OPTI 544 Final Exam, May 11, 2023 Jessen

Problem I

(a) Explain briefly and in your own words what the Ramsey Method of Separated Oscillatory Fields is about and why it is widely used in atomic clocks. If helpful, you may include a sketch and refer to it. Be as concise as possible - one paragraph of ≤ 200 words should suffice. (10%)

(b) Explain briefly and in your own words what the Hong-Ou-Mandel experiment is about and what it tells us about Photons and Quantum Beamsplitters. If helpful, you may include a sketch and refer to it. Be as concise as possible - one paragraph of ≤ 200 words should suffice. (10%)

(c) Explain briefly and in your own words what Wheeler's Delayed Choice experiment is about and what it tells us about the nature of photons. If helpful, you may include a sketch and refer to it. Be as concise as possible - one paragraph of ≤ 200 words should suffice. (10%)

Problem II

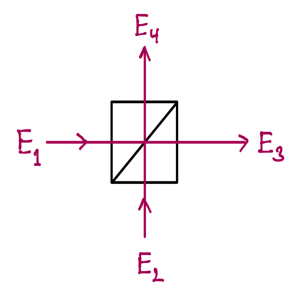
Consider a wavepacket of light with wavelength $\lambda = 1\mu m$, prepared in a coherent state $|\alpha\rangle$ and containing an average energy of 1×10^{-12} J.

(a) Based on the above, calculate the mean photon number in the wavepacket. (10%)

(b) Explain in your own words what it means when we assign a mean energy and photon number to a coherent state of light. Try arguing from both frequentist and bayesianist perspectives. (15%)

Problem III

In this problem we consider the input-output map for a 50-50 beamsplitter. To use your time efficiently, make as much use as you can of results derived in class or during homework.



(a) Given a coherent state input $|\Psi_{\text{in}}\rangle = |\alpha\rangle_1 |0\rangle_2 = e^{\alpha \hat{a}_1^\dagger - \alpha^* \hat{a}_1}|0\rangle$, with $|0\rangle$ being the two-mode vacuum, write down the output in ports 3 and 4. Is this a state in which the photon number and mode degrees of freedom are entangled? Explain your reasoning. (10%)

(b) Next, consider the input-output map for an arbitrary number of photons n in port 1 and zero photons in port 2. First, write down the familiar generic expression for the input in terms of $\hat{a}_1^\dagger, \hat{a}_2^\dagger$. Then expand to find a generic expression for the output in port 3 and port 4. You may leave the result on operator form; there is no need to explicitly calculate the output state $|\Psi_{\text{out}}\rangle$. (20%)

Hint: $$(x+\mathrm{i}y)^n = \binom{n}{0} x^n + \binom{n}{1} x^{n-1}(\mathrm{i}y) + \ldots + \binom{n}{k} (\mathrm{i}y)^{n-k} = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} (\mathrm{i}y)^{\mathrm{k}}$$

where $$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$ are the binomial coefficients

(c) Discuss the difference between the output states in III(a) and III(b). Do you expect $|\Psi_{\text{out}}\rangle$ to be photon number - mode entangled? Explain your reasoning. (10%)

(d) What do the results from (a) and (b) tell us about how the qualitative nature of coherent states and of number states change when propagating through a lossy medium? (5%)

<u>Hint</u>: Consider whether the output can be expressed as a product of a state vector for port 3 and a state vector for port 4, similar to the output in III (a) above.