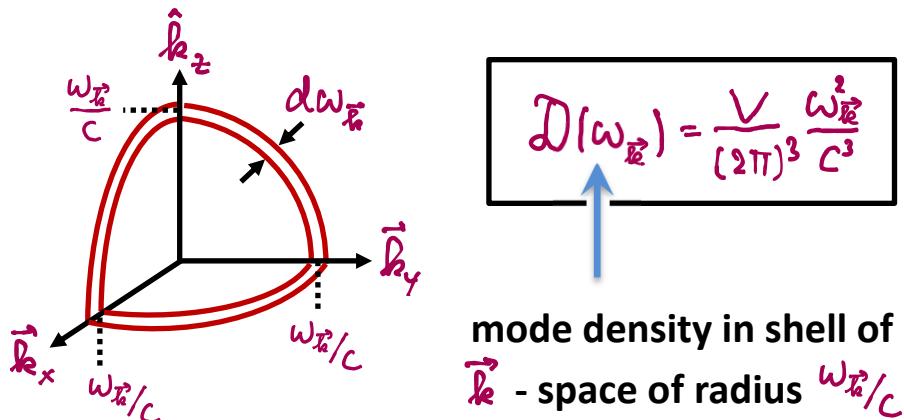
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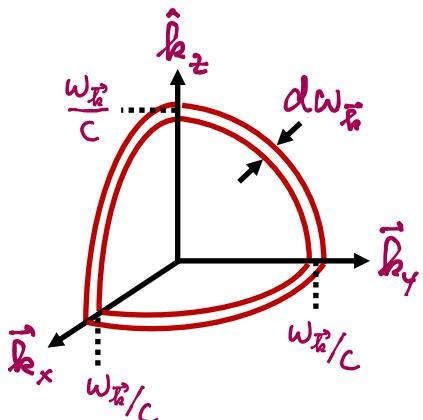
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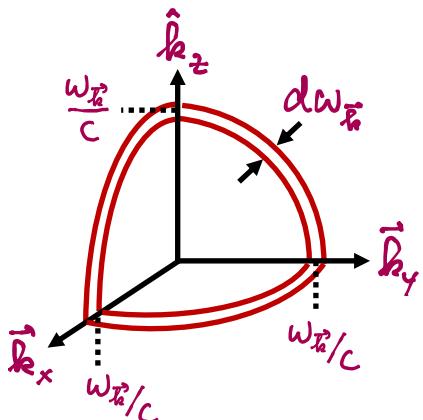
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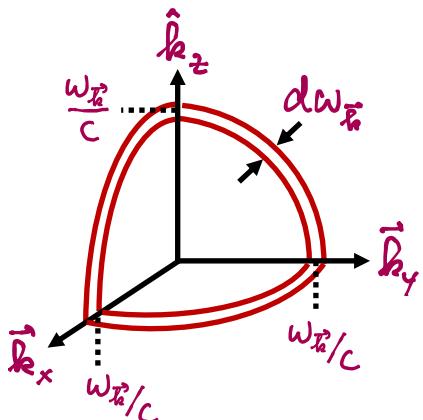
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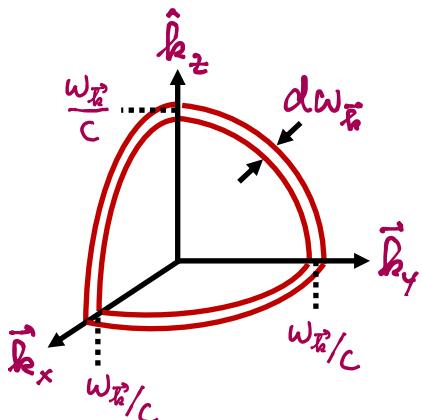
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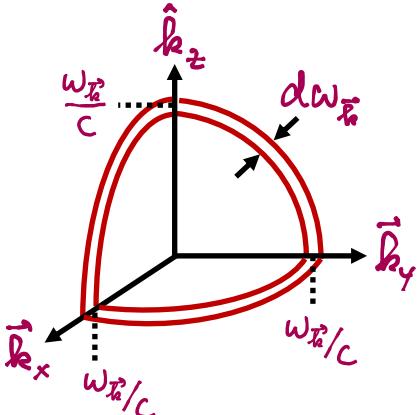
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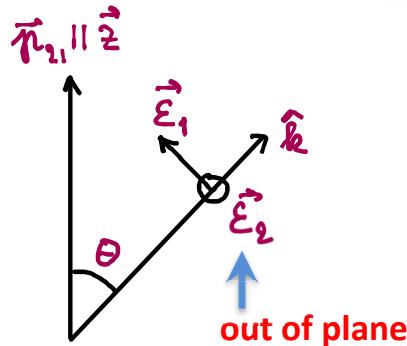
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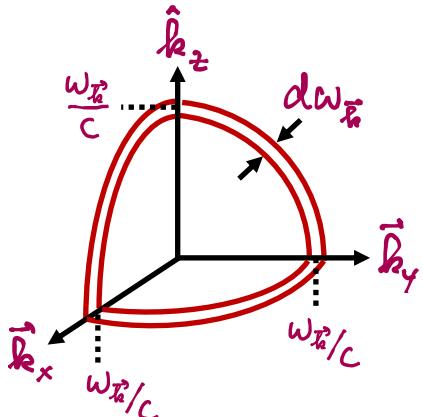
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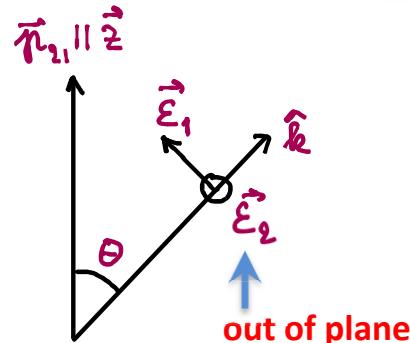
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in polar coordinates

# Wigner-Weisskopf Theory of Spontaneous Decay

$$\dot{C}_{2,0}(t) = - \sum_{\vec{k}, \lambda} |g_{\vec{k}, \lambda}|^2 \int_0^t e^{i(\omega_{eg} - \omega_{\vec{k}})(t-t')} C_{2,0}(t') dt'$$

