Atom-Light Interaction: Multi-Level Atoms

Begin 02-13-2025

General ED Selection Rules

$\Delta L = \pm 1$ $\qquad \vec{L}$: total e orbital A. M.

$\Delta F = 0, \pm 1$ $\qquad \vec{F}$: total orbital + spin A. M.

$\Delta m_F = q = 0, \pm 1$ $\qquad q$: polarization of EM field

Clebsch-Gordan coefficients ($E_{F', m_F'} > E_{F, m_F}$)

$$\langle F', m_F' | V | F, m_F \rangle \propto \langle 1, q; F, m_F | F', m_F' \rangle$$

$$\langle F, m_F | V | F', m_F' \rangle \propto \langle 1, -q; F' m_F' | F, m_F \rangle$$

Hydrogen atom

$1S - 2S$: forbidden $\qquad 1S - 2P$: allowed

Total spin: $\vec{F} = \vec{J} + \vec{I}, \quad \vec{J} = \vec{L} + \vec{S}$

nuclear orbital electron spin

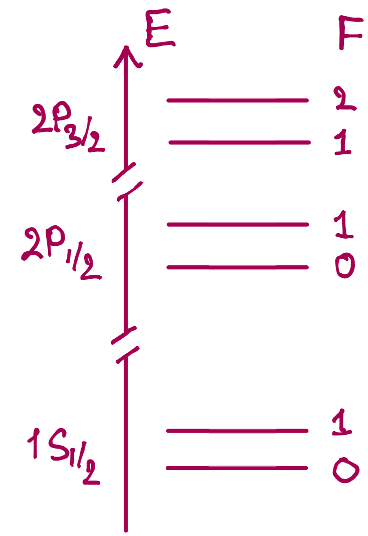
$1S$ State:

$J = 1/2, \quad F = 0, 1$

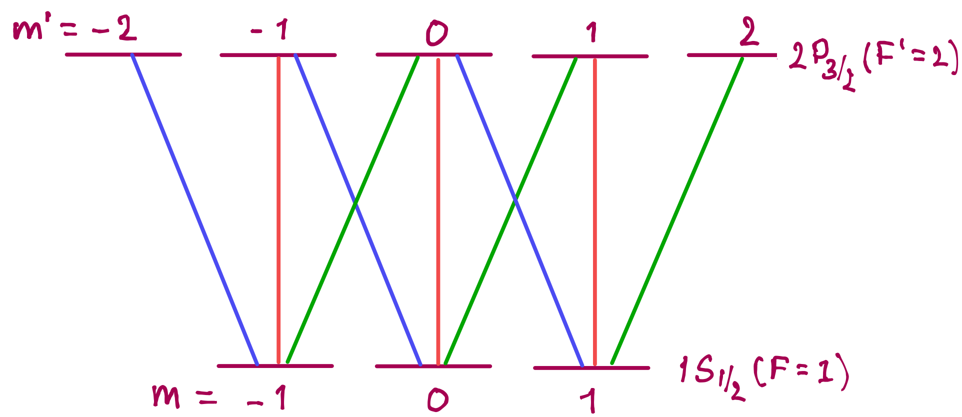
$2P$ State:

