Raman Coupling in 3-level Atoms

(5) Finally, the last term $\propto \frac{e^{-i\Delta t}}{\Delta}$ can be ignored because it averages to zero on the timescale on which b_1, b_3 evolve.

Note:

The ground state amplitudes evolve slowly Because $\chi_1/\Delta, \chi_2/\Delta \ll 1$, while the excited state amplitude evolves fast and adiabatically follows the instantaneous values of b_1, b_3

Plug the solution for $b_2(t)$ into the eqs. for b_1, b_3

$$\boxed{\begin{aligned} \dot{b}_1(t) &= i\frac{\chi_1^2}{4\Delta} b_1(t) + i\frac{\chi_1\chi_2}{4\Delta} b_3(t) \\ \dot{b}_3(t) &= -i\left(\delta - \frac{\chi_2^2}{4\Delta}\right) b_3(t) + i\frac{\chi_1\chi_2}{4\Delta} b_1(t) \end{aligned}}$$

We simplify by making a final change of variables

$$c_1(t) = b_1(t)\, e^{-i\frac{\chi_1^2}{4\Delta}t}, \quad c_3(t) = b_3(t)\, e^{-i\frac{\chi_1^2}{4\Delta}t}$$

$$\boxed{\begin{aligned} \dot{c}_1(t) &= i\frac{\chi_1\chi_2}{4\Delta} c_3(t) \\ \dot{c}_3(t) &= -i\left(\delta + \frac{\chi_1^2 - \chi_2^2}{4\Delta}\right) c_3(t) + i\frac{\chi_1\chi_2}{4\Delta} c_1(t) \end{aligned}}$$

These are two-level equations!

Physical Discussion: We have an effective 2-level atom with effective Rabi Frequency and detuning.

$$\boxed{\chi_{eff} = \frac{\chi_1\chi_2}{2\Delta}, \quad \delta_{eff} = \delta + \frac{\chi_1^2 - \chi_2^2}{4\Delta}}$$

Note that $\chi_{eff} \sim \chi^2/\Delta$ while the excited state population $P_2 \sim \chi^2/\Delta^2$. This means that for large χ, Δ we can have large χ_{eff} and no opportunity for spontaneous decay.

Coherent Rabi oscillations and long lived superposition states

Raman Coupling in 3-level Atoms

We simplify by making a final change of variables

$$C_1(t) = b_1(t)\, e^{-i\frac{\chi_1^2}{4\Delta}t}, \quad C_3(t) = b_3(t)\, e^{-i\frac{\chi_1^2}{4\Delta}t}$$

$$\dot{c}_1(t) = i\,\frac{\chi_1\chi_2}{4\Delta}\, c_3(t)$$

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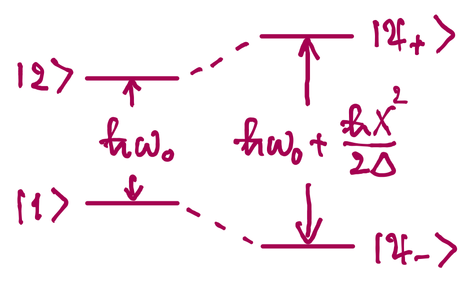
Coherent Rabi oscillations and long lived superposition states

Note also: The effective Raman detuning is shifted.

HW Set 2: Dressed-states of a 2-level atom



$$E_+ = \hbar\omega_0 + \frac{\hbar\chi^2}{4\Delta}$$

$$E_- = -\frac{\hbar\chi^2}{4\Delta}$$

3-level system ➡ ground state shifts $\frac{\chi_1^2}{4\Delta}$, $\frac{\chi_2^2}{4\Delta}$

➡ Differential ground state shift $\frac{\chi_1^2-\chi_2^2}{4\Delta}$

Final note: The atomic dipole $\langle \vec{p} \rangle$ will have components that match the frequency and polarization of both driving fields, with amplitudes that depend on both fields.

