Vector Model of the 2-Level Atom

We return to the Density Matrix Equations of Motion for a 2-Level atom

$$\dot{\rho}_{11} = -\Gamma_1 \rho_{11} + A_{21}\rho_{22} - \frac{i}{2}(\chi\rho_{12} - \chi^*\rho_{21})$$

$$\dot{\rho}_{22} = -\Gamma_2 \rho_{22} - A_{21}\rho_{22} + \frac{i}{2}(\chi\rho_{12} - \chi^*\rho_{21})$$

$$\dot{\rho}_{12} = (i\Delta - \beta)\rho_{12} + \frac{i\chi^*}{2}(\rho_{22} - \rho_{11}) = \dot{\rho}_{21}^*$$

$$\beta = \frac{1}{T} + \frac{1}{2}(\Gamma_1 + \Gamma_2 + A_{21}), \quad \chi = \vec{\mu}_{21}\cdot\hat{\varepsilon}E_0/\hbar$$

Note: In our previous iteration we studied the Rate Equation approximation, which is useful when we are looking for *steady state solutions*

Here our goal is different – we seek to recast the Density Matrix formalism in a way that is better suited to understanding and modeling *coherent evolution* and *transient phenomena*. This will also be useful when we study wave and light pulse propagation.

Optical Bloch Equations (OBE's)

Let $\Gamma_1 = \Gamma_2 = 0$ ⇨ $\rho_{11} + \rho_{22} = 1, \ \rho_{12} = \rho_{21}^*$

⇨ 3 independent, real-valued variables

Define Bloch Variables	$u = \rho_{21} + \rho_{12}$
	$v = i(\rho_{21} - \rho_{12})$
	$w = \rho_{22} - \rho_{11}$

Let $\chi = |\chi|e^{-i\varphi}$, substitute in equations for ρ, leaving out relaxation terms $A_{21}, \Gamma_1, \Gamma_2, \beta$

Optical Bloch Equations

$$\dot{u} = -\Delta v - |\chi|\sin\varphi\, w$$

$$\dot{v} = \Delta u + |\chi|\cos\varphi\, w$$

$$\dot{w} = -|\chi|(\cos\varphi\, v - \sin\varphi\, u)$$

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$$\boxed{\begin{aligned} \dot{u} &= -\Delta v - |\chi| \sin\varphi\, w \\ \dot{v} &= \Delta u + |\chi| \cos\varphi\, w \\ \dot{w} &= -|\chi| (\cos\varphi\, v - \sin\varphi\, u) \end{aligned}}$$

Define: $$\boxed{\begin{aligned} \vec{S} &= u\vec{i} + v\vec{j} + w\vec{k} \\ \vec{Q} &= -|\chi|\cos\varphi\,\vec{i} - |\chi|\sin\varphi\,\vec{j} + \Delta\vec{k} \end{aligned}}$$

$$\boxed{\dot{\vec{S}} = \vec{Q} \times \vec{S}}$$

Torque Bloch Vector

Equivalent to OBE's !

length conserved

Note: $\frac{d}{dt}(S^2) = 2\vec{S}\cdot\dot{\vec{S}} = 2\vec{S}\cdot(\vec{Q}\times\vec{S}) = 0$

From the definition of the Bloch Variables we get

$$\rho = \frac{1}{2}\begin{pmatrix} 1-w & u+iv \\ u-iv & 1+w \end{pmatrix}$$

and

$$\mathrm{Tr}\,\rho^2 = \frac{1}{2}\left[1 + u^2 + v^2 + w^2\right] = \frac{1}{2}\left[1 + |S|^2\right] \leq 1$$

$$\Rightarrow \boxed{|S|^2 \leq 1}$$

Vector Model of the 2-Level Atom

Define:

$$\boxed{\begin{aligned}\vec{S} &= u\vec{i} + v\vec{j} + w\vec{k} \\ \vec{Q} &= -|X|\cos\varphi\,\vec{i} - |X|\sin\varphi\,\vec{j} + \Delta\vec{k}\end{aligned}}$$

