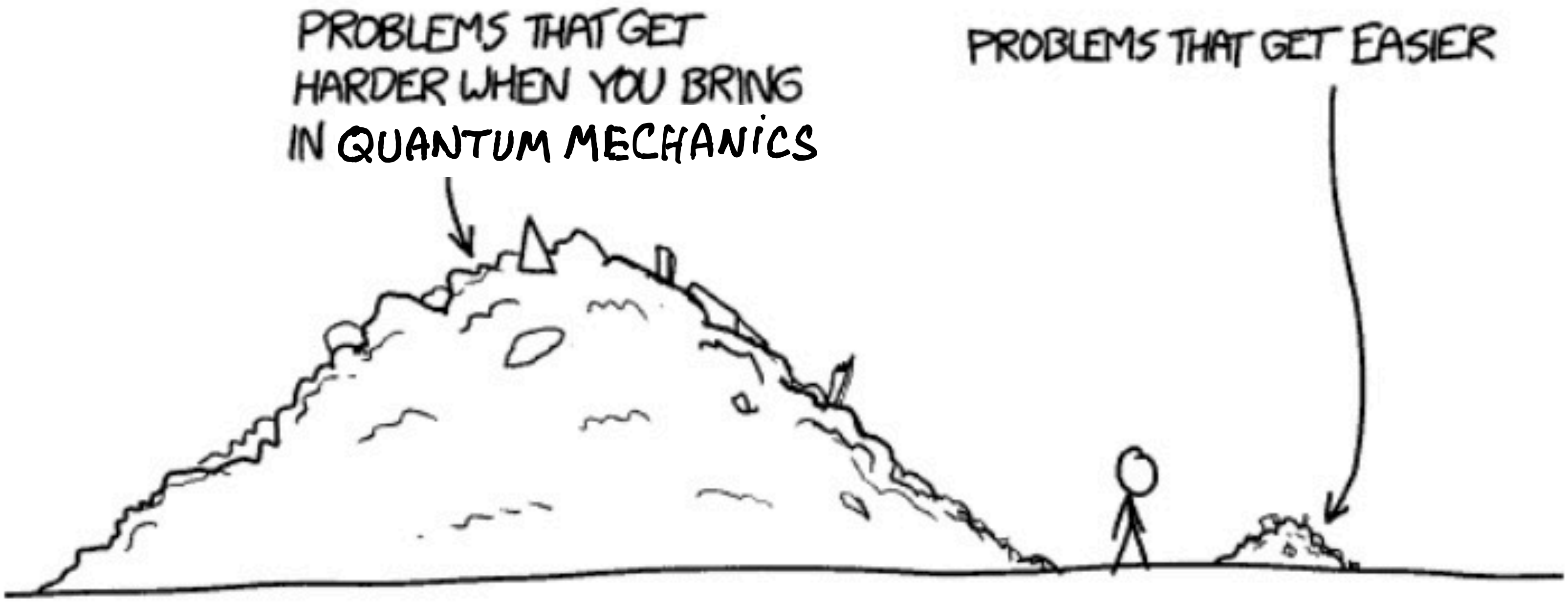
Quantum Electrodynamics – Intro to Field Theory

