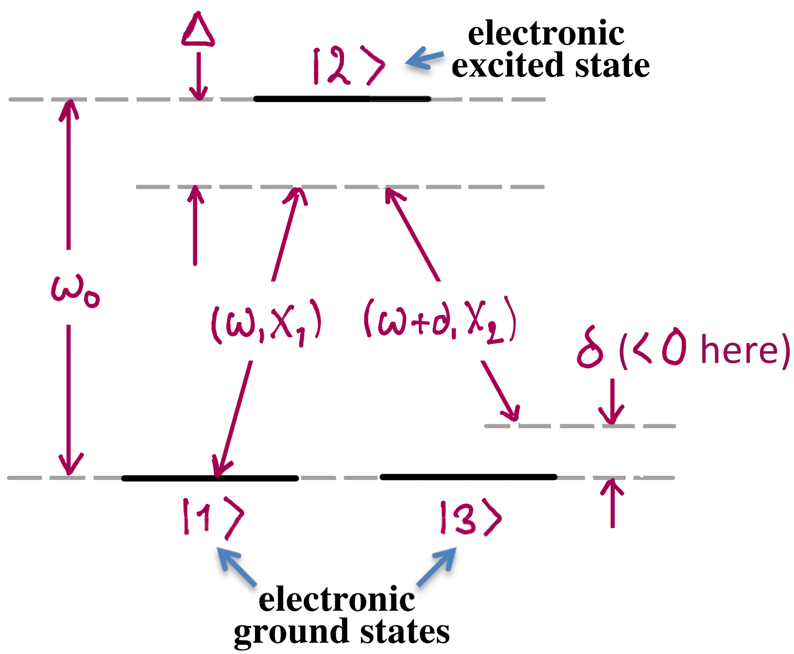
Raman Coupling in 3-level Atoms

Raman Coupling in 3-Level Systems

Consider an atom with this 3-level structure



For simplicity we set $E_1 = E_3$ **(no loss of generality)**

Fields $\begin{cases} \text{at } \omega\text{, coupling } |1\rangle, |2\rangle \text{ w/Rabi freq. } \chi_1 \\ \text{at } \omega+\delta\text{, coupling } |3\rangle, |2\rangle \text{ w/Rabi freq. } \chi_2 \end{cases}$

The Hamiltonian for this system is (χ_1, χ_2 **real)**

$$H = \hbar \begin{pmatrix} 0 & \chi_1(t) & 0 \\ \chi_1(t) & \omega_0 & \chi_2(t) \\ 0 & \chi_2(t) & 0 \end{pmatrix}$$

where

$$\chi_1(t) = \frac{\chi_1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$\chi_2(t) = \frac{\chi_2}{2}\left(e^{i(\omega+\delta)t} + e^{-i(\omega+\delta)t}\right)$$

Setting $|\psi(t)\rangle = a_1(t)|1\rangle + a_2(t)|2\rangle + a_3(t)|3\rangle$

we get a S.E.

$$\dot{a}_1 = -i\frac{\chi_1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)a_2$$

$$\dot{a}_2 = -i\omega_0 a_2 - i\frac{\chi_1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)a_1 - i\frac{\chi_2}{2}\left(e^{i(\omega+\delta)t} + e^{-i(\omega+\delta)t}\right)a_3$$

$$\dot{a}_3 = -i\frac{\chi_2}{2}\left(e^{i(\omega+\delta)t} + e^{-i(\omega+\delta)t}\right)a_2$$

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$$\dot{a}_3 = -i\frac{\chi_2}{2}\left(e^{i(\omega+\delta)t} + e^{-i(\omega+\delta)t}\right)a_2$$

Rotating Wave Approximation.

Let $a_1 = b_1, \quad a_2 = b_2 e^{-i\omega t}, \quad a_3 = b_3 e^{i\delta t}$

Plug into in S.E.

$$\dot{b}_1 = -i\frac{\chi_1}{2}\left(1 + e^{-i2\omega t}\right)b_2$$

$$\dot{b}_2 = -i(\omega_0 - \omega)b_2 - i\frac{\chi_1}{2}\left(e^{i2\omega t} + 1\right)b_1 - i\frac{\chi_2}{2}\left(e^{i2(\omega+\delta)t} + 1\right)b_3$$

$$\dot{b}_3 = -i\delta b_3 - i\frac{\chi_2}{2}\left(1 + e^{-i2(\omega+\delta)t}\right)b_2$$

Drop non-resonant terms, set $\omega_0 - \omega = \Delta$

$$\dot{b}_1 = -i\frac{\chi_1}{2}b_2$$

$$\dot{b}_2 = -i\Delta b_2 - i\frac{\chi_1}{2}b_1 - i\frac{\chi_2}{2}b_3$$

$$\dot{b}_3 = -i\delta b_3 - i\frac{\chi_2}{2}b_2$$

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Rotating Wave Approximation.