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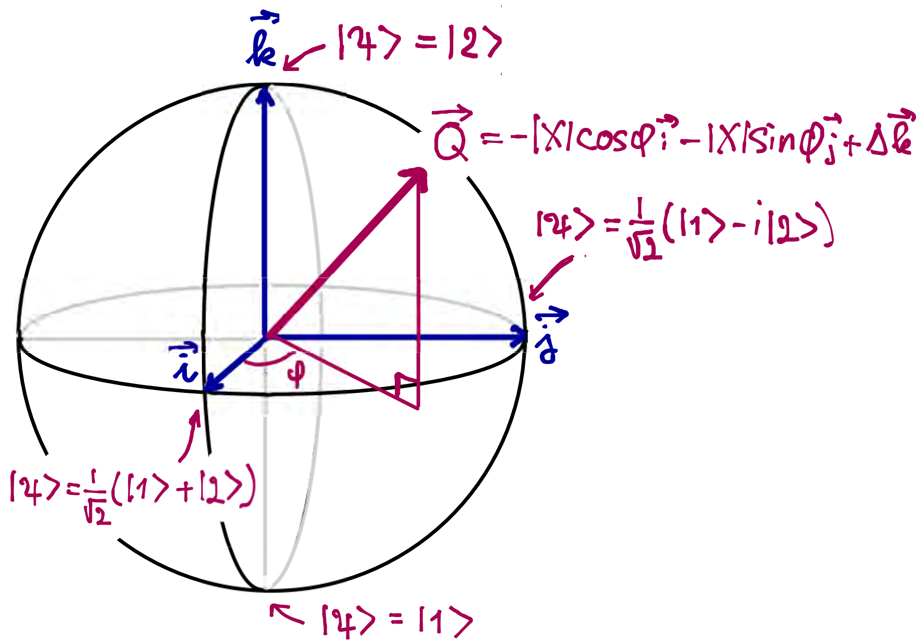
Clearly, $|S^2| = 1 \Rightarrow \mathrm{Tr}\,\rho^2 = 1 \Rightarrow$ pure state

States w/ $|\vec{S}| < 1 \Rightarrow \mathrm{Tr}\,\rho^2 < 1 \Rightarrow$ mixed state

$$|\vec{S}| = 0 \Rightarrow \mathrm{Tr}\,\rho^2 = 1/2 \Rightarrow \rho = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

maximally mixed

Note: The above suggests a physical state can be represented by a vector $\vec{S}$, whose tip lies on the surface of (pure) or inside (mixed) a sphere of unit radius, and whose length is conserved under Schrödinger evolution. This is the Bloch Sphere.



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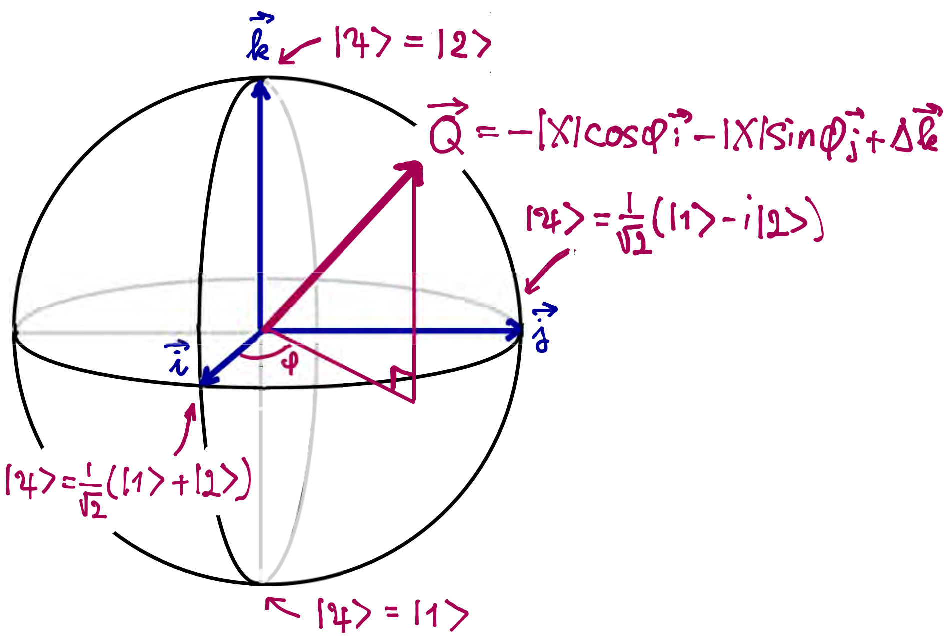

(∗) Do not confuse $\vec{S}$ with the state vector $|\psi\rangle$. $|\psi\rangle$ lives in a complex vector space. Also, do not confuse $\vec{S}$ with a vector in real, physical space. $\vec{S}$ lives in an abstract, real-valued vector space.

(∗) <u>Only</u> if the 2-level system is a physical spin-1/2 particle does $\vec{S}$ correspond to an angular momentum vector that lives in physical space. In general, $\vec{S}$ is what we call a *pseudo-spin*, not an actual physical spin.

Physical Interpretation of the Bloch Variables

We have $\langle\vec{\mu}\rangle = \mathrm{Tr}(\rho\vec{\mu}) = \rho_{12}\vec{\mu}_{21} + \rho_{21}\vec{\mu}_{12}$

where

$$\left.\begin{aligned} \rho_{12} &= \tfrac{1}{2}(u+iv)e^{i\omega t} \\ \rho_{21} &= \tfrac{1}{2}(u-iv)e^{-i\omega t} \end{aligned}\right\} \text{fast variables}$$

$$\vec{E} = \mathrm{Re}[\vec{\varepsilon}E_0 e^{-i\omega t}] \quad \text{driving field}$$

It follows that

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Thus

u is the component of $\langle\hat{\vec{\mu}}\rangle$ in-phase w/ $\vec{E}$

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Lastly, $w = \rho_{22} - \rho_{11}$ is the population inversion.