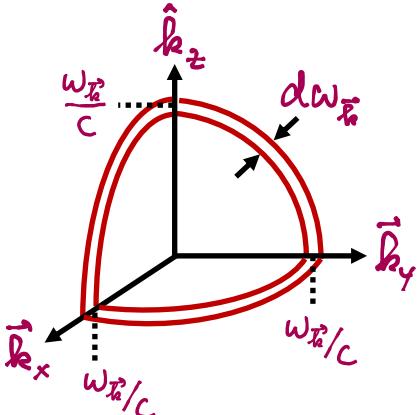
Thus, in the Continuum Limit

$$\sum_{\vec{k}, \lambda} |g_{\vec{k}, \lambda}|^2 \rightarrow \int_0^\infty d\omega_{\vec{k}} \mathcal{D}(\omega_{\vec{k}}) \sum_\lambda \int d(\hat{h}) |g_{\vec{k}, \lambda}|^2$$

Convert sum to integral over modes:

$$\begin{aligned} \sum_{\vec{k}} &\rightarrow \int d^3 \vec{k} \mathcal{D}(\vec{k}) = \int \vec{k}^2 d(\hat{k}) d\vec{k} \mathcal{D}(\vec{k}) \\ &= \int d(\hat{k}) d\omega_{\vec{k}} \frac{\omega_{\vec{k}}^2}{c^3} \mathcal{D}(\vec{k}) = \int d(\hat{k}) d\omega_{\vec{k}} \mathcal{D}(\omega_{\vec{k}}) \end{aligned}$$

where  $\hat{k} = \vec{k}/k$  is a unit vector along  $\vec{k}$  and



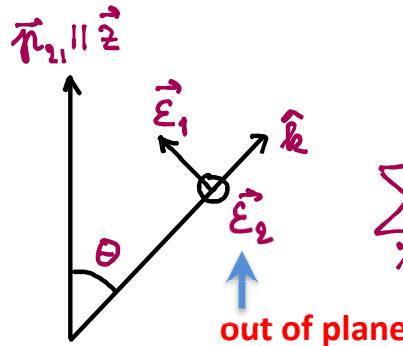
$$\mathcal{D}(\omega_{\vec{k}}) = \frac{V}{(2\pi)^3} \frac{\omega_{\vec{k}}^2}{c^3}$$

mode density in shell of  
 $\vec{k}$ -space of radius  $\omega_{\vec{k}}/c$

We define the “polarization average”

$$\begin{aligned} \overline{|g(\omega_{\vec{k}})|^2} &= \sum_\lambda \int d(\hat{k}) |g_{\vec{k}, \lambda}|^2 = \\ &= \frac{1}{\hbar^2} \left( \frac{\hbar \omega_{\vec{k}}}{2\epsilon_0 V} \right) \int d(\hat{k}) \sum_\lambda |\vec{p}_{1i} \cdot \vec{\epsilon}_{\vec{k}, \lambda}|^2 \end{aligned}$$

$\uparrow \epsilon_{\vec{k}}^2$



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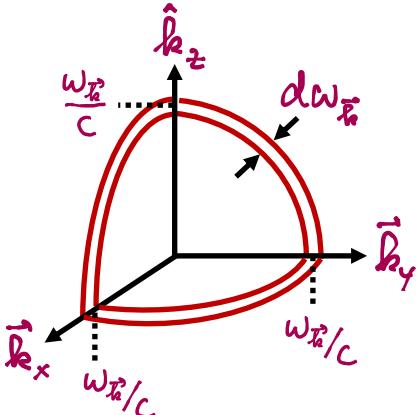
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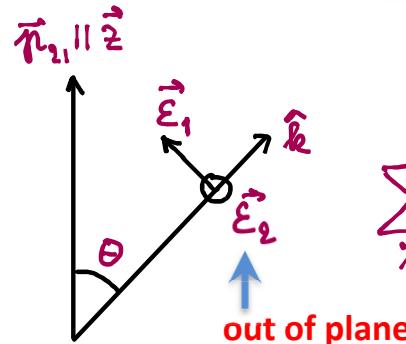
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no  $\varphi$  dependence