Let $a_1 = b_1, \quad a_2 = b_2 e^{-i\omega t}, \quad a_3 = b_3 e^{i\delta t}$

Plug into in S.E.

$$\boxed{\begin{aligned} \dot{b}_1 &= -i\frac{\chi_1}{2}\left(1+e^{-i2\omega t}\right)b_2 \\ \dot{b}_2 &= -i(\omega_0-\omega)b_2 - i\frac{\chi_1}{2}\left(e^{i2\omega t}+1\right)b_1 \\ &\quad -i\frac{\chi_2}{2}\left(e^{i2(\omega+\delta)t}+1\right)b_3 \\ \dot{b}_3 &= -i\delta b_3 - i\frac{\chi_2}{2}\left(1+e^{-i2(\omega+\delta)t}\right)b_2 \end{aligned}}$$

Drop non-resonant terms, set $\omega_0 - \omega = \Delta$

$$\boxed{\begin{aligned} \dot{b}_1 &= -i\frac{\chi_1}{2}b_2 \\ \dot{b}_2 &= -i\Delta b_2 - i\frac{\chi_1}{2}b_1 - i\frac{\chi_2}{2}b_3 \\ \dot{b}_3 &= -i\delta b_3 - i\frac{\chi_2}{2}b_2 \end{aligned}}$$

This S.E. has no explicit time dependence

Easy to solve numerically…

Now assume that $b_2(t=0) = 0$ ⇨ the atom is in the electronic ground state at $t=0$ when the fields turn on.

⇨ we can solve eq. for $b_2(t)$:

$$\dot{b}_2(t) = -i\Delta b_2 - ig(t), \quad g(t) = \left(\frac{\chi_1}{2}b_1 + \frac{\chi_2}{2}b_3\right)$$

$$b_2(t) = -e^{-i\Delta t}\int_0^t ie^{i\Delta t'}g(t')\,dt' \quad \leftarrow \text{(A)}$$

$$= -e^{-i\Delta t}\left(\left[\frac{1}{\Delta}e^{i\Delta t'}g(t')\right]_0^t - \underbrace{\int_0^t \frac{1}{\Delta}e^{i\Delta t'}\dot{g}(t')\,dt'}_{\text{(B)}}\right)$$

Reminder: Integration by parts

$$\int_a^b f(x)g(x)\,dx = \left[F(x)g(x)\right]_a^b - \int_a^b F(x)g'(x)\,dx$$

Raman Coupling in 3-level Atoms

We consider the relative magnitude of (A) & (B)

(1) Let Rabi freqs be of the same order, $\chi_1 \sim \chi_2 \sim \chi$

(2) b_1, b_3 are at most ~ 1 ⇒ $g(t)$ in (A) is $\sim \chi$

(3) In (B), the part $\frac{1}{\Delta}\dot{g}(t) = \frac{\chi}{\Delta}(\dot{b}_1 + \dot{b}_3)$

Where $\dot{b}_1, \dot{b}_3$ are $\sim \chi b_2$ and $b_2 \sim \frac{\chi}{\Delta}$

from Rabi solutions

⇒ we can solve eq. for $b_2(t)$:

$$\dot{b}_2(t) = -i\Delta b_2 - i g(t), \quad g(t) = \left(\frac{\chi_1}{2} b_1 + \frac{\chi_2}{2} b_3\right)$$

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from Rabi solutions

(4) Therefore $\frac{1}{\Delta}\dot{g}(t) \sim \frac{\chi^3}{\Delta^2}$ and $\frac{(B)}{(A)} = \frac{1}{\Delta}\frac{\dot{g}(t)}{g(t)} \sim \frac{\chi^2}{\Delta^2}$

⇒ We can ignore (B) when $\Delta^2 \gg \chi^2$

$$b_2(t) \approx -\frac{1}{\Delta} g(t) + \frac{1}{\Delta} e^{-i\Delta t} g(0)$$

$$= -\left[\frac{\chi_1}{2\Delta} b_1(t) + \frac{\chi_2}{2\Delta} b_3(t)\right]$$

$$+ e^{-i\Delta t}\left[\frac{\chi_1}{2\Delta} b_1(0) + \frac{\chi_2}{2\Delta} b_3(0)\right]$$

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(5) Finally, the last term $\propto \frac{e^{-i\Delta t}}{\Delta}$ can be ignored because it averages to zero on the timescale on which b_1, b_3 evolve.

Note:

The ground state amplitudes evolve slowly Because $\chi_1/\Delta, \chi_2/\Delta \ll 1$, while the excited state amplitude evolves fast and adiabatically follows the instantaneous values of b_1, b_3

Plug the solution for $b_2(t)$ into the eqs. for b_1, b_3

$$\boxed{\begin{aligned} \dot{b}_1(t) &= i\frac{\chi_1^2}{4\Delta}b_1(t) + i\frac{\chi_1\chi_2}{4\Delta}b_3(t) \\ \dot{b}_3(t) &= -i\left(\delta - \frac{\chi_2^2}{4\Delta}\right)b_3(t) + i\frac{\chi_1\chi_2}{4\Delta}b_1(t) \end{aligned}}$$

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We simplify by making a final change of variables

$$c_1(t) = b_1(t)e^{-i\frac{\chi_1^2}{4\Delta}t}, \quad c_3(t) = b_3(t)e^{-i\frac{\chi_1^2}{4\Delta}t}$$

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These are two-level equations!

