OPTI 544: Problem Set 6
Posted March 29, Due April 5

Electronic Submission Only, by email to Jon Pajaud (jpajaud@email.arizona.edu)

I

The Lagrangian for a chain of masses m separated by distances a and connected by springs with spring constants κ can be expressed in terms of the particle positions and velocities as

$$\mathcal{L} = \frac{1}{2}\sum_i m\dot{x}_i^2 - \kappa\left(x_{i+1} - x_i\right)^2$$

Starting from this Lagrangian derive a wave equation for the displacement field $\eta(x)$ in the continuous limit, $a \to 0$.

II

Show that

(a) $$T = \int dx \frac{1}{2}\mu\left(\frac{\eta(x,t)}{dt}\right)^2 = \sum_k \frac{1}{2}M\dot{q}_k^2$$

(b) $$V = \int dx \frac{1}{2}Y\left(\frac{\eta(x,t)}{dx}\right)^2 = \sum_k \frac{1}{2}M\omega^2 q_k^2$$

(c) Finally, write down the Lagrangian, both in terms of the field, and in terms of the dynamical variables q_k and $\dot{q}_k$. Then show that your Lagrange equation of motion yields the standard 2nd order differential equations typical of a collection of harmonic oscillators.

Note: Problem I and II(b) are a bit fiddly. If you have trouble getting started or find yourself stuck before the end, check out my Solution Set on the course website under the Homework tab.