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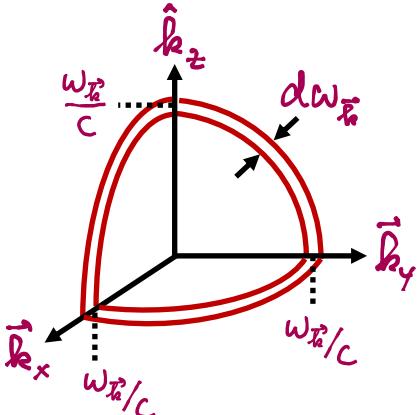
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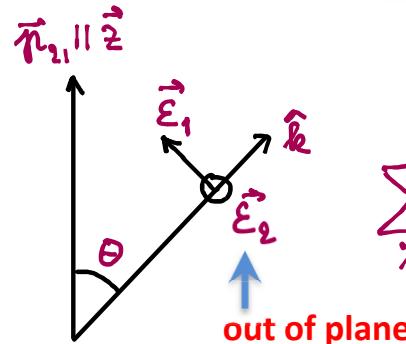
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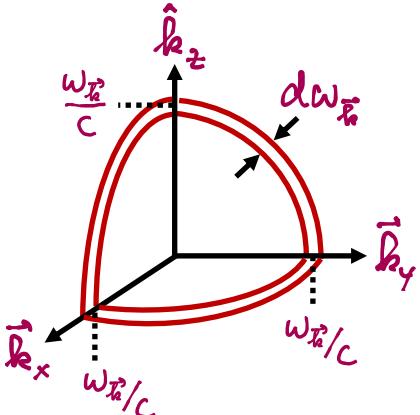
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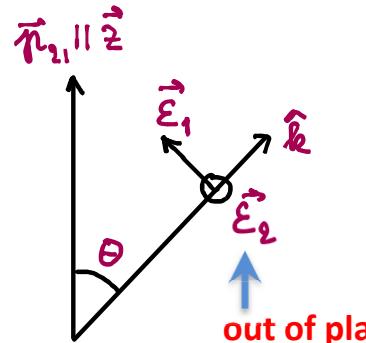
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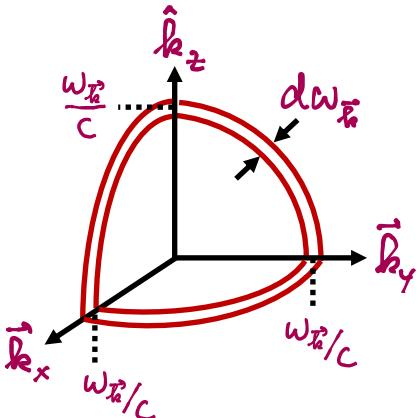
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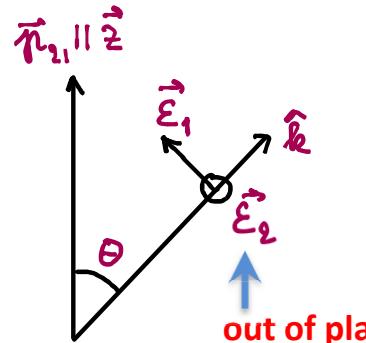
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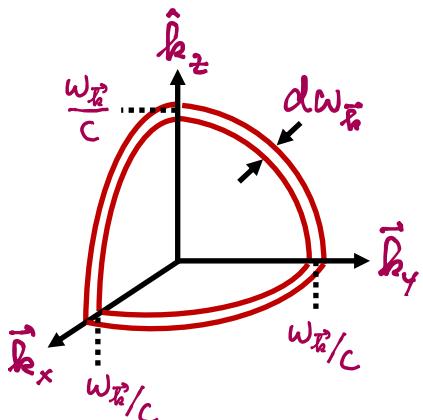
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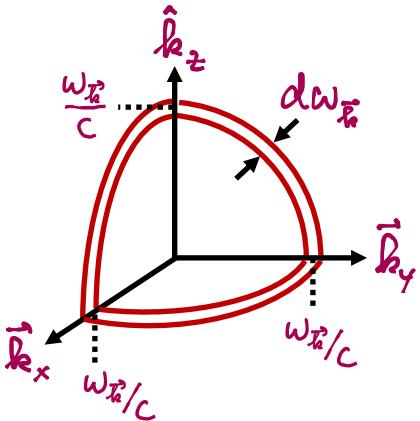
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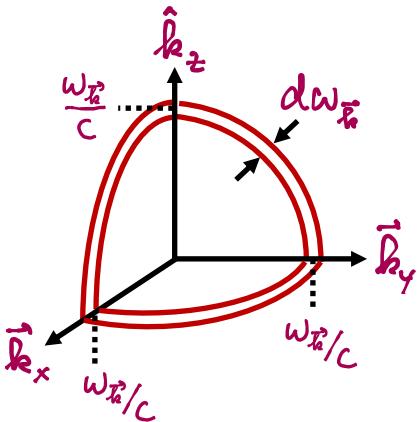
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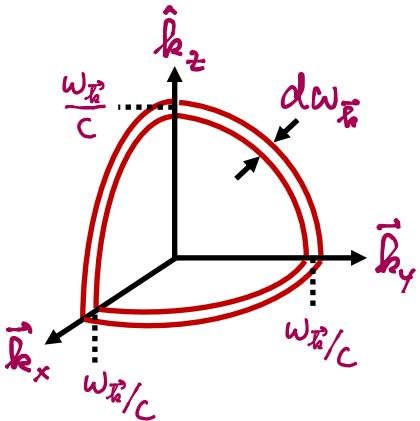
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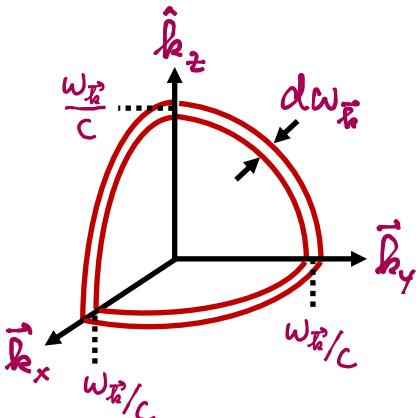
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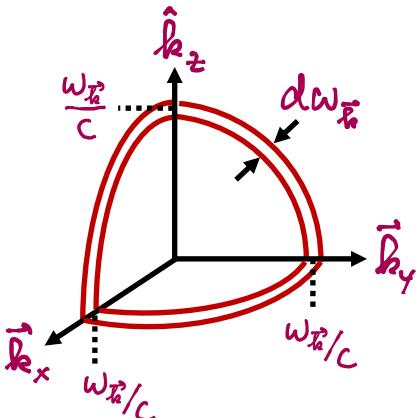
Thus, in the Continuum Limit

$$\sum_{\vec{k}, \lambda} |g_{\vec{k}, \lambda}|^2 \rightarrow \int_0^\infty d\omega_{\vec{k}} \mathcal{D}(\omega_{\vec{k}}) \sum_\lambda \int d(\hat{h}) |g_{\vec{k}, \lambda}|^2$$

Convert sum to integral over modes:

$$\begin{aligned} \sum_{\vec{k}} &\rightarrow \int d^3 \vec{k} \mathcal{D}(\vec{k}) = \int \hbar^2 d(\hat{h}) d\vec{k} \mathcal{D}(\vec{k}) \\ &= \int d(\hat{h}) d\omega_{\vec{k}} \frac{\omega_{\vec{k}}^2}{C^3} \mathcal{D}(\vec{k}) = \int d(\hat{h}) d\omega_{\vec{k}} \mathcal{D}(\omega_{\vec{k}}) \end{aligned}$$

where  $\hat{h} = \vec{k}/\hbar$  is a unit vector along  $\vec{k}$  and



$$\mathcal{D}(\omega_{\vec{k}}) = \frac{V}{(2\pi)^3} \frac{\omega_{\vec{k}}^2}{C^3}$$

mode density in shell of  
 $\vec{k}$ -space of radius  $\omega_{\vec{k}}/c$

We define the “polarization average”

$$\begin{aligned} \overline{|g(\omega_{\vec{k}})|^2} &= \sum_\lambda \int d(\hat{h}) |g_{\vec{k}, \lambda}|^2 = \\ &= \frac{1}{\hbar^2} \left( \frac{\hbar \omega_{\vec{k}}}{2\epsilon_0 V} \right) \int d(\hat{h}) \sum_\lambda |\vec{p}_{21} \cdot \vec{\varepsilon}_{\vec{k}, \lambda}|^2 \\ &= \frac{4\pi \omega_{\vec{k}}}{2\hbar \epsilon_0 V} |\vec{p}_{21}|^2 \end{aligned}$$

Putting it together:

$$\begin{aligned} \dot{c}_{2,0}(t) &= - \sum_{\vec{k}, \lambda} |g_{\vec{k}, \lambda}|^2 \int_0^t e^{i(\omega_{\text{eg}} - \omega_{\vec{k}})(t-t')} c_{2,0}(t') dt' \\ &= - \int_0^\infty d\omega_{\vec{k}} \overline{|g(\omega_{\vec{k}})|^2} \mathcal{D}(\omega_{\vec{k}}) \int_0^t dt' e^{i(\omega_{\text{eg}} - \omega_{\vec{k}})(t-t')} c_{2,0}(t') \end{aligned}$$

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$$\begin{aligned} \dot{C}_{2,0}(t) &= - \sum_{\vec{k}, \lambda} |g_{\vec{k}, \lambda}|^2 \int_0^t e^{i(\omega_{\text{rf}} - \omega_{\vec{k}})(t-t')} C_{2,0}(t') dt' \\ &= - \int_0^\infty d\omega_{\vec{k}} \overline{|g(\omega_{\vec{k}})|^2} \mathcal{D}(\omega_{\vec{k}}) \int_0^t dt' e^{i(\omega_{\text{rf}} - \omega_{\vec{k}})(t-t')} C_{2,0}(t') \end{aligned}$$

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## Wigner-Weisskopf approximation:

The integral over time reduces to

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Thus, the integral that gives us  $\dot{c}_{2,0}(t)$  is only sensitive to the value of  $c_{2,0}(t')$  at times  $t'$  infinitesimally close to  $t$ .