Solution of the OBE's

Let $\Delta = 0$ and χ real and positive ⇨ $\begin{cases}\vec{Q} = -|\chi|\vec{i}\\ \varphi = 0\end{cases}$

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simplified equations :

$$\boxed{\begin{aligned}\dot{u} &= 0\\ \dot{v} &= \chi w\\ \dot{w} &= -\chi v\end{aligned}}$$

choose global phase so u(0) = 0

Solution

Rabi Oscillations

$$\boxed{\begin{aligned}u &= 0\\ v &= -\sin\theta\,, \quad \theta = \chi t\\ w &= -\cos\theta\end{aligned}}$$

Vector Model of the 2-Level Atom

Thus

u is the component of $\langle \hat{\vec{\wp}} \rangle$ in-phase w/ $\vec{E}$

v is the component of $\langle \hat{\vec{\wp}} \rangle$ in-quadrature w/ $\vec{E}$

Lastly, $w = \rho_{22} - \rho_{11}$ is the population inversion.

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Let $\Delta = 0$ and χ real and positive ⇨ $\begin{cases} \vec{Q} = -|\chi|\hat{i} \\ \varphi = 0 \end{cases}$

⇩

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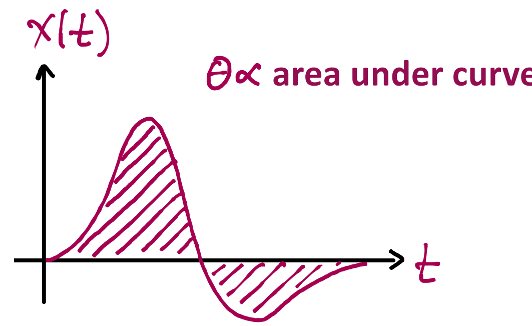
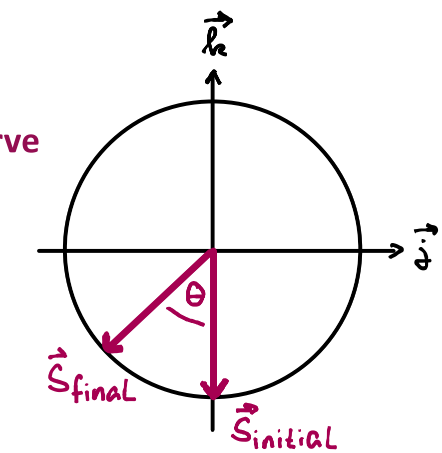
Pulse Area Theorem!

This is a very important result ⇨ We can deal with pulses !





Note: $\chi(t) = -\overbrace{\vec{\wp}_{21}\cdot\hat{\varepsilon}}^{\mu}\, E_0/\hbar \equiv \mu\,\mathcal{E}(t)/\hbar$

complex amplitude

Cannot remain real if the complex phase of $\mathcal{E}(t)$ is changing with time

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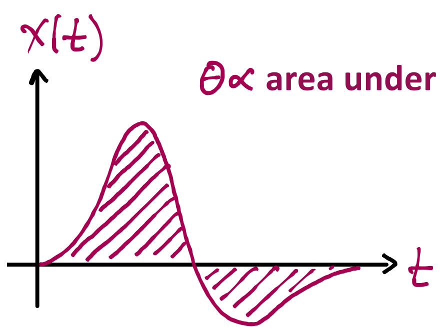