Non-Linear wave mixing, Breakdown of superposition principle

Begin 02-08-2024

Raman Coupling in 3-level Atoms

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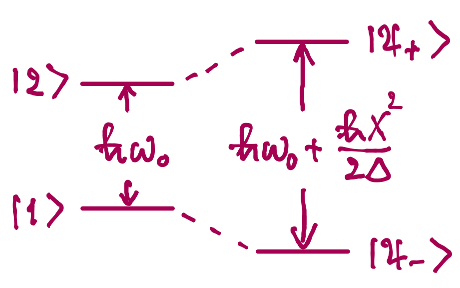

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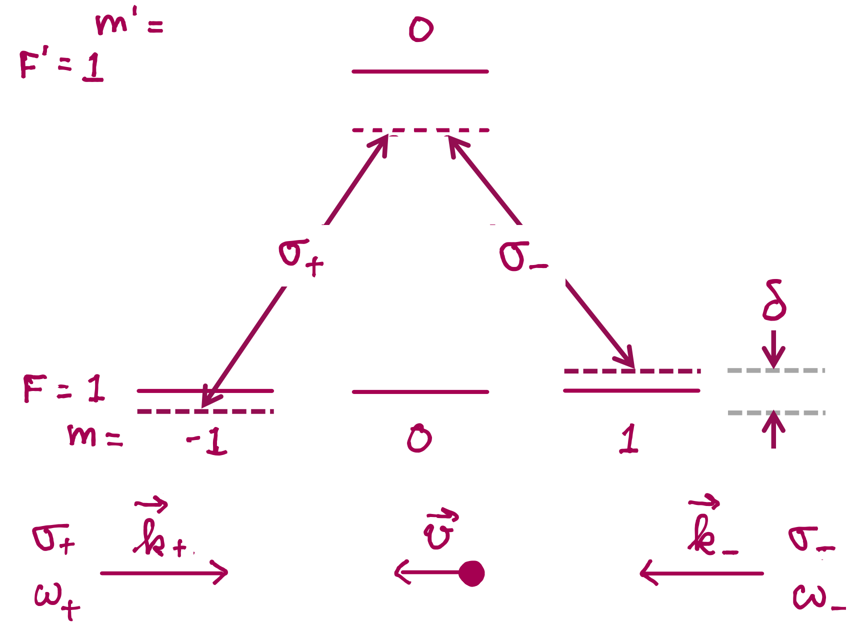
⇩

Non-Linear wave mixing,
Breakdown of superposition principle

End 02-06-2024

Example: Velocity dependent Raman Coupling



field freqs. in co-moving frame

velocity dependent Raman detuning

$$\left.\begin{aligned} \omega_+ &= \omega + kv \\ \omega_- &= \omega - kv \end{aligned}\right\} \Rightarrow \delta = 2kv$$

Applications:

- Doppler velocimetry
- Raman Cooling by velocity selective momentum transfer
- What if we apply a $\pi/2$ Raman pulse?
- Atom Interferometry

Density Matrix Description of 2-Level Atoms

Mental Warmup: What is a probability?

(1) Example: Coin toss

- We can describe physical states by probability distributions
- Probabilities are assigned based on prior knowledge, updated when additional info becomes available
- As such, probability distributions are subjective (states of knowledge)

(2) Example: Quincunx

https://www.mathsisfun.com/data/quincunx.html

- We can describe physical states by probability distributions
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This is the Bayesian Interpretation of Probability

(3) Example: Quantum Quincunx

- We can describe physical states by quantum wavefunctions (state vectors)
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(4) Mixed Quantum & Classical Case

- We can easily envision a hybrid Quincunx that is part classical, part quantum.
- Physics needs an efficient description these kinds of intermediate situations

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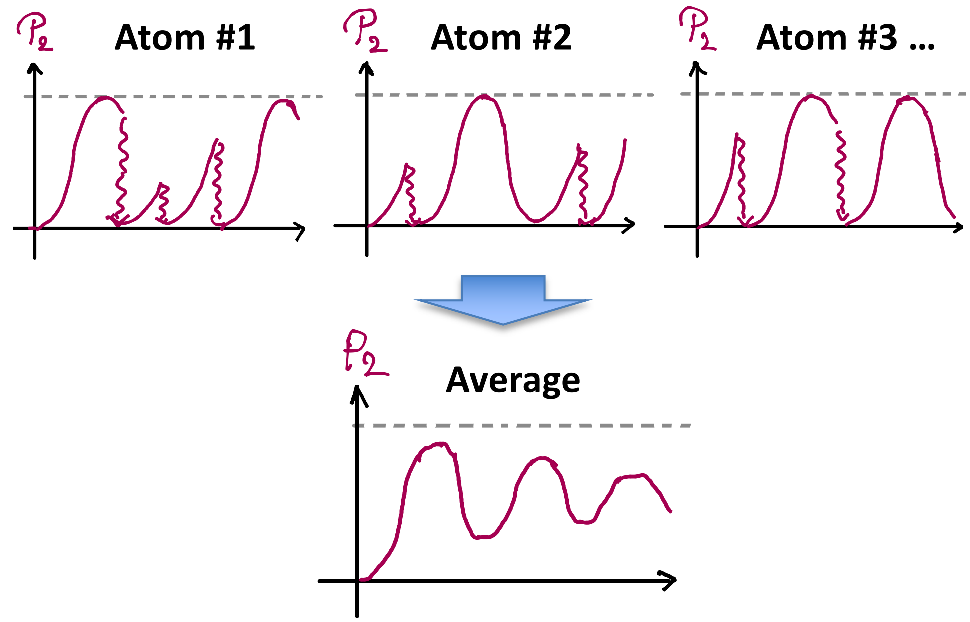
(5) Example: Quantum Trajectories