03-27-2025



Source: xkcd.com

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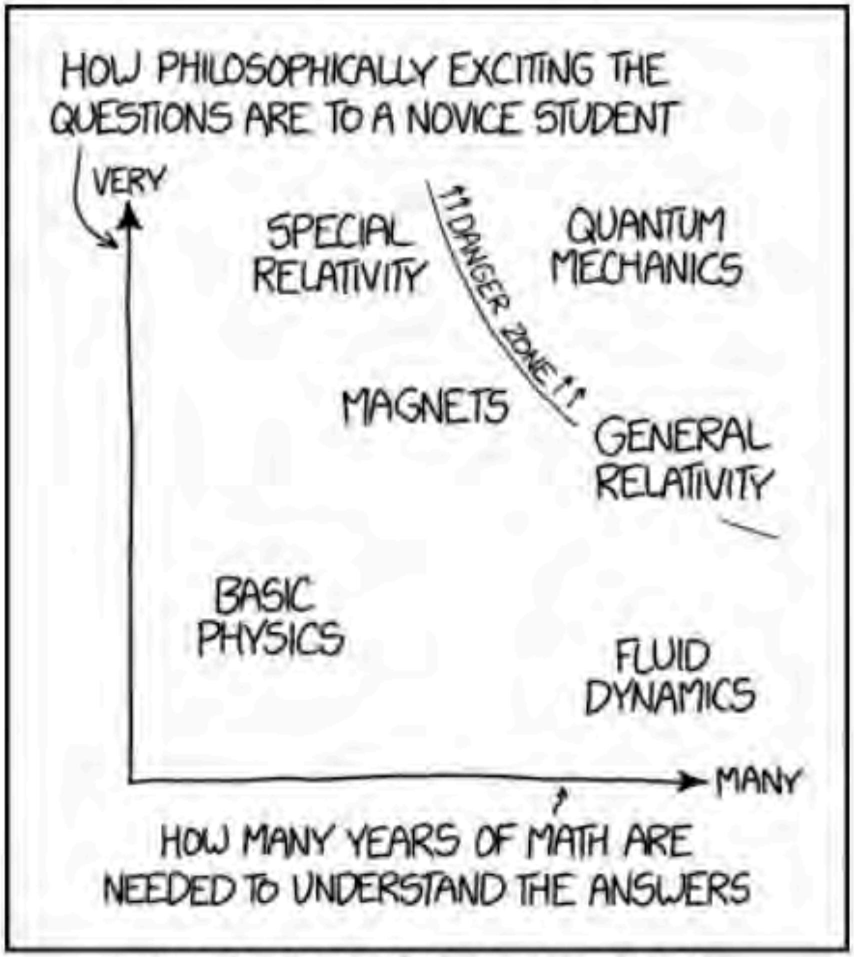
QUANTUM



WHY SO MANY PEOPLE HAVE WEIRD IDEAS ABOUT QUANTUM MECHANICS

Quantum Electrodynamics – Intro to Field Theory

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(∗) **Primary goal of OPTI 544: Quantum description of EM field**

(∗) **Challenge: 1st semester Grad level QM (OPTI 570) does not tell how to do this**

(∗) **Warm-up: Quantum field theory for vibrations (sound) in elastic rod**

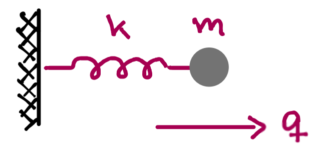
(∗) **This is in part a review of the <u>classical</u> Lagrange/Hamilton-Jacobi description of continuous systems**

(∗) **Here we present the formalism as a Cookbook Recipe for how we get from Classical to Quantum Physics**

See, e. g., Cohen-Tannoudji Vol. 2, Appendix III, Sections 1-3.

Classical Simple Harmonic Oscillator (SHO)

Particle on a spring



dynamical coordinate.

$\omega = \sqrt{k/m}$

osc. freq.

Kinetic Energy: $T = \frac{1}{2} m \dot{q}^2$

Potential Energy: $V = \frac{1}{2} k q^2 = \frac{1}{2} m \omega^2 q^2$

<u>**Lagrangian**</u>**:** $\mathcal{L} = T - V = \frac{1}{2} m \dot{q}^2 - \frac{1}{2} m \omega^2 q^2$

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = 0 \quad \Rightarrow \quad \ddot{q} + \omega^2 q = 0$$

usual eq. of motion

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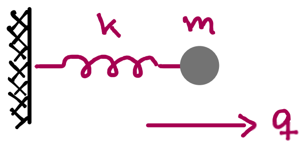

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usual eq. of motion

Conjugate momentum

$$p = \frac{\partial \mathcal{L}}{\partial \dot{q}} = m \dot{q}$$

Hamiltonian

$$\mathcal{H} = T(\dot{q} = p/m) + V(q) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2$$

$$\left. \begin{aligned} \dot{q} &= \frac{\partial \mathcal{H}}{\partial p} = p/m \\ \dot{p} &= -\frac{\partial \mathcal{H}}{\partial q} = -m\omega^2 q \end{aligned} \right\} \Rightarrow \quad \ddot{q} + \omega^2 q = 0$$