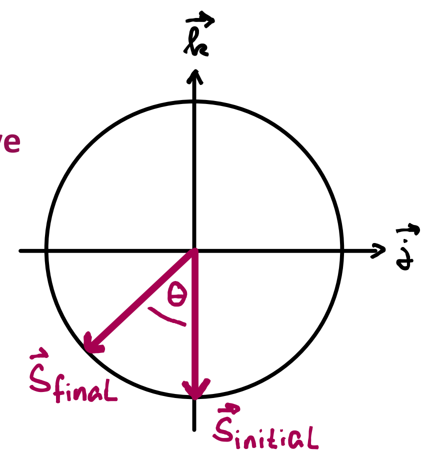

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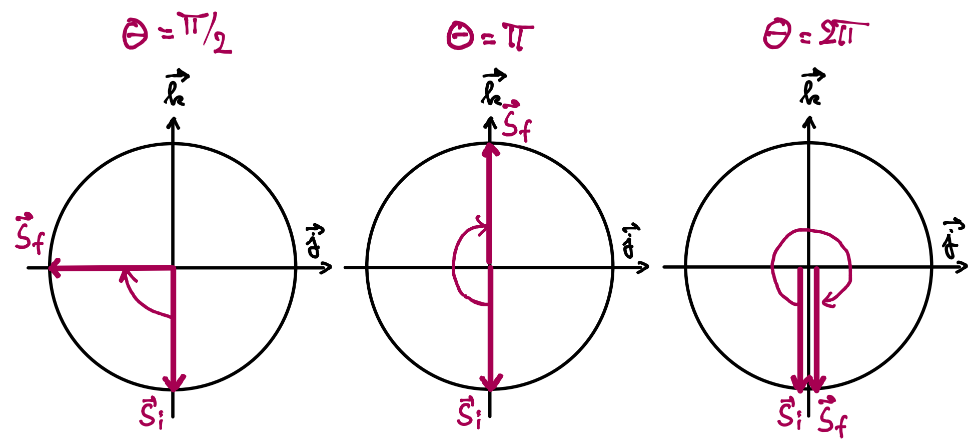
Different phase ⇨ different axis of rotation

The Pulse Area Theorem is only valid if the <u>direction</u> of the torque vector $\vec{Q}$ is constant.

<u>Note</u>: The RWA is valid only in the Slowly Varying Envelope Approx. $\Big\}$ $\frac{d\mathcal{E}}{dt} \ll \omega$

This may not hold for modern Ultrafast Lasers !

Some examples of θ-pulses: (quantum gates)



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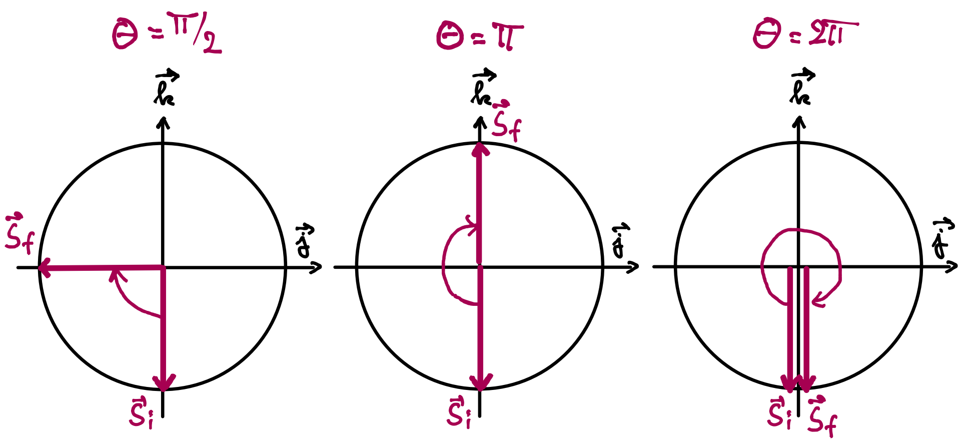
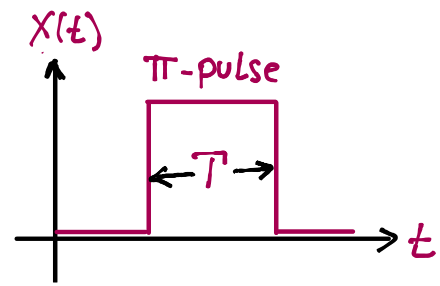
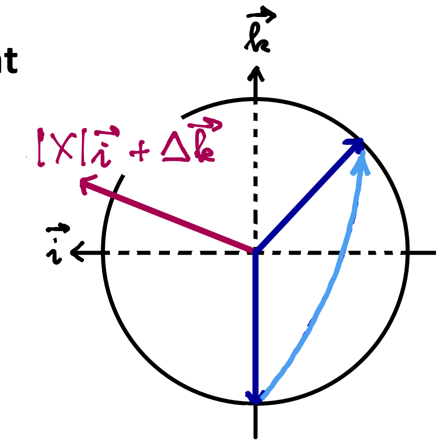


Ramsey Method of Separated Oscillatory Fields

(The Ramsey "trick", 1989 Nobel in Physics)

Single pulse measurement of ω_{21} in Atomic Clock:





Idea is to measure population of $|2\rangle$ as function of $\Delta = \omega_{21} - \omega$ which is maximized for $\omega = \omega_{21}$.

The frequency resolution is $\delta\omega \propto 1/T$, so very long pulses are required. The atom is perturbed by interaction with the light during the entire interrogation and problems can occur due to phase or amplitude noise on the light field. This is not good since the clock is supposed to link to the transition frequency of an unperturbed atom.

