

OPTI 544: Homework Set #6
Posted March 24, Due April 3.

I

The Lagrangian for a chain of masses m separated by distances a and connected by springs with spring constants κ can be expressed in terms of the particle positions and velocities as

$$\mathcal{L} = \frac{1}{2} \sum_i m \dot{x}_i^2 - \kappa (x_{i+1} - x_i)^2$$

Starting from this Lagrangian derive a wave equation for the displacement field $\eta(x)$ in the continuous limit, $a \rightarrow 0$.

II

Show that

(a) $T = \int dx \frac{1}{2} \mu \left(\frac{\eta(x,t)}{dt} \right)^2 = \sum_k \frac{1}{2} M \dot{q}_k^2$

(b) $V = \int dx \frac{1}{2} Y \left(\frac{\eta(x,t)}{dx} \right)^2 = \sum_k \frac{1}{2} M \omega^2 q_k^2$

(c) Finally, write down the Lagrangian, both in terms of the field, and in terms of the dynamical variables q_k and \dot{q}_k . Then show that your Lagrange equation of motion yields the standard 2nd order differential equations typical of a collection of harmonic oscillators.

Note: Problem I and II(b) are a bit fiddly. If you have trouble getting started or find yourself stuck before the end, check out my Solution Set on the course website under the Homework tab.