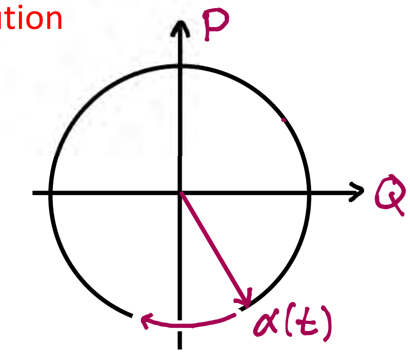
solution

Scaled variables

$Q = q/q_0, \quad P = p/p_0$

$$\alpha = Q + iP \left\{ \begin{aligned} Q &= \mathrm{Re}[\alpha] \\ P &= \mathrm{Im}[\alpha] \\ \mathcal{H} &= E_0 \alpha^* \alpha \end{aligned} \right.$$

Phase plane



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Conjugate momentum $\quad p = \dfrac{\partial \mathcal{L}}{\partial \dot{q}} = m\dot{q}$

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$$Q \equiv q/q_0, \quad P = p/p_0$$

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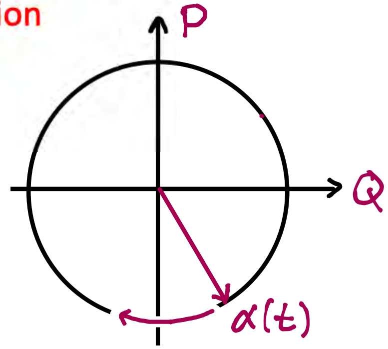
Phase plane



Quantum Harmonic Oscillator

Formal Quantization Procedure:

$$\boxed{q \to \hat{q}, \quad p \to \hat{p}, \quad [\hat{q}, \hat{p}] = i\hbar}$$

Choose $E_0 = \hbar\omega \Rightarrow q_0 = \sqrt{\dfrac{2\hbar}{m\omega}}, \quad p_0 = \sqrt{2m\hbar\omega}$

natural scale

$$\alpha \to \hat{a} = \hat{Q} + i\hat{P} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{q} + i\frac{\hat{p}}{m\omega}\right)$$

$$[\hat{a}, \hat{a}^+] = 1$$

Rewrite:

$$\hat{H} = \hbar\omega(\hat{Q}^2 + \hat{P}^2) = \hbar\omega(\hat{a}^+\hat{a} + 1/2)$$

$\hat{N} = \hat{a}^+\hat{a}$ **(number operator)**

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Quantum Harmonic Oscillator

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$\hat{N} = \hat{a}^+\hat{a}$ (number operator)

Commutator $[\hat{H}, \hat{N}] = 0$

⇒ joint energy/number states $|n\rangle$

$$\hat{H}|n\rangle = \hbar\omega(n + 1/2)|n\rangle$$

$$\hat{N}|n\rangle = n|n\rangle$$

Commutators

$$\left.\begin{array}{l}[\hat{N}, \hat{a}^+] = \hat{a}^+ \\ [\hat{N}, \hat{a}] = -\hat{a}\end{array}\right\} \Rightarrow \begin{array}{l}\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle \\ \hat{a}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle \\ \hat{a}|0\rangle = 0\end{array}$$

Generating excited states

$$\boxed{|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^+)^n|0\rangle}$$

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Commutator $[\hat{H}, \hat{N}] = 0$

➡ **joint energy/number states** $|n\rangle$

$$\boxed{\begin{aligned}\hat{H}|n\rangle &= \hbar\omega(n+1/2)|n\rangle \\ \hat{N}|n\rangle &= n|n\rangle\end{aligned}}$$

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Generating excited states

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Expectation values for $\hat{q}$ and $\hat{p}$ in number states