# Wigner-Weisskopf Theory of Spontaneous Decay

Thus, in the Continuum Limit

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Putting it together:

$$\begin{aligned} \dot{c}_{2,0}(t) &= - \sum_{\vec{k}, \lambda} |g_{\vec{k}, \lambda}|^2 \int_0^t e^{i(\omega_{21} - \omega_{\vec{k}})(t-t')} c_{2,0}(t') dt' \\ &= - \int_0^\infty d\omega_{\vec{k}} \overline{|g(\omega_{\vec{k}})|^2} D(\omega_{\vec{k}}) \int_0^t dt' e^{i(\omega_{21} - \omega_{\vec{k}})(t-t')} c_{2,0}(t') \end{aligned}$$

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$c_{2,0}(t)$  changes slowly on timescale  $\omega_u^{-1}$ , evolving at a rate  $\Gamma$  to be determined.

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Next: Atoms couple weakly to the vacuum

$C_{2,0}(t)$  changes slowly on timescale  $\omega_{21}^{-1}$ , evolving at a rate  $\Gamma$  to be determined.

That means we can let  $t \rightarrow \infty$  on timescale  $\omega_{21}^{-1}$  while still keeping  $t \ll \Gamma^{-1}$

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$$\int_0^t dt' e^{i(\omega_{21} - \omega_{\text{R}})(t-t')} \propto \delta(t-t')$$

on time scales  $t-t' \gg |\omega_{21} - \omega_{\text{R}}|^{-1}$  

$$\begin{aligned}\dot{c}_{2,0}(t) &= - \sum_{\text{R}, \lambda} |g_{\text{R}, \lambda}|^2 \int_0^t e^{i(\omega_{21} - \omega_{\text{R}})(t-t')} c_{2,0}(t') dt' \\ &= - \int_0^\infty d\omega_{\text{R}} |\overline{g(\omega_{\text{R}})}|^2 D(\omega_{\text{R}}) \int_0^t dt' e^{i(\omega_{21} - \omega_{\text{R}})(t-t')} c_{2,0}(t')\end{aligned}$$

## Rest of Lecture: Work with this Equation

First: This eq. is of the form  $\dot{c}_{2,0}(t) = \beta c_{2,0}(t)$

  $\left\{ \begin{array}{l} \text{solutions oscillate at freq. } \text{Im}[\beta] \\ \text{and grow or decay at rate } \text{Re}[\beta] \end{array} \right.$

Next: Atoms couple weakly to the vacuum

$c_{2,0}(t)$  changes slowly on timescale  $\omega_{21}^{-1}$ , evolving at a rate  $\Gamma$  to be determined.

That means we can let  $t \rightarrow \infty$  on timescale  $\omega_{21}^{-1}$  while still keeping  $t \ll \Gamma^{-1}$

Defining  $-i\mathcal{G}(\omega_{\text{R}} - \omega_{21}) = \int_0^\infty dt' e^{i(\omega_{21} - \omega_{\text{R}})(t-t')}$

# Wigner-Weisskopf Theory of Spontaneous Decay

## Regular Perturbation Theory

Validity is limited to short times, such that the probability amplitude of the initial state does not change much. That allows us to approximate  $C_{2,0}(t') \sim C_{2,0}(0)$ . That does not work here.

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We can then rewrite

$$\dot{C}_{2,0}(t) = - \int_0^\infty d\omega_{22} |g(\omega_{22})|^2 D(\omega_{22}) [-i\zeta(\omega_{22} - \omega_{21})] C_{2,0}(t)$$

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Math Details: Consider the function

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$$\mathcal{S}(\Omega)$$

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Defining  $-i\tilde{\xi}(\omega_{21} - \omega_{21}) = \int_0^\infty dt' e^{i(\omega_{21} - \omega_{21})(t-t')}$

We can then rewrite

$$\dot{C}_{2,0}(t) = - \int_0^\infty d\omega_{21} |g(\omega_{21})|^2 D(\omega_{21}) [-i\tilde{\xi}(\omega_{21} - \omega_{21})] C_{2,0}(t)$$

Math Details: Consider the function

$$\tilde{\xi}(\omega) = i \int_0^\infty d\tau e^{-i\omega\tau}$$

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Math Details: Consider the function

$$\mathfrak{S}(\Omega) = i \int_0^\infty d\tau e^{-i\Omega\tau}, \quad \Omega = \omega_{21} - \omega_{21}$$

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Math Details: Consider the function

$$\mathcal{S}(\Omega) = i \int_0^\infty d\tau e^{-i\Omega\tau}, \quad \Omega = \omega_{21} - \omega_{21}$$

Approximate the integral as

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Defining  $-i\mathfrak{F}(\omega_{21} - \omega_{21}) = \int_0^\infty dt' e^{i(\omega_{21} - \omega_{21})(t-t')}$

We can then rewrite

$$\dot{C}_{2,0}(t) = - \int_0^\infty d\omega_{21} |g(\omega_{21})|^2 D(\omega_{21}) [-i\mathfrak{F}(\omega_{21} - \omega_{21})] C_{2,0}(t)$$

Math Details: Consider the function

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Approximate the integral as

$$\mathfrak{F}(\Omega) = \lim_{\varepsilon \rightarrow 0^+} i \int_0^\infty d\tau e^{-i\Omega\tau - \varepsilon\tau}$$

