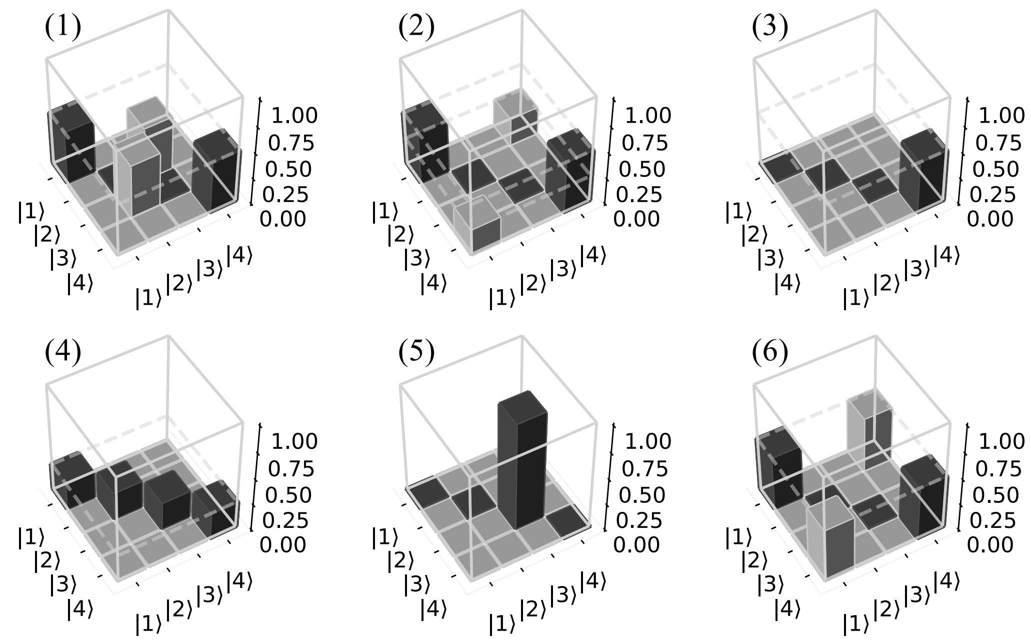
2nd Midterm 2024 - Problem Set

I

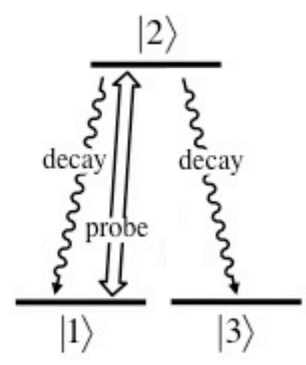


The above figure shows a number of "density matrices" for a 4-level system. The dark grey elements along the diagonal are populations; the light gray off-diagonal elements are coherences. The height of the columns indicate the absolute values of the relevant populations and coherences.

(a) Some of the density matrices correspond to pure states, some to mixed states, and some are unphysical. Indicate which are which. and explain why. Hint: The assignments can be made by visual inspection.

II

Consider a medium of atoms with the level structure shown at right. Spontaneous decay from the excited state into each ground state is equally likely, with decays $|1\rangle \to |2\rangle$ and $|3\rangle \to |2\rangle$ happening at rates A_{21} each. The transitions are at $\lambda = 1.0\,\mu m$ and have associated photon scattering cross sections identical to a similar two-level atom. The medium is a slab of length $l = 2cm$ with atom number density $N = 10^{15}\,m^{-3}$.



We send an optical probe beam through the medium. It is resonant with the $|1\rangle \to |2\rangle$ transition, and the photon flux Φ is well below saturation, $\sigma\Phi << A_{21}$. At $t = 0$ all atoms are in state $|1\rangle$, $\rho_{11}(t=0) = 1$. There is no field present to drive the $|3\rangle \to |2\rangle$ transition.