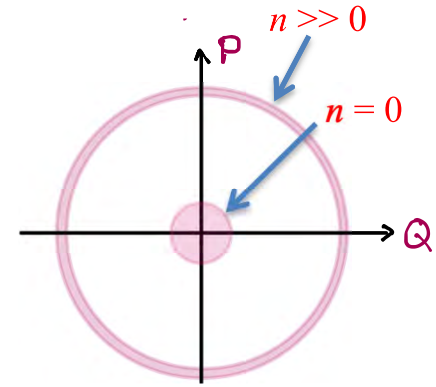
$$\langle n|\hat{q}|n\rangle = \langle n|\hat{p}|n\rangle = 0$$

$$\langle n|\hat{q}^2|n\rangle = \frac{q_0^2}{2}(n+1/2) \neq 0$$

$$\langle n|\hat{p}^2|n\rangle = \frac{p_0^2}{2}(n+1/2) \neq 0$$

$$\Delta q\,\Delta p = \frac{q_0 p_0}{2}(n+1/2) = \hbar(n+1/2)$$

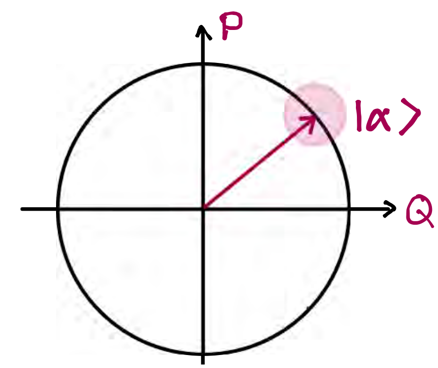
Phase space visualization of number states



Quasi-classical (coherent) state

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_i \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

$$\Delta q\,\Delta p = \hbar/2, \quad \Delta Q = \Delta P$$



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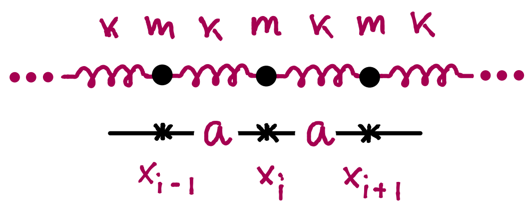
Lagrange formulation of 1D Scalar Field

"vibronium"



Configuration space = $\{x_i\}$ (set of *N* osc. positions)

$$T = \sum_{i=1} \tfrac{1}{2} m \dot{x}_i^2, \quad V = \sum_{i=1}^{N} \tfrac{1}{2} \kappa (x_{i+1} - x_i)^2$$

Lagrangian, equations of motion

Continuum limit ➡ **Elastic rod**

$N \to \infty$ $\quad m/a \to \mu$ ← linear mass density

$a \to dx$ $\quad \kappa a \to Y$ ← Youngs modulus

$\{x_i\} \to y(x)$ ← displacement field (sound)

Rewrite

$$\boxed{\begin{aligned} T &= \lim_{N\to\infty} \sum_{i=1}^{N} a \tfrac{1}{2} \left(\frac{m}{a}\right) \dot{x}_i^2 = \int dx \, \tfrac{1}{2} \mu \left(\frac{\partial y}{\partial t}\right)^2 \\ V &= \lim_{N\to\infty} \sum_{i=1}^{N} a \tfrac{1}{2} \kappa a \left(\frac{x_{i+1} - x_i}{a}\right)^2 = \int dx \, \tfrac{1}{2} Y \left(\frac{\partial y}{\partial x}\right)^2 \end{aligned}}$$

Lagrangian:

$$\boxed{\mathcal{L} = T - V = \int dx \, \tfrac{1}{2} \mu \left(\frac{\partial y}{\partial t}\right)^2 - \int dx \, \tfrac{1}{2} Y \left(\frac{\partial y}{\partial x}\right)^2}$$

Notes, Homework ➡ **Scalar wave equation**

$$\boxed{\frac{\partial^2 y}{\partial t^2} - \frac{Y}{\mu} \frac{\partial y^2}{\partial x^2} = 0}$$

– Not yet ready for Quantization –

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Normal Mode Decomposition

Field in cavity:

Solutions to wave eq.

