OPTI 544: Problem Set 4
Posted February 21, Due February 29

Electronic Submission only, by email to Jon Pajaud (jpajaud@email.arizona.edu)

I

Starting from the equation of motion for the density matrix (including relaxation but setting $\Gamma_1 = \Gamma_2 = 0$ to make things interesting), find the steady state values $\rho_{11}(\infty)$, $\rho_{22}(\infty)$, $\rho_{12}(\infty)$ and $\rho_{21}(\infty)$ as functions of the resonant Rabi frequency and detuning.

II

(a) The ^{87}Rb $5S_{1/2} \rightarrow 5P_{3/2}$ transition has a wavelength of $\lambda = 780.0nm$ and an excited state lifetime of $27.0\,ns$. Assuming there is no collision broadening and that the driving field is polarized, compute the saturation intensity I_{SAT} for this transition. Your final answer must be an accurate numerical value with correct units.

(b) Find an expression relating $|\chi|^2 / A_{21}^2$ (the resonant Rabi frequency in units of the natural linewidth) to the intensity and saturation intensity.

(c) Derive an expression for the steady state population of the excited state $\rho_{22}(\infty)$ as a function of detuning and intensity. Make a plot showing $\rho_{22}(\infty)$ as a function of I in the interval $0 \le I \le 10 I_{SAT}$, for $\Delta = 0$ and for $\Delta = 5A_{21}$.

III

In the following we examine a simple scheme for optical switching, i. e. how to use a strong counter-propagating control laser beam to control the transmission of a weak signal laser beam through a saturable absorber. Both laser beams are resonant with the absorbing transition ($\Delta = 0$), which has a wavelength of $\lambda = 1\mu m$. The absorbing medium is broadened only by the natural lifetime of the absorber excited state ($2\beta = A_{21}$).

(a) Write down an expression for the steady state inversion $\Delta N = N[\rho_{22}(\infty) - \rho_{11}(\infty)]$ that involves only N (the density of absorbers) and $\tilde{I} = I/I_{SAT}$ (the light intensity in units of the saturation intensity). You do not have to derive it from scratch.

(b) For a beam propagating through the medium, the change $d\Phi$ in photon flux over a distance dz is given by the number of stimulated emission events into the beam minus the number of absorption events out of the beam, per unit area and time. Use this to obtain an equation for $d\Phi$ in terms of the scattering cross section and the steady state inversion ΔN.

(c) Use your results from (a) and (b) to obtain a differential equation for $\tilde{I}$ for a beam propagating through the absorbing medium in the positive direction along the z-axis. Solve it for the cases $\tilde{I} << 1$ and $\tilde{I} >> 1$.

(d) Now consider the transmission of the signal beam on its own, assuming $\tilde{I}_S << 1$. Find the value of the product NL (the number density of absorbers times the optical path length through the medium) that yields a transmission $T_{off} = 0.01$.

(e) Next, consider the transmission of the signal beam in the presence of the strong control beam, assuming $\tilde{I}_C >> 1$. You may assume that the inversion depends only on the control beam intensity, and that it remains constant along the optical path. Find the control intensity (in units of I_{SAT}) required to obtain a transmission $T_{on} = 0.99$.

(f) Check that the result in (d) is consistent with the assumptions about the control intensity.