$J = 1/2, \quad F = 0, 1$

$J = 3/2, \quad F = 1, 2$



Level diagram for transitions $1S_{1/2}(F=1) \rightarrow 2P_{3/2}(F=2)$



Polarization: $q = 0$ $\quad q = 1$ $\quad q = -1$

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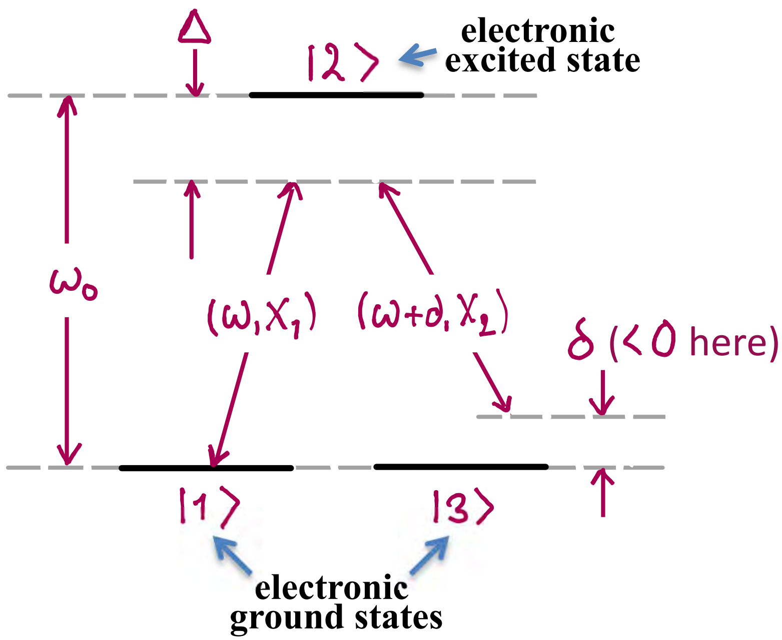
Raman Coupling in 3-level Atoms

Raman Coupling in 3-Level Systems

Consider an atom with this 3-level structure



For simplicity we set $E_1 = E_3$ **(no loss of generality)**

Fields $\begin{cases} \text{at } \omega\text{, coupling } |1\rangle, |2\rangle \text{ w/Rabi freq. } \chi_1 \\ \text{at } \omega+\delta\text{, coupling } |3\rangle, |2\rangle \text{ w/Rabi freq. } \chi_2 \end{cases}$

The Hamiltonian for this system is (χ_1, χ_2 **real)**

$$H = \hbar \begin{pmatrix} 0 & X_1(t) & 0 \\ X_1(t) & \omega_0 & X_2(t) \\ 0 & X_2(t) & 0 \end{pmatrix}$$

where

$$X_1(t) = \frac{\chi_1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$X_2(t) = \frac{\chi_2}{2}\left(e^{i(\omega+\delta)t} + e^{-i(\omega+\delta)t}\right)$$

Setting $|\psi(t)\rangle = a_1(t)|1\rangle + a_2(t)|2\rangle + a_3(t)|3\rangle$
we get a S.E.

$$\dot{a}_1 = -i\frac{\chi_1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)a_2$$

$$\dot{a}_2 = -i\omega_0 a_2 - i\frac{\chi_1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)a_1 - i\frac{\chi_2}{2}\left(e^{i(\omega+\delta)t} + e^{-i(\omega+\delta)t}\right)a_3$$

$$\dot{a}_3 = -i\frac{\chi_2}{2}\left(e^{i(\omega+\delta)t} + e^{-i(\omega+\delta)t}\right)a_2$$

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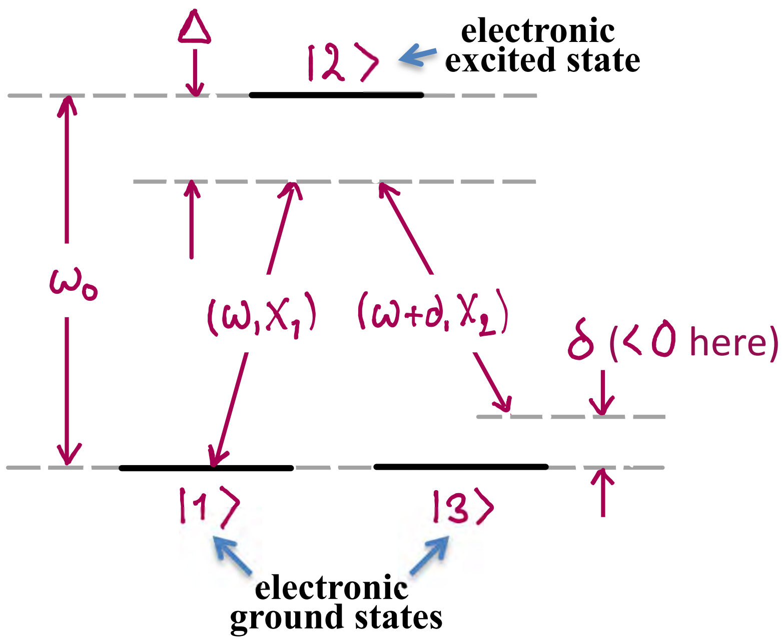

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Rotating Wave Approximation.

