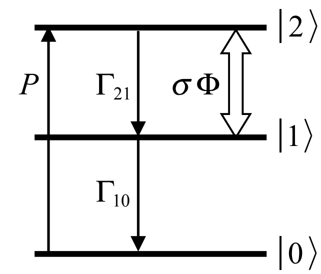
I

Consider in the following a laser with a 3-level pumping scheme as indicated on the figure to the left, with lasing occurring on the transition between levels $|1\rangle$ and $|2\rangle$.



(a) Write down a set of rate equations for the number densities N_0, N_1, and N_2 in terms of the pumping rate, the spontaneous decay rates, and the rates of absorption and stimulated emission. (5%)

(b) What is the steady state value of the intra-cavity photon flux Φ below threshold? (5%)

(c) Use your answers from (a) and (b) to determine the condition for achieving a steady state population inversion on the lasing transition. Then write down expressions for the small-signal gain, the saturated gain, and the saturation flux. There is no need to derive any expressions already in the Class Notes and/or Homework Problem Sets. (5%)

(d) Lasers with this type of level structure are rarely if ever encountered. Suggests one or more potential problems with it. (5%)

II

Consider in the following a Rubidium atom confined to a pair of Rydberg states that can be considered stable against decay on the time scales relevant here. That makes it effectively a two-level atom with a transition frequency of $\nu = 51\,\mathrm{GHz}$ and a dipole moment of $\vec{p}_{12} = 1.6\times10^{-29}\,\mathrm{C\,m}$. It is located at the antinode of a 1D standing wave inside a cylindrical cavity of length $\lambda/2$, designed so the mode polarization is parallel to $\vec{p}_{12}$ and the *effective* mode volume is λ^3. The cavity field frequency is tuned to be exactly resonant with the atomic transition.

(a) To what temperature must we cool the cavity in order to keep the mean photon number in the mode below 10^{-3}? Your final answer must be a temperature in degrees K. (10%)

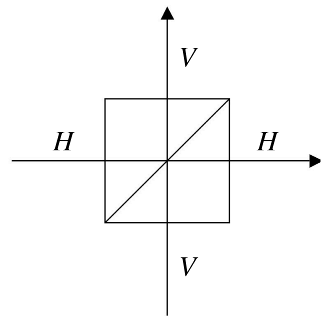
(b) Assume the atom is in the upper Rydberg state and the cavity mode in the vacuum state at $t = 0$. Calculate the frequency of oscillation between the Rydberg states. Your final answer must be a number. (15%)

(c) What is this phenomenon called? What might prevent us from observing it in a real experiment? Given the large dipole moment, why can we ignore spontaneous decay from the upper to the lower Rydberg level? Explain! (10%)

(continued below)

III

Consider in the following a symmetric beamsplitter with transmission and reflection coefficients t, r. As we have seen previously, we can think of the beamsplitter as applying a unitary transformation that maps the two-mode input state to a two-mode output state. We emphasize this by labeling the modes H and V. In the simplest case when the beamsplitter is absent, ($t=1, r=0$), the input V mode maps directly to the output V mode, and similar for the H mode.



(a) Assuming $t=1/\sqrt{2}$, $r=i/\sqrt{2}$, find the two-mode output states $|\Psi_{\rm out}^{(\pm)}\rangle$ for the following two-mode input states:

$$|\Psi_{\rm in}^{(+)}\rangle=\frac{1}{\sqrt{2}}(|1_H,0_V\rangle+i|0_H,1_V\rangle)\,,\qquad |\Psi_{\rm in}^{(-)}\rangle=\frac{1}{\sqrt{2}}(|1_H,0_V\rangle-i|0_H,1_V\rangle)\qquad (15\%)$$

Clearly, for a two-mode system and exactly one photon, we are dealing with a two-level system for which all states can be expressed in the basis $|1_H,0_V\rangle$, $|0_H,1_V\rangle$. That means all possible states of the two-modes-and-one-photon system can be visualized using the Bloch sphere.

(b) Make a sketch of the Bloch sphere showing the location of the input and output states from (a). From this, deduce the nature of the transformation implemented by the beamsplitter on the Bloch sphere. (15%)

(c) For the same t, r, consider the two-mode output state $\hat{\rho}_{\rm out}$ for the two mode input state

$$\hat{\rho}_{\rm in}=p|1_H,0_V\rangle\langle 1_H,0_V|+(1-p)|0_H,1_V\rangle\langle 0_H,1_V|$$

On your sketch from (b), indicate the location of the input and output states when $p=0$ and when $p=1/2$. (10%)

(d) Would the above representation of states on the Bloch sphere be possible if we had more than one photon present in the two-mode system? Explain! (5%)

Some numbers that might be useful above

$$c=2.998\times10^{8}\ m/s \qquad k_B=1.381\times10^{-23}\ \mathrm{J/K}$$
$$\mu_0=4\pi\times10^{-7}\ N/A^2 \qquad e=1.602\times10^{-19}\ C$$
$$\varepsilon_0=8.854\times10^{-12}\ F/m \qquad m_e=9.11\times10^{-31}\ kg$$
$$\hbar=1.055\times10^{-34}\ Js \qquad m_{Rb}=1.42\times10^{-25}\ kg$$