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Approximate the integral as

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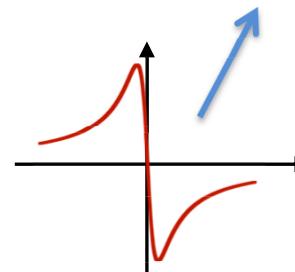
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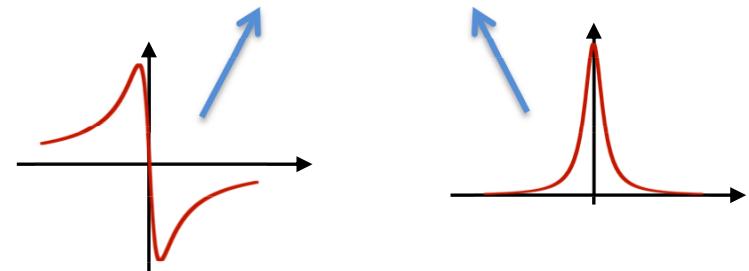
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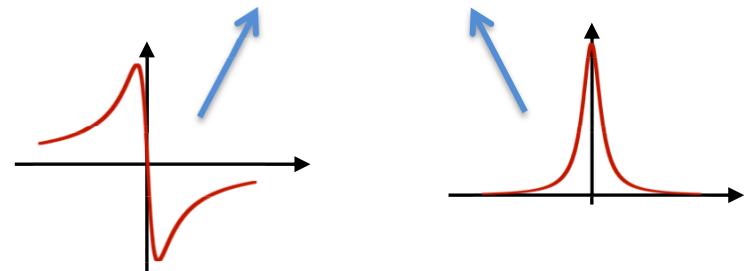
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$$= P\left(\frac{1}{\Omega}\right) + i\pi\delta(\Omega)$$

# Wigner-Weisskopf Theory of Spontaneous Decay

Rest of Lecture: Work with this Equation

First: This eq. is of the form  $\dot{C}_{2,0}(t) = \beta C_{2,0}(t)$

  $\left\{ \begin{array}{l} \text{solutions oscillate at freq. } \text{Im}[\beta] \\ \text{and grow or decay at rate } \text{Re}[\beta] \end{array} \right.$

Next: Atoms couple weakly to the vacuum

$C_{2,0}(t)$  changes slowly on timescale  $\omega_{21}^{-1}$ ,  
evolving at a rate  $\Gamma$  to be determined.

That means we can let  $t \rightarrow \infty$  on timescale  $\omega_{21}^{-1}$   
while still keeping  $t \ll \Gamma^{-1}$

Defining  $-i\zeta(\omega_{21} - \omega_{21}) = \int_0^\infty dt' e^{i(\omega_{21} - \omega_{21})(t-t')}$

We can then rewrite

$$\dot{C}_{2,0}(t) = - \int_0^\infty d\omega_{21} |g(\omega_{21})|^2 D(\omega_{21}) [-i\zeta(\omega_{21} - \omega_{21})] C_{2,0}(t)$$

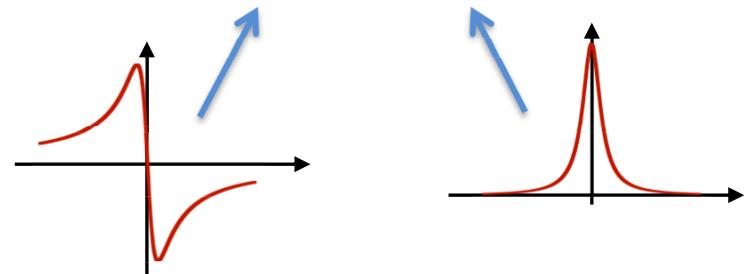
Math Details: Consider the function

$$\xi(\Omega) = i \int_0^\infty d\tau e^{-i\Omega\tau}, \quad \Omega = \omega_{21} - \omega_{21}$$

Approximate the integral as

$$\xi(\Omega) = \lim_{\varepsilon \rightarrow 0^+} i \int_0^\infty d\tau e^{-i\Omega\tau - \varepsilon\tau} = \lim_{\varepsilon \rightarrow 0^+} \left( \frac{i}{-i\Omega - \varepsilon} \right)$$

$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= P\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$

 Cauchy's Principal Part

# Wigner-Weisskopf Theory of Spontaneous Decay

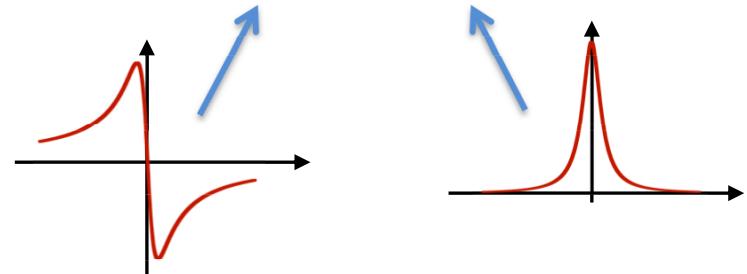
Math Details: Consider the function

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Approximate the integral as

$$\xi(\Omega) = \lim_{\varepsilon \rightarrow 0^+} i \int_0^\infty d\tau e^{-i\Omega\tau - \varepsilon\tau} = \lim_{\varepsilon \rightarrow 0^+} \left( \frac{i}{-i\Omega - \varepsilon} \right)$$

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Cauchy's Principal Part

# Wigner-Weisskopf Theory of Spontaneous Decay

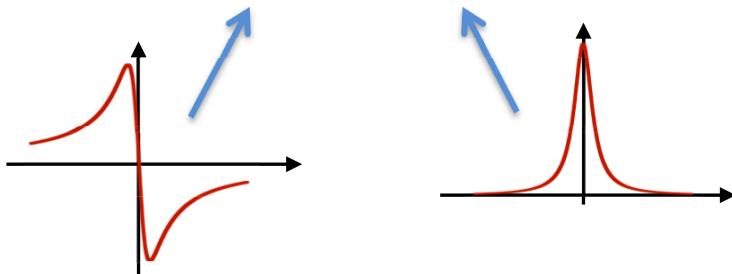
Math Details: Consider the function

$$\xi(\Omega) = i \int_0^\infty d\tau e^{-i\Omega\tau}, \quad \Omega = \omega_k - \omega_L$$

Approximate the integral as

$$\xi(\Omega) = \lim_{\varepsilon \rightarrow 0^+} i \int_0^\infty d\tau e^{-i\Omega\tau - \varepsilon\tau} = \lim_{\varepsilon \rightarrow 0^+} \left( \frac{i}{-i\Omega - \varepsilon} \right)$$

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Cauchy's Principal Part

# Wigner-Weisskopf Theory of Spontaneous Decay

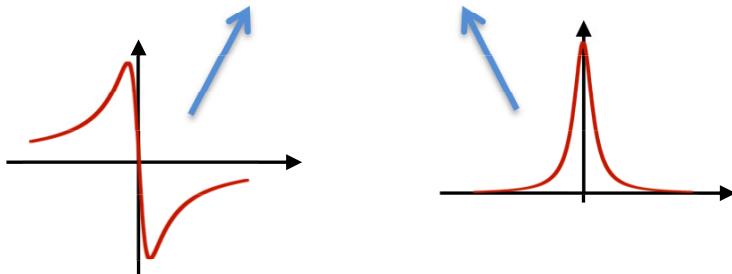
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$$= P\left(\frac{1}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