Let $y(x,t) = g(t)u(x) = g_0 e^{i\omega t} u(x)$

$$\ddot{y} - v^2 y'' = -\omega^2 g(t) u(x) - v^2 g(t) u''(x) = 0$$

$$\boxed{u''(x) = -k^2 u(x), \quad k = \omega/v}$$

Solutions in cavity:

$$\boxed{u_k(x) = \sqrt{\frac{2}{L}} \sin(kx), \quad k = \frac{n\pi}{L}}$$

These standing waves are a set of Normal Modes

These modes are orthonormal and complete

$$\boxed{y(x,t) = \sqrt{L} \sum_k q_k(t) u_k(x)}$$

Normal mode expansion of $y(x,t)$ **in basis** $u_k(x)$

Lagrangian for the acoustic field:

$$T = \int dx \frac{1}{2} \mu \left(\frac{\partial y}{\partial t}\right)^2 = \sum_{k,k'} \frac{1}{2} \underbrace{\mu L}_{M} \dot{q}_k \dot{q}_{k'} \underbrace{\int dx\, u_k(x) u_{k'}(x)}_{\delta_{kk'}}$$
$$= \sum_k \frac{1}{2} M \dot{q}_k^2$$

$$V = \int dx \frac{1}{2} Y \left(\frac{\partial y}{\partial x}\right)^2 = \sum_{kk'} \frac{1}{2} Y L q_k q_{k'} \int dx \left(\frac{\partial u_k}{\partial x}\right)\left(\frac{\partial u_{k'}}{\partial x}\right)$$
$$= \sum_k \frac{1}{2} M \omega_k^2 q_k^2$$

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The rest now follows from the Lagrangian

$$\mathcal{L} = T - V = \sum_k \left(\tfrac{1}{2} M \dot{q}_k^2 - \tfrac{1}{2} M \omega_k^2 q_k^2 \right) = \sum_k \mathcal{L}_k$$

Canonical Momentum

$$p_k = \frac{\partial \mathcal{L}}{\partial \dot{q}_k} = M \dot{q}_k$$

Hamiltonian

$$\mathcal{H}(\{p_k, q_k\}) = T + V = \sum_k \left(\frac{p_k^2}{2M} + \frac{1}{2} M \omega_k^2 q_k^2 \right)$$

(collection of SHO's, one for each normal mode)

Following the standard recipe…

$$E_{0,k} = \hbar\omega_k \ , \quad q_{0,k} = \sqrt{2\hbar / M\omega_k} \ , \quad p_{0,k} = \sqrt{2M\hbar\omega_k}$$

$$Q_k = q_k / q_{0,k} \ , \quad P_k = p_k / p_{0,k} \ , \quad \alpha_k = Q_k + i P_k$$

… we get solutions

$$\alpha_k(t) = Q_k(t) + i P_k(t) = \alpha_k(0)\, e^{-i\omega_k t}$$

This finally gives us

$$\mathcal{H} = \sum_k \hbar\omega_k \left(Q_k^2 + P_k^2 \right) = \sum_k \hbar\omega_k \alpha_k^* \alpha_k$$

$$y(x,t) = \sqrt{L} \sum_k q_k(t)\, u_k(x)$$

$$= \frac{1}{2} \sum_k \sqrt{L q_{0,k}^2} \left(\alpha_k(t)\, u_k(x) + \alpha_k^*(t)\, u_k^*(x) \right)$$

allows real or complex $u_k(x)$

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$$= \frac{1}{2} \sum_k \sqrt{L q_{0,k}^2} \left(\alpha_k(t) u_k(x) + \alpha_k^*(t) u_k(x) \right)$$

$$\alpha = Q + iP \qquad Q = \tfrac{1}{2}(\alpha + \alpha^*) \qquad q = \tfrac{q_0}{2}(\alpha + \alpha^*)$$

$$\alpha^* = Q - iP \qquad P = \tfrac{1}{2i}(\alpha - \alpha^*) \qquad p = \tfrac{p_0}{2}(\alpha - \alpha^*)$$

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… we get solutions

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Formal Quantization Procedure:

$$q_k \to \hat{q}_k \,, \quad p_k \to \hat{p}_k \,, \quad \alpha_k \to \hat{a}_k$$

$$[\hat{q}_k, \hat{p}_{k'}] = i\hbar\,\delta_{kk'} \,, \quad [\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'} \,, \quad [\hat{a}_k, \hat{a}_{k'}] = 0$$

Note: $k \neq k'$ $\Rightarrow$ **operators commute**
(normal modes = independent degs. of freedom)

