OPTI 544: Homework Set #9
Posted April 21, Due May 1.

I

A Cs atom with transition wavelength $\lambda = 852nm$ and excited state decay rate $A_{21} = 3.28\times 10^7/s$ is trapped at an antinode of the field in a resonant optical cavity with length $l = 100\mu m$ and effective cross sectional area $A = \pi\,(2\mu m)^2$. We assume the field polarization is parallel to the electric dipole matrix element, and that the cavity mirrors are perfect, lossless reflectors.

(a) Calculate the vacuum Rabi frequency g and compare to A_{21}. Is the system in a regime that permits the observation of coherent vacuum Rabi oscillations?

(b) A weak probe beam is passed through the cavity. Sketch the transmission as the probe frequency ω is tuned around the atom/cavity resonance frequency ω_0. What phenomenon are you seeing?

II

In the following, consider a normal mode of the electromagnetic field in a cavity that contains *two* identical 2-level atoms. For simplicity we assume the cavity mode is resonant with the atomic transition frequency, $\omega = \omega_{21}$, and that the interaction strength g is the same for both atoms.

(a) Write down the single-mode, two-atom Jaynes-Cummings Hamiltonian in terms of creation and annihilation operators for the field, and atomic operators $\hat{\sigma}^{(i)}_{\pm,z}$, where the index $i = 1,2$ refers to the two atoms.

(b) The Hamiltonian from (a) has the form $\hat{H} = \hat{H}_0 + \hat{H}_{AF}$. Use the notation $|\,j,k,n\rangle$ for the eigenstates of $\hat{H}_0$, where j,k are the states of the first and second atom and n is the photon number, respectively. Find the eigenvalues λ and $\lambda_q, q = 0, \pm 1$ of $\hat{H}$ corresponding to coupled atom-cavity states that have zero or one quantum of excitation, respectively. Then draw a level diagram showing the dressed-state energies for the zero- and one-excitation parts of the two-atom Jaynes-Cummings ladder.

(c) Sketch the expected transmission of a weak probe beam through the coupled atom-cavity system as the probe frequency is tuned from well below to well above the cavity mode frequency ω. No formal calculation is required; rely instead on your physical intuition and your results in (b).

(d) Returning to (c), find the eigenstate associated with the eigenvalue λ_0 in the single-excitation manifold. Based on your result, how might you revise your sketch from (d)?