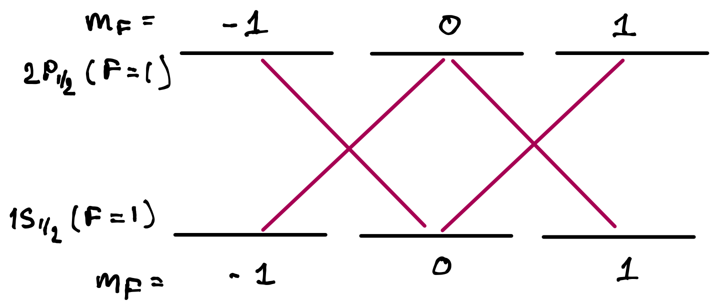
OPTI 544 1st Midterm Exam, March 4, 2022

Problem 1

(a) There are 3 magnetic sublevels, $m_F = 0, \pm 1$, in a state with total angular momentum $F = 1$. Thus, the level diagram looks like this:



(b) The driving field is $\vec{\varepsilon}_x$ polarized. But $\vec{\varepsilon}_x$ is an equal superposition of $\vec{\varepsilon}_+$ and $\vec{\varepsilon}_-$, so it drives all the available $\Delta m_F = \pm 1$ transitions, indicated by the red lines above.

Problem 2

(a) From the note set on the classical electron oscillator, aka the Lorentz atom, we find in the near-resonance, weakly polarizable limit, the following simplified expression for the imaginary index of refraction:

$$n_I(\omega_0) = \frac{Ne^2}{4\varepsilon_0 m\omega_0\beta}$$

From elsewhere in the same note set we have for the absorption coefficient

$$a(\omega_0) = 2n_I(\omega_0)\frac{\omega_0}{c} = \frac{Ne^2}{2\varepsilon_0 mc\beta}$$

(b) From the note set on Rate Equations we have for 2-level atom in the collision free and low saturation regimes that

$$a(\omega_{21}) = N\frac{3\lambda^2}{2\pi}$$

Problem 3

(a) A unique quantum state (pure or mixed) is described by a unique density operator (density matrix) ρ, where $\mathrm{Tr}[\rho]=1$.

(b) Let A be an observable with eigenvalues a and eigenvectors $|a\rangle$. Given a density operator ρ, the expectation value of this observable is $\langle A\rangle = \mathrm{Tr}[\rho A]$.

(c) The probability of getting the outcome a when measuring A is $\mathcal{P}(a) = \mathrm{Tr}[\rho|a\rangle\langle a|]$, where $|a\rangle\langle a|$ is the projector onto the eigenstate $|a\rangle$.

(d) When a measurement of the observable A yields the outcome a, the density operator immediately afterwards has the form $\rho = |a\rangle\langle a|$.

(e) The density operator evolves according to the Schrödinger Equation, $i\hbar\frac{d}{dt}\rho = [H,\rho]$.