OPTI 544 **Final Exam, May 7, 2021** **Jessen**

Note: In the following you may make use of prior results from lectures and homework whenever possible and without rederiving them, provided that you acknowledge the source.

I

In a Homework Problem we studied a laser designed around a 4-level gain medium with lasing wavelength λ and atom density N, uniformly distributed throughout a cavity with length L and cross sectional area A. The mirror reflectivities were $R_1 = 1$ and $R_2 < 1$, and there were no other losses besides transmission through the output coupler. We found an expression for the threshold inversion ΔN_t and calculated that for the given parameters $\Delta N_t / N \sim 10^{-5}$.

Here we consider the behavior of the same laser for a different hierarchy of pumping and decay rates, i. e., $P >> \Gamma_{21}, \Gamma_{10}$, while Γ_{10} and Γ_{21} are of the same order.

(a) Write down an expression for the steady state inversion ΔN present in the gain medium in terms of $P, \Gamma_{21}, \Gamma_{10}$ and $\sigma\phi$, valid for any values of these rates. From this, find a simpler approximate expression for ΔN when $P >> \Gamma_{21}, \Gamma_{10}$. (5%)

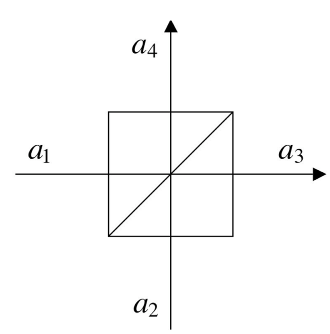
(b) Based on (a) and assuming $N / \Delta N_t >> 1$, find an expression for the intracavity photon flux ϕ when the laser is operated far above threshold. Given this, how do you expect the laser output power to scale with the pump rate P? (10%)

(c) Now assume you can control the decay rate Γ_{10}. Find an expression for the output power P_{out}. Then draw a sketch showing P_{out} as function of Γ_{10}, keeping everything else constant. (10%)

(d) In HeNe lasers the decay $|1\rangle \rightarrow |0\rangle$ occurs when atoms diffuse out of the lasing mode and strike the wall of the discharge tube. As result $\Gamma_{10} = \eta / A$, where η is some constant. Also, the upper lasing level is metastable, $\Gamma_{21} \sim 0$. In that case, how does P_{out} scale with A? In this situation, how might we go about increasing the output power? Hint: $g_t = (1 - R_2)/2L$ (15%)

II

Consider in the following a 4-port beamsplitter with transmission and reflection coefficient $t = 1/\sqrt{2}$ and $r = i/\sqrt{2}$.



(a) Let the input be $|\Psi_{in}\rangle = |\alpha_1\rangle|\alpha_2\rangle$, i. e. a product of coherent states in port 1 and 2. Write down an expression for the output state $|\Psi_{out}\rangle$, showing that it is a product of coherent states $|\alpha_3\rangle$ and $|\alpha_4\rangle$ in ports 3 and 4, and giving expressions for α_3, α_4 in terms of α_1, α_2. (5%)

(b) We measure a signal $\hat{S} = \hat{N}_4 - \hat{N}_3$ at the output. Write down expressions for $\langle \hat{S} \rangle$ in terms of α_3, α_4 as well as α_1, α_2. (5%)

A lengthy but straightforward calculation shows that $\Delta S^2 = |\alpha_1|^2 + |\alpha_2|^2 = |\alpha_3|^2 + |\alpha_4|^2$. This is (not surprisingly) the shot noise for a mean photon number $\bar{n} = |\alpha_1|^2 + |\alpha_2|^2 = |\alpha_3|^2 + |\alpha_4|^2$.

(c) Now let $\alpha_1 = \beta$, $\alpha_2 = \beta e^{i\varphi} \sim \beta(1+i\varphi)$, where φ is real-valued and $<<1$. Find expressions for $\langle \hat{S} \rangle$ and ΔS^2, keeping only terms to leading order in φ. (10%)

(d) Find the sensitivity with which the differential phase φ can be measured. By definition, the sensitivity is determined as the smallest value of a given parameter for which the resulting signal $|\hat{S}| \geq \Delta S$. (10%)

III

In this problem we explore a quantum version of the electron oscillator model, i. e. an atom where the internal motion of the electron relative to the nucleus is described by a one-dimensional quantum mechanical harmonic oscillator, rather than the classical harmonic oscillator used in the Lorentz model. This quantum electron oscillator has frequency ω_0, creation and annihilation operators $\hat{b}$ and $\hat{b}^+$, position $\hat{x} = x_0(\hat{b} + \hat{b}^+)$, and dipole $\hat{p} = e\hat{x}$, where e is the electron charge.

(a) Consider the dipole matrix elements $p_{q',q} = \langle q' | \hat{p} | q \rangle$ between electron oscillator states with q and q' quanta of excitation. What are the selection rules for electric-dipole transitions between the oscillator states, i. e. which $p_{q',q}$ are zero and which are non-zero? (10%)

We put the quantum electron oscillator inside an optical cavity, where it interacts with a single quantized mode of the electromagnetic field having frequency ω, photon creation and annihilation operators $\hat{a}$ and $\hat{a}^+$, and electric field $\hat{E} = E_0(\hat{a} + \hat{a}^+)$. For simplicity, we assume the electric field is parallel to the electron oscillator motion, and define the characteristic interaction strength $\hbar g = -e x_0 E_0$.

(b) Write down the Hamiltonian for the system, including the energy of the electromagnetic field, the energy of the electron oscillator, and the electric-dipole interaction. Compare to the Jaynes-Cummings Hamiltonian for a two-level atom coupled to a quantized mode of the electromagnetic field. (10%)

(c) Now assume $\omega = \omega_0$. Based on what you found in (b), what do you expect will happen if the system at $t = 0$ has one quantum of excitation in the electron oscillator, and zero quanta of excitation in the field? Sketch the populations of the states $|q_{osc} = 1\rangle |n_{field} = 0\rangle$ and $|q_{osc} = 0\rangle |n_{field} = 1\rangle$ as function of time. (10%)