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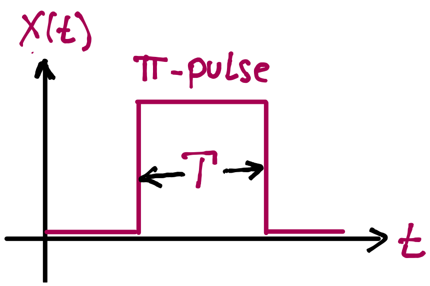

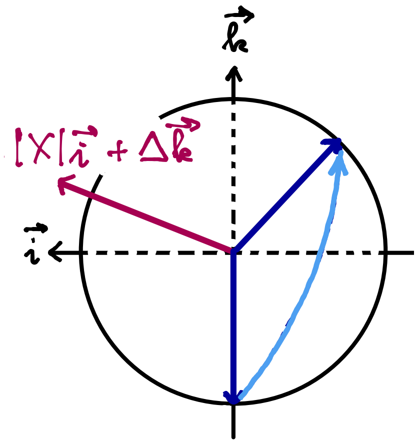

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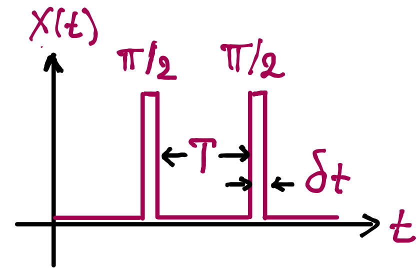
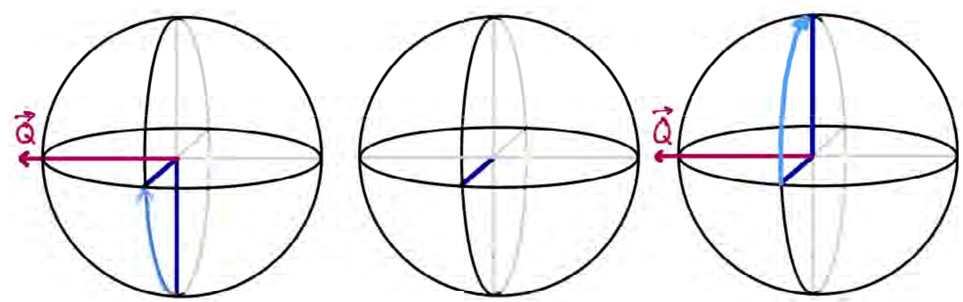
Ramsey's two-pulse strategy:



Sequence of 2 short, intense $\pi/2$ pulses, separated by a long free evolution period, So that $T \gg \delta t$

During pulses $|X| \gg \Delta$ ➡ $\vec{Q} = |X|\vec{i} + \Delta\vec{k} \sim |X|\vec{i}$

Case $\Delta = 0$, path on the Bloch sphere



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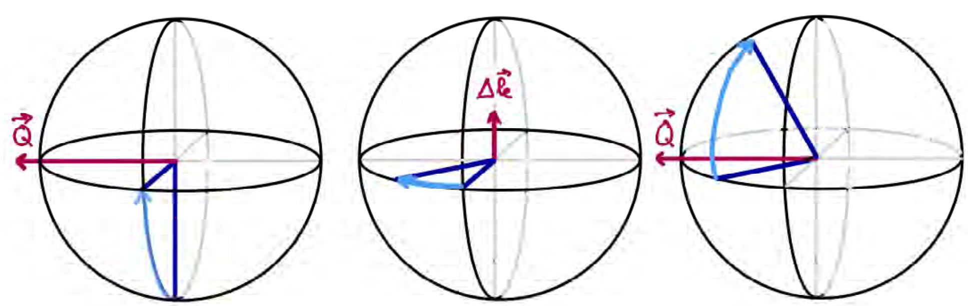

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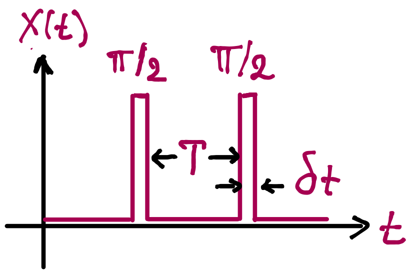

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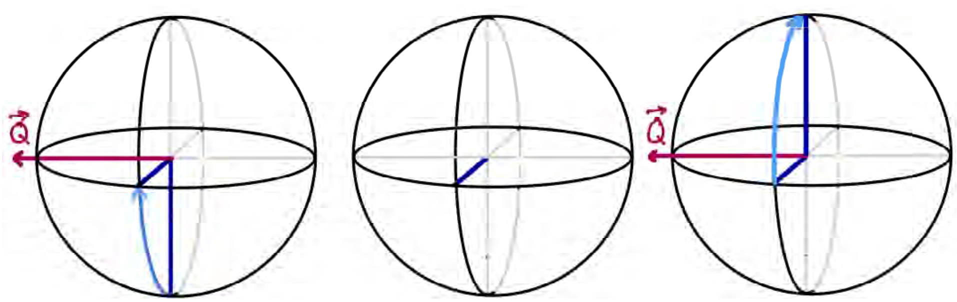