# Wigner-Weisskopf Theory of Spontaneous Decay

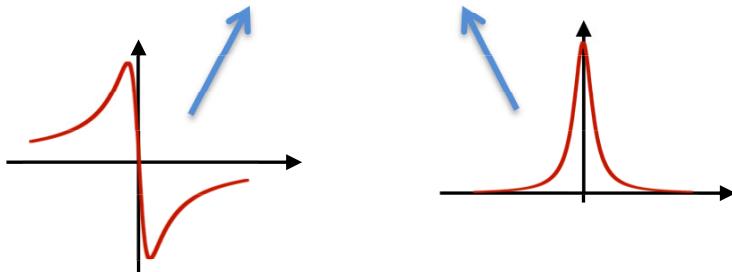
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$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= P\left(\frac{1}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx$$

# Wigner-Weisskopf Theory of Spontaneous Decay

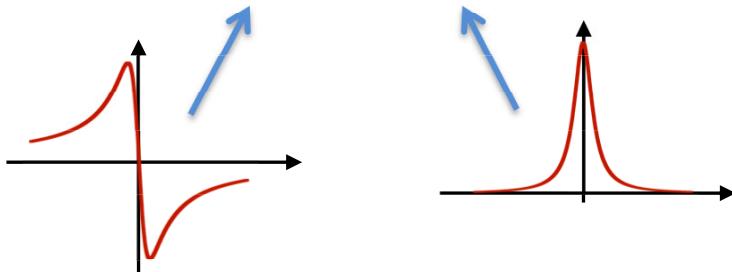
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$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= \text{P}\left(\frac{1}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\begin{aligned} \dot{c}_{2,0}(t) \approx & \\ & - \left[ \int_0^\infty d\omega_{\text{R}} |\mathcal{G}(\omega_{\text{R}})|^2 D(\omega_{\text{R}}) \left( \pi\delta(\omega_{\text{L}} - \omega_{\text{R}}) - i\text{P}\left(\frac{1}{\omega_{\text{L}} - \omega_{\text{R}}}\right) \right) \right] c_{2,0}(t) \end{aligned}$$

# Wigner-Weisskopf Theory of Spontaneous Decay

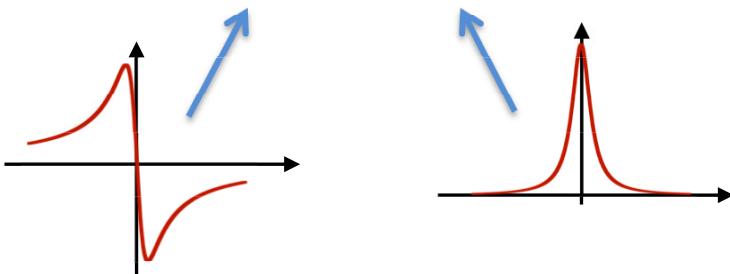
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Approximate the integral as

$$\xi(\Omega) = \lim_{\varepsilon \rightarrow 0^+} i \int_0^\infty d\tau e^{-i\Omega\tau - \varepsilon\tau} = \lim_{\varepsilon \rightarrow 0^+} \left( \frac{i}{-i\Omega - \varepsilon} \right)$$

$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= P\left(\frac{1}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_R |\mathcal{G}(\omega_R)|^2 D(\omega_R) \left( \pi\delta(\omega_2 - \omega_R) - iP\left(\frac{1}{\omega_2 - \omega_R}\right) \right) \right] c_{2,0}(t)$$

which is of the form

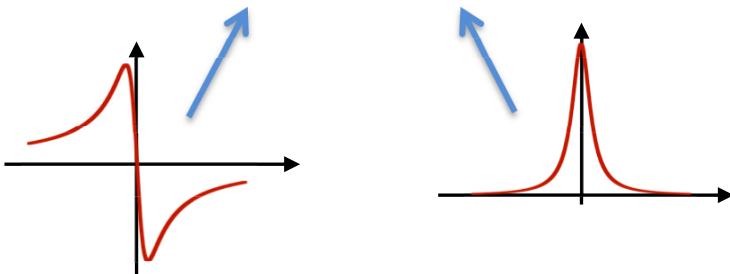
# Wigner-Weisskopf Theory of Spontaneous Decay

Math Details: Consider the function

$$\xi(\Omega) = i \int_0^\infty d\tau e^{-i\Omega\tau}, \quad \Omega = \omega_R - \omega_I$$

Approximate the integral as

$$\begin{aligned} \xi(\Omega) &= \lim_{\varepsilon \rightarrow 0^+} i \int_0^\infty d\tau e^{-i\Omega\tau - \varepsilon\tau} = \lim_{\varepsilon \rightarrow 0^+} \left( \frac{i}{-i\Omega - \varepsilon} \right) \\ &= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right) \end{aligned}$$



$$= P\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\begin{aligned} \dot{c}_{2,0}(t) &\approx \\ &- \left[ \int_0^\infty d\omega_R |\mathcal{G}(\omega_R)|^2 D(\omega_R) \left( \pi\delta(\omega_I - \omega_R) - iP\left(\frac{1}{\omega_I - \omega_R}\right) \right) \right] c_{2,0}(t) \end{aligned}$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t)$$

# Wigner-Weisskopf Theory of Spontaneous Decay

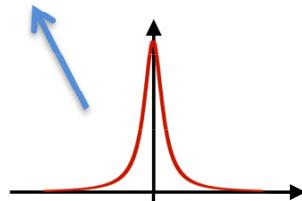
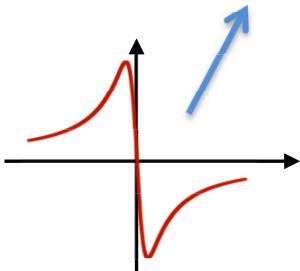
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Approximate the integral as

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$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= \text{P}\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_{\text{R}} |\mathcal{G}(\omega_{\text{R}})|^2 D(\omega_{\text{R}}) \left( \pi \delta(\omega_{\text{L}} - \omega_{\text{R}}) - i \text{P}\left(\frac{1}{\omega_{\text{L}} - \omega_{\text{R}}}\right) \right) \right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t) \quad \rightarrow$$