Hamiltonian & Quantized fields

$$\hat{H} = \sum_k \hbar\omega_k \left(\hat{a}_k^\dagger \hat{a}_k + 1/2\right)$$

$$\hat{y}(x) = \sqrt{L}\sum_k \hat{q}_k\, u_k(x) = \sum_k \sqrt{L q_{0,k}^2}\left(\hat{a}_k u_k(x) + \hat{a}_k^\dagger u_k(x)\right)$$

$$\hat{\Pi}(x) = \frac{1}{\sqrt{L}}\sum_k \hat{p}_k u_k(x) = -i\sum_k \sqrt{\frac{p_{0,k}^2}{L}}\left(\hat{a}_k u_k(x) - \hat{a}_k^\dagger u_k(x)\right)$$

field $\hat{y}(x)$ **and canonical momentum field** $\hat{\Pi}(x)$

$$\Rightarrow \left[\hat{y}(x), \hat{\Pi}(x')\right] = i\hbar\,\delta(x-x')$$

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Hamiltonian & Quantized fields

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field $\hat{\eta}(x)$ and canonical momentum field $\hat{\pi}(x)$

⇒ $$[\hat{\eta}(x), \hat{\pi}(x')] = i\hbar\delta(x - x')$$

Quantum States:

The quantum field is a collection of QSHO's

⇓

State space = tensor product of SHO spaces

SHO space

Fock Space $\mathcal{E} = \mathcal{E}_{k_1} \otimes \mathcal{E}_{k_2} \otimes \mathcal{E}_{k_3} \otimes \cdots , \mathcal{E}_{k_j}$

Fock State $|\{n_{k_1}, n_{k_2}, \ldots\}\rangle = |n_{k_1}\rangle \otimes |n_{k_2}\rangle \otimes \ldots |n_{k_j}\rangle$

$\hat{a}_{k_i}, \hat{a}^\dagger_{k_i}$ destroy/create excitations in mode k_i

Vacuum State $|0\rangle$ ← zero quanta in every mode

Favorite Question: What is a Phonon?

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Vacuum Fluctuations:

Expectation value of the Field

$$\hat{\eta}(x) = \frac{1}{2}\sum_k \sqrt{L q^2_{q_k}}\left(\hat{a}_k u_k(x) + \hat{a}^{\dagger}_k u_k(x)\right)$$

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Expectation value of the Field

$\langle 0 | \hat{\eta}(x) | 0 \rangle$

$$\hat{\eta}(x) = \frac{1}{2} \sum_k \sqrt{L q_{0,k}^2} \left(\hat{a}_k u_k(x) + \hat{a}_k^{\dagger} u_k(x) \right)$$

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SHO state

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Expectation value of the Field

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⇩

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Expectation value of the Field

$$\langle 0|\hat{\eta}(x)|0\rangle =$$

$$\sum_k \frac{1}{2}\sqrt{L q^2_{0,k}}\left(\langle 0|\hat{a}_k|0\rangle u_k(x) + \langle 0|\hat{a}^\dagger_k|0\rangle u_k(x)\right)$$

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Quantum Electrodynamics – Intro to Field Theory

Quantum States:

The quantum field is a collection of QSHO's

State space = tensor product of SHO spaces

SHO space

Fock Space $\mathcal{E} = \mathcal{E}_{k_1} \otimes \mathcal{E}_{k_2} \otimes \mathcal{E}_{k_3} \otimes \ldots, \mathcal{E}_{k_j}$

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Thus $\Delta\eta(x) \neq 0$

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$$= \sum_k \frac{1}{4} L q_{0,k}^2 |u_k(x)|^2 \neq 0$$

Thus $\Delta\eta(x) \neq 0$ with zero phonons in field

Quantum Electrodynamics – Intro to Field Theory

Quantum States:

The quantum field is a collection of QSHO's

State space = tensor product of SHO spaces

SHO space

Fock Space $\mathcal{E} = \mathcal{E}_{k_1} \otimes \mathcal{E}_{k_2} \otimes \mathcal{E}_{k_3} \otimes \dots, \mathcal{E}_{k_j}$