Physical Discussion: We have an effective 2-level atom with effective Rabi Frequency and detuning.

$$\boxed{\chi_{eff} = \frac{\chi_1\chi_2}{2\Delta}, \quad \delta_{eff} = \delta + \frac{\chi_1^2-\chi_2^2}{4\Delta}}$$

Note that $\chi_{eff} \sim \chi^2/\Delta$ while the excited state population $P_2 \sim \chi^2/\Delta^2$. This means that for large χ, Δ we can have large χ_{eff} and no opportunity for spontaneous decay.

Coherent Rabi oscillations and long lived superposition states

Raman Coupling in 3-level Atoms

Begin 02-08-2023

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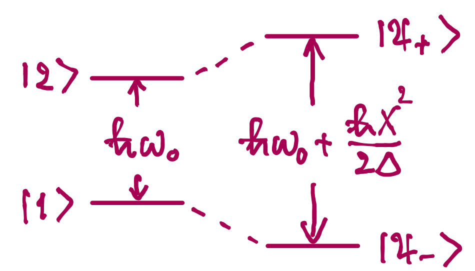
Coherent Rabi oscillations and long lived superposition states

End 02-06-2023

Note also: The effective Raman detuning is shifted.

HW Set 2: Dressed-states of a 2-level atom



$$E_+ = \hbar\omega_0 + \frac{\hbar\chi^2}{4\Delta}$$

$$E_- = -\frac{\hbar\chi^2}{4\Delta}$$

3-level system ➡ ground state shifts $\frac{\chi_1^2}{4\Delta}$, $\frac{\chi_2^2}{4\Delta}$

➡ Differential ground state shift $\frac{\chi_1^2-\chi_2^2}{4\Delta}$

Final note: The atomic dipole $\langle \hat{\vec{p}} \rangle$ will have components that match the frequency and polarization of both driving fields, with amplitudes that depend on both fields.

Non-Linear wave mixing,
Breakdown of superposition principle

Raman Coupling in 3-level Atoms

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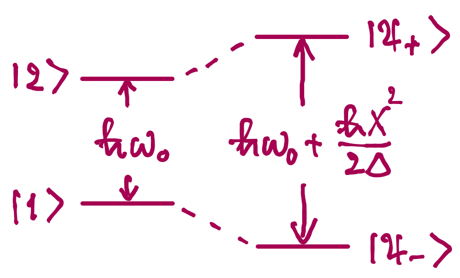

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Final note: The atomic dipole $\langle \vec{\mu} \rangle$ will have components that match the frequency and polarization of both driving fields, with amplitudes that depend on both fields.

Non-Linear wave mixing,
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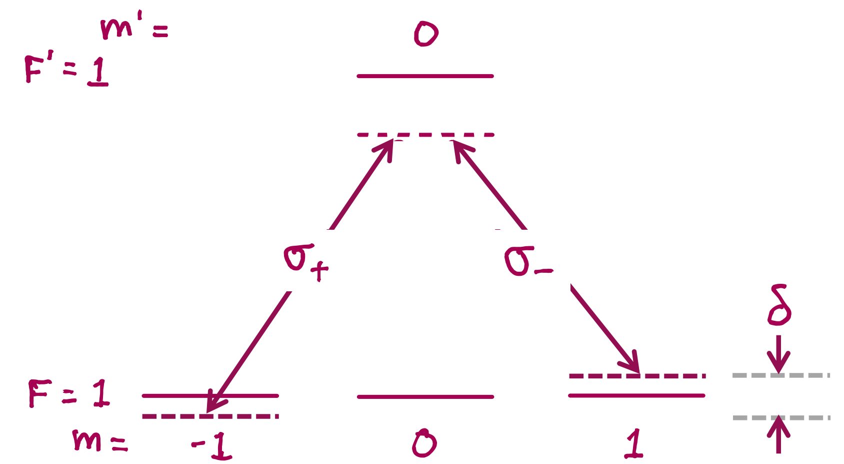
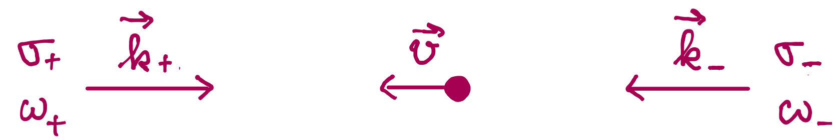
Example: Velocity dependent Raman Coupling





field freqs. in co-moving frame

$$\left.\begin{aligned} \omega_+ &= \omega + kv \\ \omega_- &= \omega - kv \end{aligned}\right\}$$

velocity dependent Raman detuning

➡ $\delta = 2kv$

Applications:

- Doppler velocimetry
- Raman Cooling by velocity selective momentum transfer
- What if we apply a $\pi/2$ Raman pulse?
- Atom Interferometry