# Wigner-Weisskopf Theory of Spontaneous Decay

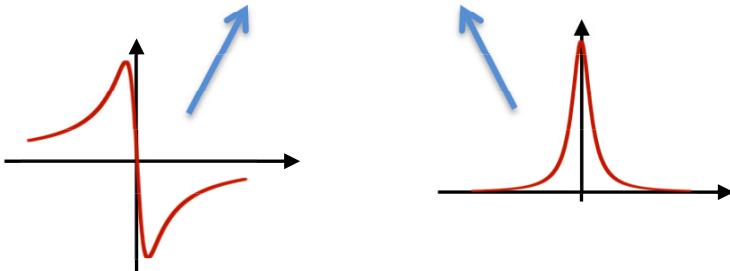
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$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= \text{P}\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_R |g(\omega_R)|^2 D(\omega_R) \left( \pi\delta(\omega_2 - \omega_R) - i\text{P}\left(\frac{1}{\omega_2 - \omega_R}\right) \right) \right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t) \quad \rightarrow$$

$$c_{2,0}(t) = e^{-A_{21}/2 t} e^{i\delta(t)} c_{2,0}(0)$$

# Wigner-Weisskopf Theory of Spontaneous Decay

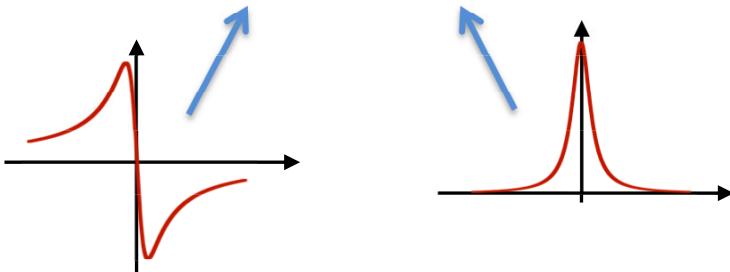
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$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= P\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \left( \pi\delta(\omega_{\vec{q}} - \omega_{\vec{k}}) - iP\left(\frac{1}{\omega_{\vec{q}} - \omega_{\vec{k}}}\right) \right) \right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t) \quad \rightarrow$$

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# Wigner-Weisskopf Theory of Spontaneous Decay

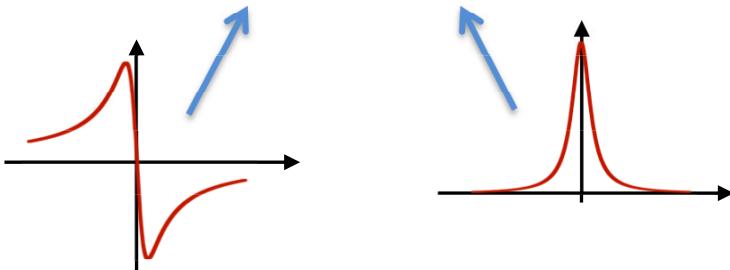
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$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= \text{P}\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_{\text{R}} |g(\omega_{\text{R}})|^2 D(\omega_{\text{R}}) \left( \pi\delta(\omega_{\text{L}} - \omega_{\text{R}}) - i\text{P}\left(\frac{1}{\omega_{\text{L}} - \omega_{\text{R}}}\right) \right) \right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t) \quad \rightarrow$$

$$c_{2,0}(t) = e^{-A_{21}/2 t} e^{i\delta(t)} c_{2,0}(0)$$

Decay Rate:

$$A_{21} = - \int_0^\infty d\omega_{\text{R}} 2\pi |g(\omega_{\text{R}})|^2 D(\omega_{\text{R}}) \delta(\omega_{\text{L}} - \omega_{\text{R}})$$

# Wigner-Weisskopf Theory of Spontaneous Decay

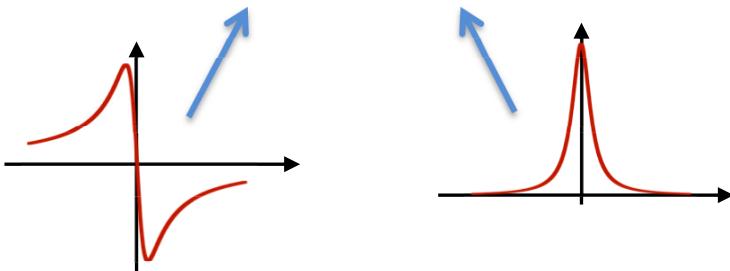
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$$= \text{P}\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_{\text{R}} |g(\omega_{\text{R}})|^2 D(\omega_{\text{R}}) \left( \pi\delta(\omega_{\text{L}} - \omega_{\text{R}}) - i\text{P}\left(\frac{1}{\omega_{\text{L}} - \omega_{\text{R}}}\right) \right) \right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t) \quad \rightarrow$$

$$c_{2,0}(t) = e^{-A_{21}/2 t} e^{i\delta(t)} c_{2,0}(0)$$

Decay Rate:

$$A_{21} = - \int_0^\infty d\omega_{\text{R}} 2\pi |g(\omega_{\text{R}})|^2 D(\omega_{\text{R}}) \delta(\omega_{\text{L}} - \omega_{\text{R}})$$

$$= 2\pi |g(\omega_{21})|^2 D(\omega_{21})$$

# Wigner-Weisskopf Theory of Spontaneous Decay

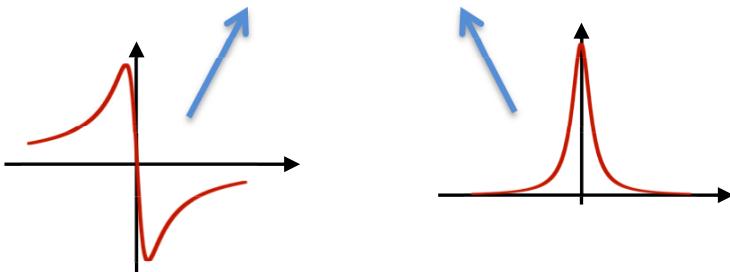
Math Details: Consider the function

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Approximate the integral as

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$$= P\left(\frac{1}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \left( \pi\delta(\omega_2 - \omega_{\vec{k}}) - iP\left(\frac{1}{\omega_2 - \omega_{\vec{k}}}\right) \right) \right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t) \quad \Rightarrow$$

$$c_{2,0}(t) = e^{-A_{21}/2 t} e^{i\delta(t)} c_{2,0}(0)$$

Decay Rate:

$$A_{21} = - \int_0^\infty d\omega_{\vec{k}} 2\pi |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \delta(\omega_2 - \omega_{\vec{k}})$$

$$= 2\pi |g(\omega_{21})|^2 D(\omega_{21})$$

$$= 2\pi \times \frac{4\pi C_{21}}{3\hbar\epsilon_0 V} |\vec{p}_{21}|^2 \times \frac{V}{(2\pi)^3} \frac{\omega_{21}^2}{C^3}$$

