OPTI 544 **2nd Midterm Exam, April 12, 2023** **Jessen**

I

(a) Draw a sketch of an Optical Bloch Sphere, including the $\vec{i}$, $\vec{j}$, $\vec{k}$ axes defining a right-handed coordinate system. Assuming the ground state $|1\rangle$ of the two-level system resides at the "south pole", indicate which states correspond to the points where the $\vec{i}$, $\vec{j}$, $\vec{k}$ axes pierce the surface of the Bloch Sphere. (5%)

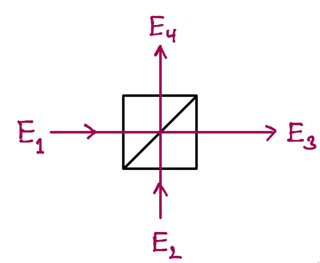
(b) Write out the Optical Bloch equations for a real and positive Rabi frequency χ , $\Delta=0$, and absent relaxation (no collisions, no spontaneous decay). (10%)

(c) Write out the Optical Bloch equations for $\chi=0$, $\Delta=0$, with no collisions but with spontaneous decay $|2\rangle \rightarrow |1\rangle$ at a rate A_{21}. (10%)

(d) Assuming the atom starts at the 'north pole", the equations in (b) and (c) will move the state of the atom along different paths with identical beginning and end points. Assuming a π -pulse in (b) and time to reach steady state in (c), sketch these trajectories on a second copy of the Bloch sphere. You may base your answer on physical intuition (explain in words), solve the OBE's above, or use a combination thereof. (10%)

II

Consider a loss-less 50/50 beam splitter with $|t|^2+|r|^2=1$ as shown in the figure on the right. The input is a 2-photon wavepacket in port 1 and vacuum in port 2.



(a) Write out an expression for the two-mode input in terms of creation and/or annihilation operators. (10%)

(b) Based on the expression in (a), write out an expression for the two-mode output in terms of creation and or annihilation operators. (15%)

(c) What will happen if we look for coincident counts as in the Hong-Ou-Mandel experiment? (10%)

III

We consider in the following a model of longitudinal sound waves in a cavity.

(a) Write out an expression for the quantum field $\eta(x,t)$ in terms of the generalized coordinates and normal modes. Then write out the standard Lagrangian for the field $\eta(x,t)$ in terms of the generalized coordinates q_k and generalized velocities $\dot{q}_k$. No derivation required, you can find the relevant expressions in the notes or in the Homework. Use this Lagrangian to derive a simpler equation of motion for the normal modes. (15%)

(b) Write out the Wave Equation for sound in this system. Then borrow an idea from a recent Homework Problem to show that substituting the expression for $\eta(x,t)$ into the wave equation leads to the exact same equations of motions for the normal modes as (a). What does this tell us about the Lagrangian? (15%)

Hint: In the wave equation, $Y/\mu = v^2$ is the speed of sound squared and $u''(x) = -(\omega^2/v^2)u(x)$.