(a) Calculate the transmission T at $t = 0$. Your final answer must be a number. (8%)

(b) Find the steady state transmission T_{SS}. Explain. (8%)

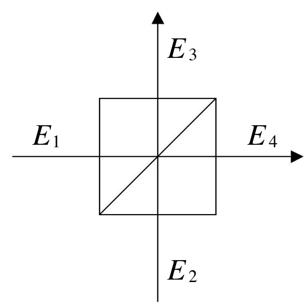
III

(a) Discuss (in your own words) the phenomenon of power broadening: What is its significance in the context of spectroscopy, and how can we manage or avoid it. Explain in your own words.

(b) Explain in your own words why the photon scattering cross section ends up being a simple function of the transition wavelength.

IV

Consider in the following a 50/50 beamsplitter with input ports 1, 2 and output ports 3, 4. Calculate the output states and the probability of coincidence detection (simultaneous detection of one or more photons in each of the ports 3, 4) for the following 2-photon input states.



(a) $|\Psi_{in}\rangle = |1\rangle_1 |1\rangle_2$

(b) $|\Psi_{in}\rangle = |2\rangle_1 |0\rangle_2$

(c) $|\Psi_{in}\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_1 |0\rangle_2 + |0\rangle_1 |2\rangle_2\right)$

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(a) Initially all atoms are in state $|1\rangle$, and the absorption coefficient for the probe is

$$a_0 = N\sigma = N\frac{1}{2}\frac{3\lambda^2}{2\pi} = 10^{15}m^{-3}\frac{3\times\left(10^{-6}m\right)^2}{4\pi} = 238.73m^{-1}$$

The transmission is then $\quad T_0 = \exp(-a_0 l) = \exp\left(-238.73m^{-1}\times 0.02m\right) = 0.00844$

(b) In steady state all atoms are optically pumped into state $|3\rangle$. With no atoms in state $|1\rangle$ the medium becomes transparent to the probe, $T_{SS} = 1$.

Problem III

(a) We have (following the lecture notes)

$$|\Psi_{in}\rangle = |1\rangle_1|1\rangle_2 = \hat{a}_1^+\hat{a}_2^+|0\rangle \Rightarrow$$
$$|\Psi_{out}\rangle = \frac{1}{\sqrt{2}}\left(\hat{a}_3^+ + i\hat{a}_4^+\right)\left(i\hat{a}_3^+ + \hat{a}_4^+\right)|0\rangle = \frac{i}{2}\left(\hat{a}_3^+\hat{a}_3^+ + \hat{a}_4^+\hat{a}_4^+\right)|0\rangle = \frac{i}{\sqrt{2}}\left(|2\rangle_3|0\rangle_4 + |0\rangle_3|2\rangle_4\right)$$

Because $|\Psi_{out}\rangle$ has zero probability amplitude for the $|1\rangle_3|1\rangle_4$ state the probability of coincidence detection is zero.

(b) We have

$$|\Psi_{in}\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_1|0\rangle_2 + |0\rangle_1|2\rangle_2\right) = \frac{1}{2}\left(\hat{a}_1^+\hat{a}_1^+ + \hat{a}_2^+\hat{a}_2^+\right)|0\rangle \Rightarrow$$
$$|\Psi_{out}\rangle = \frac{1}{4}\left[\left(\hat{a}_3^+ + i\hat{a}_4^+\right)\left(\hat{a}_3^+ + i\hat{a}_4^+\right) + \left(i\hat{a}_3^+ + \hat{a}_4^+\right)\left(i\hat{a}_3^+ + \hat{a}_4^+\right)\right]|0\rangle$$
$$= \frac{1}{4}\left(\hat{a}_3^+\hat{a}_3^+ - \hat{a}_4^+\hat{a}_4^+ + i2\hat{a}_3^+\hat{a}_4^+ - \hat{a}_3^+\hat{a}_3^+ + \hat{a}_4^+\hat{a}_4^+ + i2\hat{a}_3^+\hat{a}_4^+\right)|0\rangle$$
$$= i\hat{a}_3^+\hat{a}_4^+|0\rangle = i|1\rangle_3|1\rangle_4$$

Because $|\Psi_{out}\rangle$ has a probability amplitude with unit norm for the $|1\rangle_3|1\rangle_4$ state the probability of coincidence is one.