Light-Matter Interaction

Hierarchy of Sophistication:

- Classical — Classical light, classical matter
- Semiclassical — Classical light, quantum matter
- Quantum — Quantum light, quantum matter

Possible attitudes:

- Purist — Most complete description possible
- Minimalist — Quantum only when necessary
- Pragmatic — Quantum or classical, based on what is simplest and still works

OPTI 544: All of the above in turn

Classical Theory of Light-Matter Interaction

Self-consistent, fully classical description

Electromagnetic Field $\rightarrow$ Atom/Molecule/Solid

Motivation: We will

- Develop Concepts $\alpha(\omega), n, \chi$
- Develop Intuition
- Classical is often adequate, sometimes accurate
- A Quantum Theory has classical limits $\Rightarrow$ Identify/understand regime of validity
- The Classical description is a useful starting point for Nonlinear and Quantum Optics

Light-Matter Interaction

The Electromagnetic Field: Basic Eqs. in SI Units

Maxwell's eqs.

(no free charges, currents ⇨ dielectrics)

(i) $\nabla \cdot \vec{D} = \varrho = 0$ $\vec{D}$: Dielectric displacement

(ii) $\nabla \cdot \vec{B} = 0$ $\vec{B}$: Magnetic induction

(iii) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ $\vec{E}$: Electric field

(iv) $\nabla \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t} + \vec{J}$ $\vec{H}$: Magnetic field

Material Response:

(v) $\vec{B} = \mu_0 \vec{H} + \vec{M}$ ⇦ Non-magnetic ⇨ $\vec{M} = 0$

(vi) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ ⇦ Info about response in dipole moment density (polarization density)

We need equations that describe:

- the behavior of $\vec{E}$ for given $\vec{P}$
- the medium response $\vec{P}$ for given $\vec{E}$

Wave Equation:

Take curl of (iii), then use (iv)

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \frac{\partial^2 \vec{D}}{\partial t^2}$$

Next, use the identity

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

to obtain $\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \dfrac{\partial^2 \vec{D}}{\partial t^2}$

Finally, let $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$, use $\varepsilon_0 \mu_0 = \dfrac{1}{c^2}$,

and rearrange to obtain

$$\boxed{\nabla^2 \vec{E} - \nabla(\nabla \cdot \vec{E}) - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \vec{P}}{\partial t^2}}$$

This is the Wave Equation, still exact in this form

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Transverse Fields

Definition: a field for which $\nabla\cdot\vec{E} = 0$ is <u>Transverse</u>

Example: a plane wave, $\vec{E}(\vec{r},t) = \vec{E}(t)\, e^{i\vec{k}\cdot\vec{r}}$,

where $\vec{E}(t) \perp \vec{k}$, is transverse.

The physical field is $Re[\vec{E}(\vec{r},t)]$

For transverse fields the wave equation simplifies to

$$\boxed{\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\vec{P}}$$

This version of the wave equation can be a poor approximation in non-isotropic media!

Isotropic Media

Absent a preferred direction, the induced $\vec{P}$ must be parallel to the driving field $\vec{E}$

Regime of <u>Linear response</u>, steady state:

$\vec{D}(t) = \varepsilon_0\vec{E}(t) + \vec{P}(t)$ **where** $\vec{P}(t) = R\cdot\vec{E}(t)$

Constant, same units as ε_0, but $R \gg \varepsilon_0$

Regime of <u>Linear response</u>, transient case:

$$\vec{D}(t) = \varepsilon_0\vec{E}(t) + \int_{-\infty}^{t} dt' R(t-t')\vec{E}(t')$$

where the *response function* $R(t-t')$ is a scalar and we have $R(\tau) = 0$ for $\tau < 0$

Take divergence on both sides and use M.E. (1)

$$\nabla\cdot\vec{D}(t) = \varepsilon_0\nabla\cdot\vec{E}(t) + \int_{-\infty}^{t} dt' R(t-t')\nabla\cdot\vec{E}(t') = 0$$

and $\nabla\cdot\vec{E}(t) = -\int_{-\infty}^{t} dt' R(t-t')\nabla\cdot\vec{E}(t')$ **for all t**

It follows that $\nabla\cdot\vec{E}(t) = 0$ **for all t,**

<u>OR</u> $R(\tau) = -2\delta(\tau)$

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