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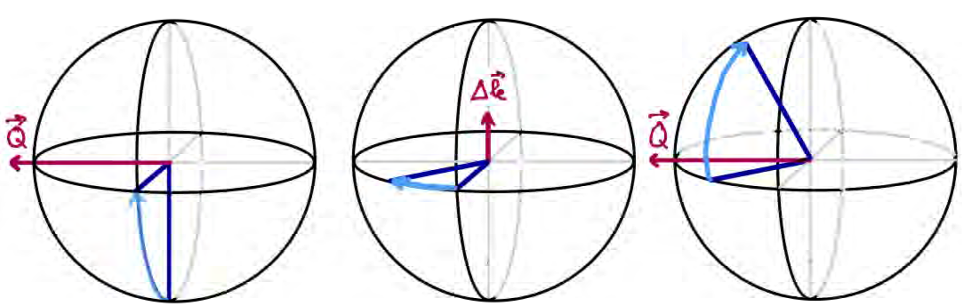

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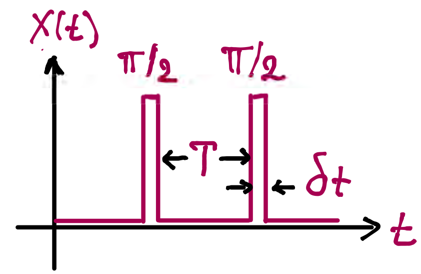

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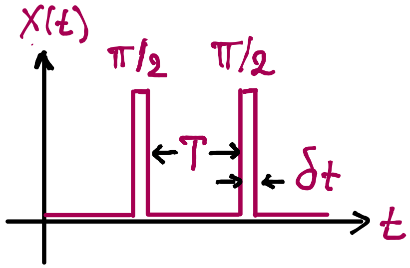

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https://www.nobelprize.org/prizes/physics/1989/summary/

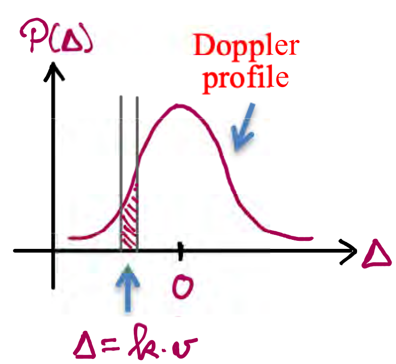
https://www.nobelprize.org/uploads/2018/06/ramsey-lecture.pdf

Spin Echo/Photon Echo

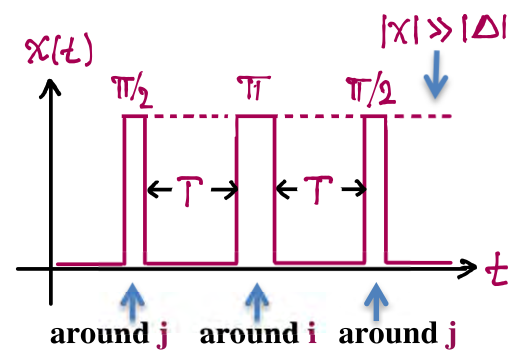
- (∗) **Spin Echo pulse sequences are related to the Ramsey "trick", but their goal is different.**
- (∗) **Typically, one seeks to suppress sensitivity to one or more uncontrolled parameters.**
- (∗) **Whole books have been written about the design of composite pulses for different purposes – google NMR Spin Choreography.**
- (∗) **We look at a classic example…**

Spin Echo in Doppler Broadened gas:

Velocity Classes ➡ different Δ



Spin Echo Sequence



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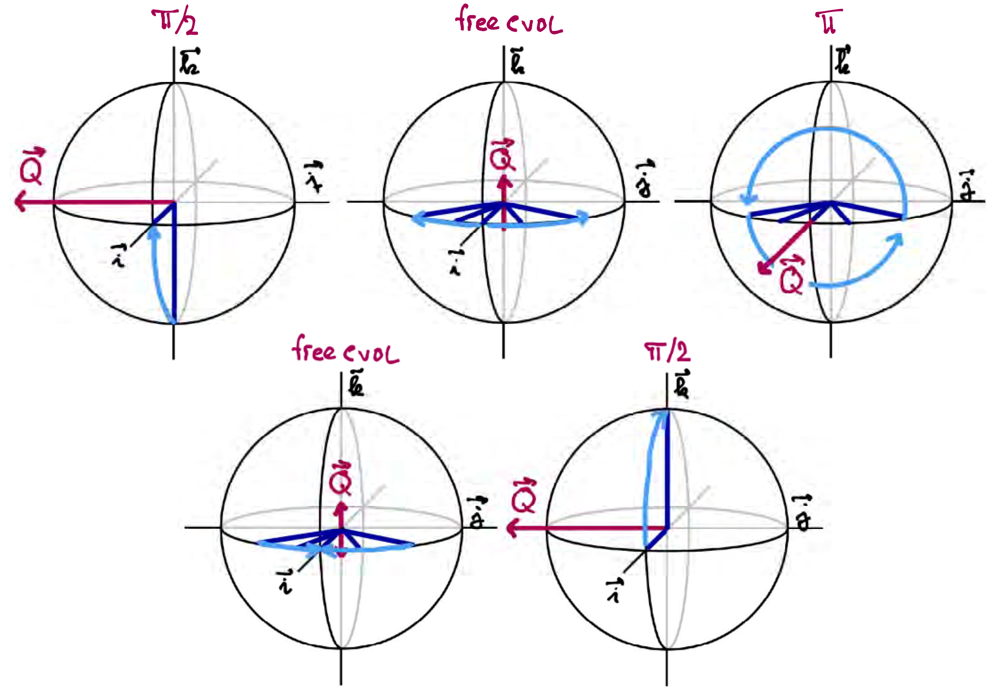