Let $a_1 = b_1,\quad a_2 = b_2 e^{-i\omega t},\quad a_3 = b_3 e^{i\delta t}$

Plug into in S.E.

$$\boxed{\begin{aligned} \dot b_1 &= -i\frac{\chi_1}{2}\left(1+e^{-i2\omega t}\right) b_2 \\ \dot b_2 &= -i(\omega_0-\omega) b_2 - i\frac{\chi_1}{2}\left(e^{i2\omega t}+1\right) b_1 \\ &\quad - i\frac{\chi_2}{2}\left(e^{i2(\omega+\delta)t}+1\right) b_3 \\ \dot b_3 &= -i\delta b_3 - i\frac{\chi_2}{2}\left(1+e^{-i2(\omega+\delta)t}\right) b_2 \end{aligned}}$$

Drop non-resonant terms, set $\omega_0 - \omega = \Delta$

$$\boxed{\begin{aligned} \dot b_1 &= -i\frac{\chi_1}{2} b_2 \\ \dot b_2 &= -i\Delta b_2 - i\frac{\chi_1}{2} b_1 - i\frac{\chi_2}{2} b_3 \\ \dot b_3 &= -i\delta b_3 - i\frac{\chi_2}{2} b_2 \end{aligned}}$$

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$$\dot b_1 = -i\frac{\chi_1}{2}\left(1+e^{-i2\omega t}\right) b_2$$

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$$\dot b_3 = -i\delta\, b_3 - i\frac{\chi_2}{2}\left(1+e^{-i2(\omega+\delta)t}\right) b_2$$

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This S.E. has no explicit time dependence
Easy to solve numerically…

Now assume that $b_2(t=0) = 0$ ⇨ **the atom is in the electronic ground state at** $t=0$ **when the fields turn on.**

⇨ **we can solve eq. for** $b_2(t)$ **:**

$$\dot b_2(t) = -i\Delta b_2 - i g(t), \quad g(t) = \left(\frac{\chi_1}{2} b_1 + \frac{\chi_2}{2} b_3\right)$$

$$b_2(t) = -e^{-i\Delta t}\int_0^t i e^{i\Delta t'} g(t')\, dt' \quad \leftarrow \quad (A)$$

$$= -e^{-i\Delta t}\left(\left[\frac{1}{\Delta} e^{i\Delta t'} g(t')\right]_0^t - \underbrace{\int_0^t \frac{1}{\Delta} e^{i\Delta t'} \dot g(t')\, dt'}_{(B)}\right)$$

Reminder: Integration by parts

$$\int_a^b f(x) g(x)\, dx = \left[F(x) g(x)\right]_a^b - \int_a^b F(x) g'(x)\, dx$$

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Now consider the relative magnitude of (A) & (B)

(1) Let Rabi freqs be of the same order, $\chi_1 \sim \chi_2 \sim \chi$

(2) b_1, b_3 are at most ~ 1 ⇨ $g(t)$ in (A) is $\sim \chi$

(3) In (B), the part $\frac{1}{\Delta}\dot{g}(t) = \frac{\chi}{\Delta}(\dot{b}_1 + \dot{b}_3)$