# Wigner-Weisskopf Theory of Spontaneous Decay

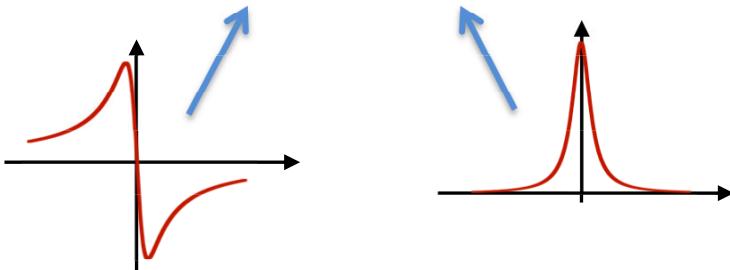
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Approximate the integral as

$$\xi(\Omega) = \lim_{\varepsilon \rightarrow 0^+} i \int_0^\infty d\tau e^{-i\Omega\tau - \varepsilon\tau} = \lim_{\varepsilon \rightarrow 0^+} \left( \frac{i}{-i\Omega - \varepsilon} \right)$$

$$= \lim_{\varepsilon \rightarrow 0^+} \left( \frac{-\Omega^2}{\Omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\Omega^2 + \varepsilon^2} \right)$$



$$= P\left(\frac{i}{\Omega}\right) + i\pi\delta(\Omega)$$



Cauchy's Principal Part

Thus,

$$\dot{c}_{2,0}(t) \approx - \left[ \int_0^\infty d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \left( \pi \delta(\omega_{21} - \omega_{\vec{k}}) - i P\left(\frac{1}{\omega_{21} - \omega_{\vec{k}}}\right) \right) \right] c_{2,0}(t)$$

which is of the form

$$\dot{c}_{2,0} = - \left( \frac{A_{21}}{2} - i\delta \right) c_{2,0}(t) \quad \Rightarrow$$

$$c_{2,0}(t) = e^{-A_{21}/2 t} e^{i\delta(t)} c_{2,0}(0)$$

Decay Rate:

$$A_{21} = - \int_0^\infty d\omega_{\vec{k}} 2\pi |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \delta(\omega_{21} - \omega_{\vec{k}})$$

$$= 2\pi |g(\omega_{21})|^2 D(\omega_{21})$$

$$= 2\pi \times \frac{4\pi C_{21}}{3\hbar\epsilon_0 V} |\vec{p}_{21}|^2 \times \frac{V}{(2\pi)^3} \frac{\omega_{21}^2}{C^3}$$



# Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{C}_{2,0}(t) \approx$$

$$-\left[ \int_0^{\infty} d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) (\pi \delta(\omega_{21} - \omega_{\vec{k}}) - i \Phi\left(\frac{1}{\omega_{21} - \omega_{\vec{k}}}\right)) \right] C_{2,0}(t)$$

which is of the form

$$\dot{C}_{2,0} = -\left(\frac{A_{21}}{2} - i\delta\right) C_{2,0}(t) \quad \rightarrow$$

$$C_{2,0}(t) = e^{-A_{21}/2 t} e^{i\delta(t)} C_{2,0}(0)$$

Decay Rate:

$$A_{21} = - \int_0^{\infty} d\omega_{\vec{k}} 2\pi |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \delta(\omega_{21} - \omega_{\vec{k}})$$

$$= 2\pi |g(\omega_{21})|^2 D(\omega_{21})$$

$$= 2\pi \times \frac{4\pi\omega_{21}}{3\hbar\epsilon_0 V} |\vec{p}_{21}|^2 \times \frac{V}{(2\pi)^3} \frac{\omega_{21}^2}{C^3}$$



# Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

$$\dot{C}_{2,0}(t) \approx - \left[ \int_0^{\infty} d\omega_{\vec{k}} |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \left( \pi \delta(\omega_{21} - \omega_{\vec{k}}) - i \Phi \left( \frac{1}{\omega_{21} - \omega_{\vec{k}}} \right) \right) \right] C_{2,0}(t)$$

which is of the form

$$\begin{aligned}\dot{C}_{2,0} &= - \left( \frac{A_{21}}{2} - i\delta \right) C_{2,0}(t) \quad \rightarrow \\ C_{2,0}(t) &= e^{-A_{21}/2 t} e^{i\delta(t)} C_{2,0}(0)\end{aligned}$$

Decay Rate:

$$\begin{aligned}A_{21} &= - \int_0^{\infty} d\omega_{\vec{k}} 2\pi |g(\omega_{\vec{k}})|^2 D(\omega_{\vec{k}}) \delta(\omega_{21} - \omega_{\vec{k}}) \\ &= 2\pi |g(\omega_{21})|^2 D(\omega_{21}) \\ &= 2\pi \times \frac{4\pi c \omega_{21}}{3\hbar \epsilon_0 V} |\vec{p}_{21}|^2 \times \frac{V}{(2\pi)^3} \frac{\omega_{21}^2}{c^3}\end{aligned}$$

# Wigner-Weisskopf Theory of Spontaneous Decay

Thus,

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$A_{21}$

which is of the form

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Lamb shift

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Hydrogen 2P lifetime:

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Lamb shift

Hydrogen 2P lifetime:

- accurate calculations for

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Lamb shift

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$$(A_{21})^{-1} = \tau_{2p} = 1.59 \text{ ns}$$

Measured:  $\tau_{2p} = 1.60 \text{ ns}$

# Wigner-Weisskopf Theory of Spontaneous Decay

$$A_{21} = \frac{\omega_{21}^3 |\vec{p}_{21}|^2}{3\pi \epsilon_0 \hbar c^3} = \frac{8\pi^2 |\vec{p}_{21}|^2}{3\pi \hbar c^3}$$

This is the value used earlier!

Frequency shift:

$$\delta = \lim_{\varepsilon \rightarrow 0} \left( i \int_0^\infty d\omega_{kk} [\overline{g(\omega_{kk})}]^2 D(\omega_{kk}) \frac{\omega_{21} - \omega_{kk}}{(\omega_{21} - \omega_{kk})^2 + \varepsilon^2} \right)$$

Integral is not well behaved!

Further development by Willis Lamb



Lamb shift

Hydrogen 2P lifetime:

- accurate calculations for

$$\lambda = 121.57 \text{ nm}$$

$$|\vec{p}_{21}| = 0.745 a_0 e = \frac{4\pi \epsilon_0 \hbar^2}{m_e e^2} \times 0.745 e$$



$$(A_{21})^{-1} = \tau_{2p} = 1.59 \text{ ns}$$

Measured:  $\tau_{2p} = 1.60 \text{ ns}$