(∗) The term Spin Echo comes from NMR, where the expectation value of the transverse spin component decays due to dephasing of the u, v variables. The middle pulse causes them to rephase and restores the transverse spin.

(∗) How might this translate to a Photon Echo?

(∗) What else can we do with Composite pulses? – Some examples discussed in class…

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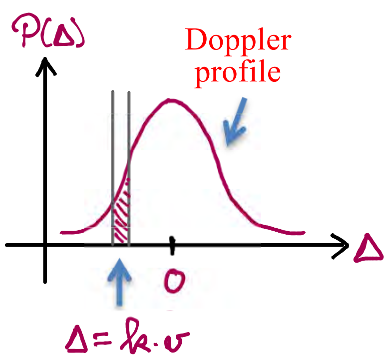
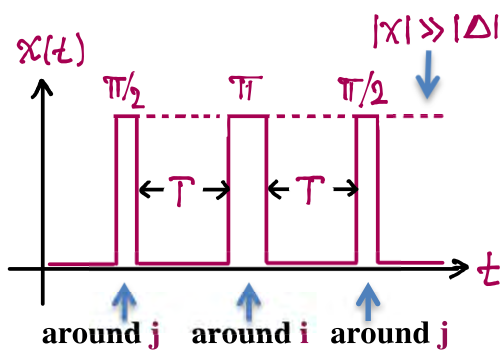


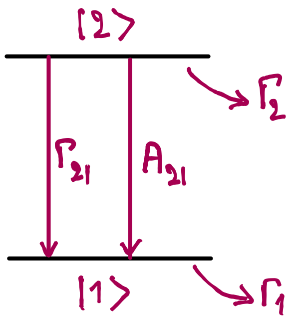
Spin Echo Sequence



Vector Model of the 2-Level Atom

Optical Bloch Equations including Relaxation

We pick χ real for simplicity, and carry over the relaxation terms from the Density Matrix Eqs.:



Convention from NMR/ESR

$$\frac{1}{T_1} = A_{21} + \Gamma_{21}$$ Longitudinal relaxation rate

$$\beta = \frac{1}{T_2} = \frac{1}{\tau} + \frac{1}{2T_1}$$ Transverse relaxation rate

$$\boxed{\begin{aligned} \dot{u} &= -\beta u - \Delta v \\ \dot{v} &= -\beta v + \Delta u + \chi w \\ \dot{w} &= -\frac{1}{T_1}(w+1) - \chi v \end{aligned}}$$

(∗) No closed-form solutions, numerics is easy.

(∗) Relaxation ⇨ $|\vec{S}|$ is not conserved. Without a driving field, $\vec{S}$ decays to south pole

Steady State Solutions to OBE's:

$$\boxed{\begin{aligned} u(\infty) &= \frac{\Delta\chi}{\Delta^2+\beta^2+\chi^2\beta T_1} \\ v(\infty) &= \frac{-\chi\beta}{\Delta^2+\beta^2+\chi^2\beta T_1} \\ w(\infty) &= 1+\frac{\chi^2\beta T_1}{\Delta^2+\beta^2+\chi^2\beta T_1} \end{aligned}}$$

Transient response:

(1) Steady State for given χ, Δ, turn off @ $t = 0$

⇨ analytic solution

OBE's

$$\boxed{\begin{aligned} \dot{u} &= -\beta u - \Delta v \\ \dot{v} &= -\beta v + \Delta u \\ \dot{w} &= -\frac{1}{T_1}(1+w) \end{aligned}}$$

⇨

Free Induction Decay

$$\boxed{\begin{aligned} u(t) &= [u_0 \cos\Delta t - v_0 \sin\Delta t]\, e^{-\beta t} \\ v(t) &= [v_0 \cos\Delta t + u_0 \sin\Delta t]\, e^{-\beta t} \\ w(t) &= -[1-(w_0-1)\, e^{-t/T_1}] \end{aligned}}$$