Where $\dot{b}_1, \dot{b}_3$ are $\sim \chi b_2$ and $b_2 \sim \frac{\chi}{\Delta}$ ← from Rabi solutions

$$\dot{b}_1 = -i\frac{\chi_1}{2} b_2$$
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$$c_1(t) = \left(\cos\frac{\Omega t}{2} + i\frac{\Delta}{\Omega}\sin\frac{\Omega t}{2}\right) e^{-i\Delta t/2}$$

$$c_2(t) = \left(i\frac{\chi}{\Omega}\sin\frac{\Omega t}{2}\right) e^{-i\Delta t/2}$$

$$\Omega \equiv \sqrt{\chi^2 + \Delta^2}$$

Rabi Solutions

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(1) Let Rabi freqs be of the same order, $\chi_1 \sim \chi_2 \sim \chi$

(2) b_1, b_3 **are at most** ~ 1 ⇨ $g(t)$ **in** (A) **is** $\sim \chi$

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(5) Finally, the last term $\propto \frac{e^{-i\Delta t}}{\Delta}$ can be ignored because it averages to zero on the timescale on which b_1, b_3 evolve.

Note:

The ground state amplitudes evolve slowly Because $\chi_1/\Delta, \chi_2/\Delta \ll 1$, while the excited state amplitude evolves fast and adiabatically follows the instantaneous values of b_1, b_3

Plug the solution for $b_2(t)$ into the eqs. for b_1, b_3

$$\boxed{\begin{aligned}\dot{b}_1(t) &= i\frac{\chi_1^2}{4\Delta}b_1(t) + i\frac{\chi_1\chi_2}{4\Delta}b_3(t)\\ \dot{b}_3(t) &= -i\left(\delta - \frac{\chi_2^2}{4\Delta}\right)b_3(t) + i\frac{\chi_1\chi_2}{4\Delta}b_1(t)\end{aligned}}$$

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$$\dot{b}_1(t) = i\frac{\chi_1^2}{4\Delta} b_1(t) + i\frac{\chi_1\chi_2}{4\Delta} b_3(t)$$

$$\dot{b}_3(t) = -i\left(\delta - \frac{\chi_2^2}{4\Delta}\right) b_3(t) + i\frac{\chi_1\chi_2}{4\Delta} b_1(t)$$

We simplify by making a final change of variables

$$c_1(t) = b_1(t) e^{-i\frac{\chi_1^2}{4\Delta}t}, \quad c_3(t) = b_3(t) e^{-i\frac{\chi_1^2}{4\Delta}t}$$

$$\dot{c}_1(t) = i\frac{\chi_1\chi_2}{4\Delta} c_3(t)$$

$$\dot{c}_3(t) = -i\left(\delta + \frac{\chi_1^2 - \chi_2^2}{4\Delta}\right) c_3(t) + i\frac{\chi_1\chi_2}{4\Delta} c_1(t)$$

These are two-level equations!

Physical Discussion: We have an effective 2-level atom with effective Rabi Frequency and detuning.

$$\chi_{eff} = \frac{\chi_1\chi_2}{2\Delta}, \quad \delta_{eff} = \delta + \frac{\chi_1^2 - \chi_2^2}{4\Delta}$$

Note that $\chi_{eff} \sim \chi^2/\Delta$ while the excited state population $P_2 \sim \chi^2/\Delta^2$. This means that for large χ, Δ we can have large χ_{eff} and no opportunity for spontaneous decay.

Coherent Rabi oscillations and long lived superposition states

Begin 02-13-2025

Raman Coupling in 3-level Atoms

We simplify by making a final change of variables

$$c_1(t) = b_1(t)\, e^{-i\frac{\chi_1^2}{4\Delta}t}, \quad c_3(t) = b_3(t)\, e^{-i\frac{\chi_1^2}{4\Delta}t}$$

$$\boxed{\begin{aligned} \dot{c}_1(t) &= i\frac{\chi_1\chi_2}{4\Delta}c_3(t) \\ \dot{c}_3(t) &= -i\left(\delta + \frac{\chi_1^2-\chi_2^2}{4\Delta}\right)c_3(t) + i\frac{\chi_1\chi_2}{4\Delta}c_1(t) \end{aligned}}$$