- Ensemble of 2-level atoms undergoing Rabi oscillation with random decays



<u>Definition</u>: A system for which we know only the probabilities p_k of finding the system in state $|\psi_k\rangle$ is said to be in a statistical mixture of states. Shorthand: <u>mixed state</u>.

Shorthand for non-mixed state: <u>pure state</u>

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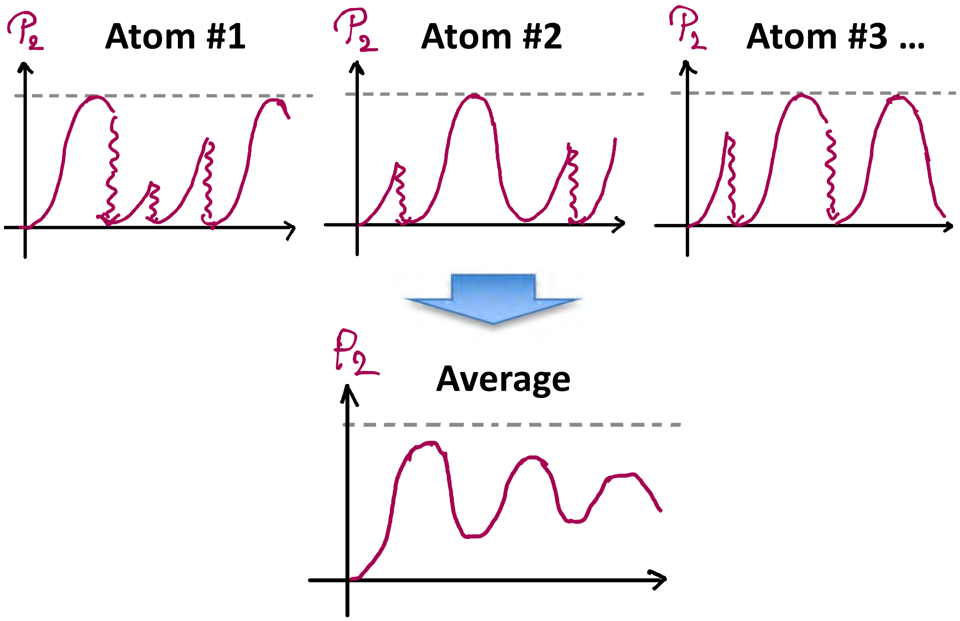

Definition: A system for which we know only the probabilities η_k of finding the system in state $|\psi_k\rangle$ is said to be in a statistical mixture of states. Shorthand: mixed state.

Shorthand for non-mixed state: pure state

Definition: Density Operator for pure states

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

Definition: Density Matrix

$$|\psi(t)\rangle = \sum_n C_n(t)|u_n\rangle \Rightarrow$$

$$\rho_{pn}(t) = \langle u_p|\rho(t)|u_n\rangle = C_p(t)\,C_n^*(t)$$

Definition: Density Operator for mixed states

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Note: A pure state is just a mixed state for which one $\eta_k = 1$ and the rest are zero.

The terms Density Operator and Density Matrix are used interchangeably

Density Matrix Description of 2-Level Atoms

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Let A be an observable w/eigenvalues a_n

Let P_n be the projector on the eigen-subspace of a_n

For a pure state, $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$, we have

(∗) $\mathrm{Tr}\,\rho(t) = \sum_n \rho_{nn}(t) = \sum_n |C_n|^2 = 1$

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