Vector Model of the 2-Level Atom

Steady State Solutions to OBE's:

$$\boxed{\begin{aligned} u(\infty) &= \frac{\Delta\chi}{\Delta^2+\beta^2+\chi^2\beta T_1} \\ v(\infty) &= \frac{-\chi\beta}{\Delta^2+\beta^2+\chi^2\beta T_1} \\ w(\infty) &= 1+\frac{\chi^2\beta T_1}{\Delta^2+\beta^2+\chi^2\beta T_1} \end{aligned}}$$

Transient response:

(1) Steady State for given χ, Δ, turn off @ $t = 0$

⇨ analytic solution

OBE's

$$\boxed{\begin{aligned} \dot{u} &= -\beta u - \Delta v \\ \dot{v} &= -\beta v + \Delta u \\ \dot{w} &= -\frac{1}{T_1}(1+w) \end{aligned}}$$

⇨

Free Induction Decay

$$\boxed{\begin{aligned} u(t) &= [u_0 \cos\Delta t - v_0 \sin\Delta t]e^{-\beta t} \\ v(t) &= [v_0 \cos\Delta t + u_0 \sin\Delta t]e^{-\beta t} \\ w(t) &= -[1-(w_0-1)e^{-t/T_1}] \end{aligned}}$$

Radiation by $\langle\hat{\vec{\mu}}\rangle$ (choose $\vec{\mu}_{21}$ real)

$$E_{Rad} \propto \mathrm{Tr}\,[\rho\hat{\vec{\mu}}] = \vec{\mu}_{21}(u\cos\omega t + v\sin\omega t)$$

⇨ Radiation at freqs. ω, $\omega \pm \Delta$

Decay at rate β

Transient response:

(2) System is in state $|1\rangle$ at $t<0$

Integrate numerically ⇨ Damped Rabi Osc. (Optical Nutation)

Vector Model of the 2-Level Atom

Radiation by $\langle \hat{\vec{\mu}} \rangle$ **(choose** $\vec{\mu}_{21}$ **real)**

$$E_{Rad} \propto \mathrm{Tr}(\rho \hat{\vec{\mu}}) = \vec{\mu}_{21}(u\cos\omega t + v\sin\omega t)$$

$\Rightarrow$ $\begin{cases} \text{Radiation at freqs. } \omega,\ \omega \pm \Delta \\ \text{Decay at rate } \beta \end{cases}$

Transient response:

(2) **System is in state** $|1\rangle$ **at** $t<0$

Integrate numerically $\Rightarrow$ **Damped Rabi Osc. (Optical Nutation)**

From Milloni & Eberly, p. 206

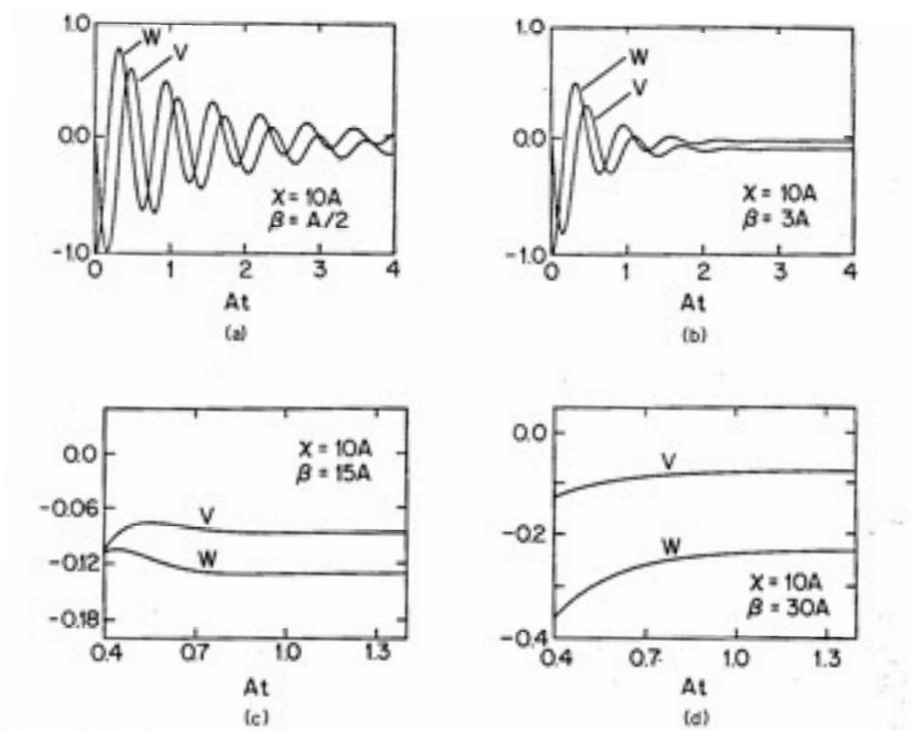


Figure 6.6 Numerical solutions of the v, w equations (6.5.21) for a range of collisional damping rates. Note scale changes.