SHO state

Fock State $|\{n_{k_1}, n_{k_2}, \dots\}\rangle = |n_{k_1}\rangle \otimes |n_{k_2}\rangle \otimes \dots |n_{k_j}\rangle$

$\hat{a}_{k_i}, \hat{a}^\dagger_{k_i}$ destroy/create excitations in mode k_i

Vacuum State $|0\rangle$ ← zero quanta in every mode

Favorite Question: **What is a Phonon?**

Vacuum Fluctuations:

Expectation value of the Field

$$\langle 0|\hat{\eta}(x)|0\rangle =$$

$$\sum_k \frac{1}{2}\sqrt{L q^2_{0,k}}\left(\langle 0|\hat{a}_k|0\rangle u_k(x) + \langle 0|\hat{a}^\dagger_k|0\rangle u^*_k(x)\right) = 0$$

Zero Point Fluctuations

$$\langle 0|\hat{\eta}(x)^2|0\rangle$$

$$= \sum_k \frac{1}{4} L q_{0,k} q_{0,k} \langle 0|\hat{a}^\dagger_k \hat{a}_k + 1|0\rangle u_k(x) u^*_k(x)$$

$$= \sum_k \frac{1}{4} L q^2_{0,k} |u_k(x)|^2 \neq 0$$

Thus $\Delta\eta(x) \neq 0$ with zero phonons in field

Note the famous divergence:

$$E_{vac} = \langle 0|\hat{H}|0\rangle = \sum_k \frac{\hbar\omega_k}{2} \rightarrow \infty \text{ for } k \rightarrow \infty$$

Quantum Electrodynamics – Intro to Field Theory

03-27-2025

Vacuum Fluctuations:

Expectation value of the Field

$$\langle 0|\hat{y}(x)|0\rangle =$$

$$\sum_k \frac{1}{2}\sqrt{L q_{0,k}^2}\left(\langle 0|\hat{a}_k|0\rangle u_k(x) + \langle 0|\hat{a}_k^{+}|0\rangle u_k(x)\right) = 0$$

Zero Point Fluctuations

$$\langle 0|\hat{y}(x)^2|0\rangle$$

$$= \sum_k \frac{1}{4} L q_{0,k} q_{0,k} \langle 0|\hat{a}_k^{+}\hat{a}_k + 1|0\rangle u_k(x) u_k(x)$$

$$= \sum_k \frac{1}{4} L q_{0,k}^2 |u_k(x)|^2 \neq 0$$

Thus $\Delta y(x) \neq 0$ **with zero phonons in field**

Note the famous divergence:

$$E_{vac} = \langle 0|\hat{H}|0\rangle = \sum_k \frac{\hbar\omega_k}{2} \to \infty \text{ for } k \to \infty$$

Are our Phonons waves or particles?

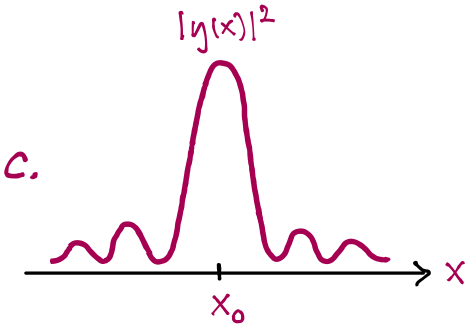
Extended

Localized

Particle-like Phonons

Classical Wavepacket

$$y(x) = \sum_k f_k u_k(x) + c.c.$$



Define $\hat{A}^{+} = \sum_k f_k \hat{a}_k^{+}$, $\sum_k |f_k|^2 = 1$

$$\Rightarrow \hat{A}^{+}|0\rangle = f_{k_1}|1_{k_1}, 0_{k_2}, \ldots\rangle + f_{k_2}|0_{k_1}, 1_{k_2}, \ldots\rangle + \ldots$$

Localized excitation in the field.

These Particle-like Phonons are Bosons

$$\hat{A}'^{+}\hat{A}^{+}|0\rangle = \hat{A}^{+}\hat{A}'^{+}|0\rangle$$

1^{st} particle @ x
2^{nd} particle @ x'

1^{st} particle @ x'
2^{nd} particle @ x