Solution Set, OPTI 544 1st Midterm Exam, March 15, 2023

Problem I

This is radiation by the dipole moment of a two-level atom, with amplitude modulation as the atom Rabi oscillates between energy eigenstates states and coherent superposition states that have zero and maximum dipole amplitude, respectively. The large central peak is at the frequency ω_0 of the optical driving field, and the sidebands are centered at frequencies $\omega = \omega_0 \pm \Omega$, where $\Omega = \sqrt{\chi^2+\Delta^2}$ is the generalized Rabi frequency. The width of the peaks is the natural linewidth β associated with the finite phase-memory time determined by elastic collisions and spontaneous decay. Going beyond what is expected here, we might look at Homework Set 2, Problem II-b, where we note that sidebands of roughly equal amplitude occur when $\chi >> \Delta$, i. e., when the resonant Rabi frequency is much larger than the detuning and thus $\Omega - \Delta \approx \Omega + \Delta$.

Problem II

(a) The Electric Dipole selection rules are as follows: $\Delta L = \pm 1$, $\Delta F = 0, \pm 1$, $\Delta m_F = q = 0, \pm 1$

Here $q = 0$ is light that is linearly polarized along the quantization (z) axis, and $q = \pm 1$ is circularly polarized with respect to the z-axis. (Level diagram to be added)
If we drive this system with σ_+ polarized light, the cycles of absorption and spontaneous emission will add spin-angular momentum until the atom falls into the $F = 2, m_F = 2$ ground state. At that point no further excitation is possible.

(b) (Level diagram to be added) In this case the optical pumping cycles continue until the atom reaches either $F = 2, m_F = 1$ or $F = 2, m_F = 2$ and further excitation is not possible. To avoid atoms getting permanently stuck in $F = 2, m_F = 1$ state we can add a small amount of $q = 0$ polarized light. This drives the transition to $F' = 2, m_F = 1$, from which the atom can decay into $F = 2, m_F = 2$ where it is stuck.

Problem III

To prepare this particular mixed state, Alice can time a Rabi oscillation such that the driving field turns off when the atom is in the state $|\Psi\rangle = \sqrt{1/4}\,|1\rangle + \sqrt{3/4}\,|2\rangle$. This is her basic starting point, from which she can make the desired mixed state in several ways:

(1) Alice can prepare two sub-ensembles, one in the state $|1\rangle$ and one in state $|2\rangle$. She then mixes the atoms in a ratio of one part in $|1\rangle$ to three parts $|2\rangle$. An atom randomly picked from the mixed ensemble will then be in the desired state ρ.

(2) Alice can prepare the state $|\Psi\rangle$, then gives it to Bob who performs the measurement that collapses the atom into state $|1\rangle$ or $|2\rangle$ with the appropriate probabilities, and hands it back to Alice without revealing the measurement outcome. Variant: If the computer is running the apparatus, Alice can grab the atom post-measurement and then permanently erase the measurement outcome from the computer's memory without looking at it.

(3) Alice can apply a randomly fluctuating magnetic field. In her system this mimics the effect of inelastic collisions, which will eventually randomize the complex phase in the superposition $|\Psi\rangle$, reduce the coherence to zero, and leave her with the desired state ρ. Note that if Alice kept a record of the modulation history she could in principle predict which coherent superposition she has at any given moment. Thus, like in (2), the process depends on Alice deliberately throwing away information that is in principle available to her.

Problem IV

(a) The rate equations for this system have the form Check: adding the 3 equations together gives zero, indicating that the total probability is conserved.

$$\dot{\rho}_{11} = A\rho_{22} + \sigma\phi(\rho_{22} - \rho_{11})$$
$$\dot{\rho}_{22} = -2A\rho_{22} - \sigma\phi(\rho_{22} - \rho_{11})$$
$$\dot{\rho}_{33} = A\rho_{22}$$

(b) As time goes on, atoms excited to state $|2\rangle$ gradually decay into state $|3\rangle$ from which they cannot escape. As a result, the steady state will have populations $\rho_{11} = \rho_{22} = 0$, $\rho_{33} = 1$

(c) Sketches showing the time dependence of the three populations. These are only a rough guess, but it is in principle straightforward to numerically integrate the three coupled first-order differential equations to make sketches that are quantitatively correct.

Note: Along any vertical line corresponding to a given time, the three populations must add up to one. This is clearly not the case for my sketches. If I have time I will make up better ones.