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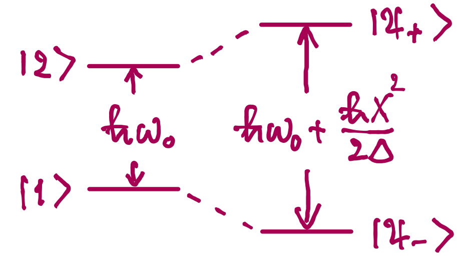
Coherent Rabi oscillations and long lived superposition states

Note also: The effective Raman detuning is shifted.

HW Set 2: Dressed-states of a 2-level atom



$$E_+ = \hbar\omega_0 + \frac{\hbar\chi^2}{4\Delta}$$

$$E_- = -\frac{\hbar\chi^2}{4\Delta}$$

3-level system → ground state shifts $\frac{\chi_1^2}{4\Delta}, \frac{\chi_2^2}{4\Delta}$

→ Differential ground state shift $\frac{\chi_1^2-\chi_2^2}{4\Delta}$

Final note: The atomic dipole $\langle\vec{p}\rangle$ **will have components that match the frequency and polarization of both driving fields, with amplitudes that depend on both fields.**

Non-Linear wave mixing,
Breakdown of superposition principle

Raman Coupling in 3-level Atoms

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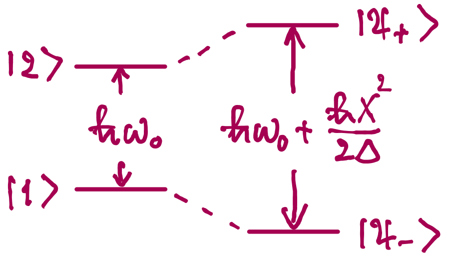

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⬇

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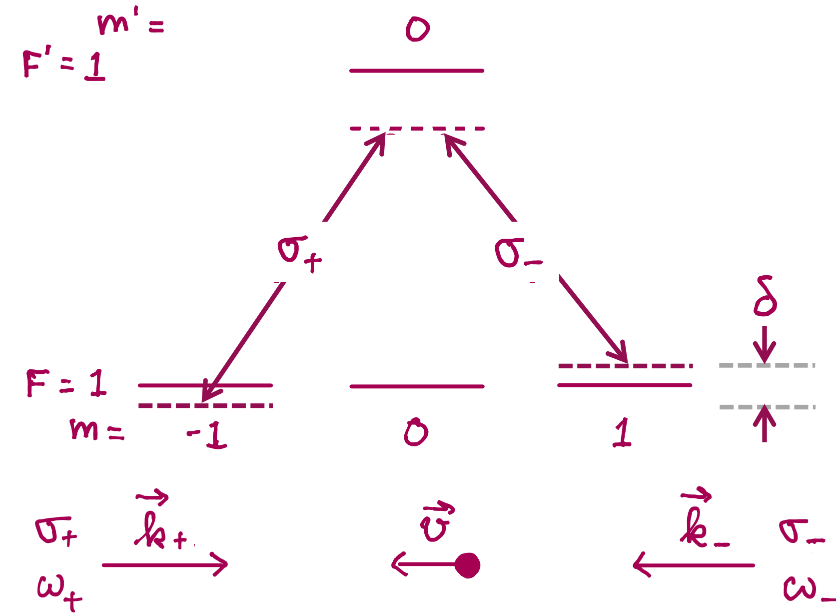
Example: Velocity dependent Raman Coupling



field freqs. in co-moving frame

$$\left.\begin{aligned} \omega_+ &= \omega + kv \\ \omega_- &= \omega - kv \end{aligned}\right\}$$

velocity dependent Raman detuning

➡ $\delta = 2kv$

Applications:

- Doppler velocimetry
- Raman Cooling by velocity selective momentum transfer
- What if we apply a $\pi/2$ Raman pulse?
- Atom Interferometry